COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	X	Ɛ	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	136	Five ft. nine in.	5.9'
31	694	Three ft. two in.	3.2'
96	3Ɛ2	Two ft. eight in.	2.8'
19Ɛ	1000	Eleven ft. seven in.	Ɛ.7'

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2Ɛ*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
12 ) 30  + 5
12 )  2  + 6
      0  + 2     Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *X*, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *X*.

Numerical Progression

1	One		
10	Do	.1	Edo
100	Gro	.01	Egro
1,000	Mo	.001	Emo
10,000	Do-mo	.000,1	Edo-mo
100,000	Gro-mo	.000,01	Egro-mo
1,000,000	Bi-mo	.000,001	Ebi-mo
1,000,000,000	Tri-mo	and so on.	

Multiplication Table

1	2	3	4	5	6	7	8	9	X	Ɛ
2	4	6	8	X	10	12	14	16	18	1X
3	6	9	10	13	16	19	20	23	26	29
4	8	10	14	18	20	24	28	30	34	38
5	X	13	18	21	26	2Ɛ	34	39	42	47
6	10	16	20	26	30	36	40	46	50	56
7	12	19	24	2Ɛ	36	41	48	53	5X	65
8	14	20	28	34	40	48	54	60	68	74
9	16	23	30	39	46	53	60	69	76	83
X	18	26	34	42	50	5X	68	76	84	92
Ɛ	1X	29	38	47	56	65	74	83	92	X1

E. M. Scifres

The Duodecimal Bulletin

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THE DUODECIMAL SOCIETY OF AMERICA
20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y.

THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve-base arithmetic. The lessons and examinations are free to those whose entrance applications are accepted. Remittance of $6, covering initiation fee ($3) and one year's dues ($3), must accompany applications.

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WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numbers universally used. *"Why even try to learn to use these heathenish scrawly symbols, with their stubborn propensity for error, in place of the beautiful clear numbers which our fathers have used for untold generations? Think of the needless waste. We would have to change all of our counting boards and abaci. X is X, isn't it? And why do we need a symbol for nothing? You can't count it! No! Let us keep to our simple tried and true numerals, and let the barbarians scratch their heads, and rub themselves out. It will all come to 0 anyhow."*

Yet, although it took D years, the new notation became generally used, and man's thinking leapt forward like an arrow sped from a bow. The early years of the Renaissance marked a new stage in the use of symbols, with the advent of algebra, fractionals (decimals?), logarithms, analytical geometry, and the calculus. Can you imagine what it would be like to try to express the coordinates of points on a curve in Roman notation?

Mathematicians became conscious of a new dimension in symbolism, and the fundamental concepts of numbers were re-examined. Man awoke to the fact that different number bases could be used, and Simon Stevin stated in 1585 that the duodecimal base was to be preferred to the decimal.

The new Arabic notation accomodated mathematical statement better, and facilitated ideation. All thinking accelerated when released from the drag of the cumbrous Roman notation.

The parallel seems tenable. The notation of the dozen base accomodates mathematical statement better, and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base embodies a concurrent analysis and definition of numbers that stimulates classification and generalization. Yet this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis-factory because it has "not-enough-factors."

Then shouldn't we change? No! No change should be made, and we urge no change. All the world uses decimals. But people of understanding should learn to use duodecimals to facilitate their thinking, and to ease the valuative processes of their minds. Duodecimals should be man's second mathematical language. They should be taught in all the schools. In any operation, that base should be used which is most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity. But no change should be made. Perhaps by the year 2000, or maybe by *1200*, which is *14* years later, duodecimals may be the more popular base. But then no change need be made, because people will already be using the better base.

When one is familiar with duodecimals, a number of accessory advantages become apparent. Percentage is a very useful tool, but many percentages come out in awkward figures because of the inflexibility of decimals. When based on the gross, twice as many ratios come out in even figures, and among them are some of those most used, as thirds, sixths, and twelfths, - eighths and sixteenths. There are advantages associated with time and the calendar. Monthly interest rates or charges are derived from annual rates, or the reverse, by simply moving the unit (decimal?) point. The price of a single item bears the same relation to the price of the dozen, and so does the inch to the foot.

The proper correlation of weights and measures has always been one of the world's serious problems. None of the present systems is completely satisfactory. The American and English standards are convenient to use since they are the final result of a long process of practical evolution in which many inconvenient measures have been adjusted or abandoned. The French decimal metric measures have the advantage of being set upon the same base as the number system, and are well systemized. But many of the units are awkward because of their arbitrary sizes, and because their decimal scale does not accomodate division into thirds and fourths readily.

The duodecimal system of weights and measures, based on the inch and yard, the pint and the pound, has the desirable elements of both systems, and few of their faults. This Do-Metric System retains the familiar units of the American and British standards in approximately their present size, and arranges them into an ordered metric system using the scale of twelve. This fits perfectly into the duodecimal notation, and the combination accomodates the inclusion of the units of time and of angular measure within the system, which hitherto has not been possible.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have thought staid and established, and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is cordially invited.

The Duodecimal Bulletin

All figures in italics are duodecimal.

GEORGE BERNARD SHAW APPROVES

George Bernard Shaw, now in his ninety-fourth year (even duodecimally he is past *79*) is still producing plays and accepting iconoclastic ideas. He has in the past commented favorably on the duodecimal system, and copies of our BULLETIN have been sent him, including the recent issue in which Mr. Velizar Godjevatz explained his new musical notation. A British friend of Mr. Godjevatz also saw to it that Shaw received a personal copy of his privately published booklet, *The New Musical Notation.* As a result, its author and the Society have a remarkable G.B.S. letter, wholeheartedly approving both the Godjevatz musical notation and the duodecimal arithmetic, which, says Mr. Shaw, "is a coming reform."

The letter follows:

From
Bernard Shaw

Ayot Saint Lawrence, Welwyn Herts — 21st October 1949
Station: Welwyn Garden City, 5 Miles

Dear Sirs:

I am much obliged to you for sending me Mr. Godjevatz's book entitled The New Musical Notation. I have read it through and followed its argument attentively. I know most of the new notations from that of Rousseau to the latest, as their inventors sent them to me because I was a prominent professional critic of music 60 years ago. I am greatly taken by Mr. G's plan. It is enormously more readable, writable, logical, graphic, and labor saving than any I can remember. Its adoption would save a world of trouble.

Wagner in his last days was still complaining that he could get nobody to transpose his clarinet parts for him. Reading music is said to be like riding: unless it is learnt in childhood it is never learnt at all.

Mr. G's plan would teach people to count duodecimally with two new digits: eight nine deck ell ten; and this by itself would recommend it, as duodecimal arithmetic is a coming reform.

I am no longer a reviewer; but if my valuation of the plan will help to call attention to it, you may quote this letter as much as you please.

(Signed) G. Bernard Shaw

THE ROUNDING OFF OF UNCIALS

by William Shaw Crosby

When a decimal is rounded off, the present practice is simply to discard the superfluous figures if they amount to less than a half unit of the last decimal place to be retained, but if they amount to more than a half unit, then also to increase the last retained figure by one unit.

When the discarded figure or figures happens to be a 5 or a 5 followed only by zeros, the error of rounding upward is the same as that of rounding downward, and at first sight it appears indifferent which practice should be followed. But to prevent a systematic tendency for sums of rounded numbers to be too high or too low, a rule for such cases arranges that numbers discarding 5 are rounded upward and downward with roughly equal frequency. The rule is that if the discarded figure is exactly 5, make the last retained figure even: e.g., 0.75 is rounded up to 0.8, but 0.25 is rounded down to 0.2.

The purpose of making this change in the rule is to reduce the error made in certain cases when a number is rounded off twice or more in succession. In the last example, if *0.556* were to be rounded up to *0.56*, and this were again to be rounded upward (for example, by someone who had no access to the original three-place uncial), the result would be *0.6* - actually not so close to the original value as the one-place figure, *0.5*, obtained by twice applying the rule in the form I have recommended.

A slight modification of this practice is proposed in adapting it to the rounding off of uncials. As heretofore, leave the last retained figure unaltered if the discarded figures "spell" less than a half unit, and increase it by a unit if they spell more. But if the discarded figure or figures is a *6* of a *6* followed only by zeros, make the last retained figure odd: e.g., *0.666* should be rounded up to *0.67*, but *0.556* should be rounded down to *0.55*.

In any number system in which the case occurs (i.e., in any number system whose base is even), when the discarded figures spell exactly a half unit, the last retained figure should be made even or odd according as one-half the base of the system is odd or even, respectively.

DUODENAL ARITHMETIC AND METROLOGY

by John W. Nystrom, C.E.

Note: Mr. Nystrom published this paper as an appendix to his treatise on the Elements of Mechanics, printed in 1875. The following pages are reproductions of that appendix, with some material ommitted. Mr. Nystrom uses italic figures to represent decimals, which reverses our common practice.

Charles XII. of Sweden proposed to introduce a duodenal system of arithmetic and metrology. The king complained of ten as a base, and said, "It can be divided only once by 2, and then stops." The number *12* can be divided by 2, 3, 4 and 6 without leaving fractions; and divided by 8 gives 2/3, by 9 gives 3/4, and by 10 gives 5/6, all convenient fractions for calculation.

The number *12* has always been a favorite base in metrology.

The old French foot was divided into *12* inches, the inch into *12* lines, and the line into *12* points. The *dozen* is a well-known base adopted all over the world; *12 dozens* is a *gross*, and *12 gross* is a *great-gross*. We have *12* months in a year, *12* hours in a day, *12* signs in the zodiac, *12* musical notes in an octave. The old Roman metrology was based on *12*, like the English foot and the Troy pound.

A writer in the Edinburgh Review (Jan., *1807*, vol. *9*, page *376*) regrets that the philosophers of France, when engaged in making so radical a change in the measures and standards of the nation, did not attempt a reform in the popular *arithmetic*. He, being in favor of a duodenal system, says, "The property of the number *12* which recommends it so strongly for the purpose we are now considering is its divisibility into so many more aliquot parts than ten, or any other number that is not much greater than itself. Twelve is divisible by *2, 3, 4* and *6;* and this circumstance fits it so well for the purpose of arithmetical computation that it has been resorted to in all times as the most convenient number into which any unit either of weight or measure could be divided. The divisions of the Roman *as*, the *libra*, the *jugerum*, and the modern foot, are all proofs of what is here asserted; and this advantage, which was perceived in rude and early times, would have been found of great value in the most improved eras of mathematical science. . . . We regret therefore that the experiment of this new arithmetic was not attempted. Another opportunity of trying it is not likely to occur soon.

"In the ordinary course of human affairs such improvements are not thought of, and the moment may never again present itself when the wisdom of a nation shall come up to the level of this species of reform."

If man had been created with six fingers on each hand, we would have had in arithmetic a duodenal instead of the present cumbrous decimal system.

A uniform duodenal system of metrology, even with decimal arithmetic, would be much better in the shop and market than the French metrical system.

A duodenal system would be equally applicable in all branches of metrology, and it would include those which are excluded by the metrical system - namely, astronomy, geography, navigation, time and the circle.

The duodenal system would require two new characters to represent *10* and *11*, so as to place *10* at *12*. This change in the figures would appear strange at the first glance, but a little reflection, with due consideration, would soon lead to the satisfaction that these two new figures simplify the arithmetic and render it much easier for mental calculation than decimal arithmetic.

The base in the duodenal system is *12*, instead of *10* in the decimal system.

The Arabic system of notation is composed of ten simple digits, or characters - namely, *0, 1, 2, 3, 4, 5, 6, 7, 8, 9*, and the base *10*.

These same characters can be used in the duodenal system by adding two numbers to complete the base - namely, *11* and *12*; then all the units of weights and measures should be divided and multiplied by *12*, but in order to render the system simple for calculation, it will be necessary to substitute new characters for the numbers *10, 11* and *12* - namely,

Decimal system, *1,2,3,4,5,6,7,8,9,10,11,12;*
Duodenary system, 1,2,3,4,5,6,7,8,9,φ,ɤ,10,

in which 10 denotes the base *12*, φ stands for *10*, and ɤ stands for *11*.

The Italic figures mean decimal numbers, and the Roman figures mean duodenal numbers.

In order to distinguish the two systems from one another, it will be necessary to give new names to the duodenary figures.

A duodenary system of arithmetic cannot be adopted by only one nation, but the whole civilized world ought to agree upon such a scheme. Different nations have different languages and names for the decimal figures and numbers; but in the adoption of a

duodenary system of arithmetic, one common nomenclature might be agreed upon.

The new figures and nomenclature appear to be the greatest objection to the introduction of the duodenal system of arithmetic and metrology.

There is no difficulty in convincing the public of the utility of the duodenal system, and with that impression, a pride will be taken in using the new nomenclature, which could be taught in every school; and each individual would attempt to follow up the time of education.

The following table contains the names of the figures and numbers up to twelve in different languages. (See page *6*.)

METROLOGY The utility of a duodenary system of arithmetic consists in its combination with a similar system of metrology-namely, that all units of measure should be divided and multiplied by the same base, twelve.

Units of measure are required for the following fifteen quantities.

Length	Weight	Heat	Force	Power
Surface	Mass	Light	Velocity	Space
Volume	Money	Electricity	Time	Work

MEASUREMENT OF LENGTH Assume the mean circumference of the earth to be the primary unit of length, and divide it by *twelve* repeatedly until the divisions are reduced to a length which would be a convenient unit to handle in the shop and in the market.

The mean circumference of the earth is about *24851.64* miles, which, multiplied by *5280*, will be

		Duodenal
0	*131216659.2* feet............	1 circum.
1	*10934721.6* feet............	1 hour.
2	*911226.8* feet............	1 grad.
3	*75935.56* feet...........	1 minute.
4	*6327.96* feet...........	1 mile.
5	*527.33* feet...........	1 cable.
6	*43.944* feet..........	1 chain.
7	*3.772* feet..........	1 metre.
The required unit of length	*43.944* inches........	1 metre.

The duodenal metre to be divided into twelve equal parts of *3.772* inches each, and called metons. The meton into twelve equal parts of *0.31433* of an inch each, called mesans. The mesan into twelve equal parts of *0.0262* of an inch each, called metos.

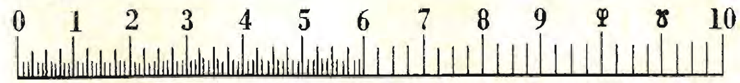
Nomenclature of Numbers in Different Languages.

No.	English.	French.	German.	Swedish.	Spanish.	Latin.	Greek.
0	Naught.	Zéro.	Null.	Noll.	Cero.	Nihil.	Zero.
1	One.	Un.	Eins.	En.	Uno.	Unus.	Eis, En.
2	Two.	Deux.	Zwei.	Två.	Dos.	Duo.	Duo.
3	Three.	Trois.	Drei.	Tre.	Tres.	Tres.	Treis.
4	Four.	Quatre.	Vier.	Fyra.	Cuatro.	Quatuor.	Tessares.
5	Five.	Cinq.	Funf.	Fem.	Cinco.	Quinque.	Pente.
6	Six.	Six.	Sechs.	Sex.	Sois.	Sex.	Hex.
7	Seven.	Sept.	Siben.	Sju.	Siete.	Septem.	Hepta.
8	Eight.	Huit.	Acht.	Åtta.	Ocho.	Octo.	Okto.
9	Nine.	Neuf.	Neun.	Nio.	Nueve.	Novem.	Ennea.
10	Ten.	Dix.	Zehn.	Tio.	Diez.	Decem.	Deka.
11	Eleven.	Onze.	Elf.	Elva.	Once.	Undecem.	Endeka.
12	Twelve.	Douze.	Zwölf.	Tolf.	Doce.	Duodecem.	Dodeka.

No.	Russian.	Finnish.	Welsh.	Gaelic.	Gothic.	Turkish.	Hebrew.
0	Nill.	Nolli.				Zuffer.	Ayin.
1	Odna.	Yksi.	Una.	Unah.	Ains.	Bier.	Aleph.
2	Dva.	Kaksi.	Dou.	Gha.	Tvai.	Icki.	Beth.
3	Tri.	Koleme.	Tri.	Trec.	Threis.	Utch.	Gimel.
4	Tcheterl.	Nelja.	Pedvar.	Cheir.	Fidvor.	Duert.	Daleth.
5	Piat.	Viisi.	Pump.	Coag.	Finf.	Bach.	He.
6	Shest.	Kuusi.	Chewech.	Seach.	Saihs.	Altoe.	Vau.
7	Sem.	Seitsemän.	Saith.	Sheach.	Sibun.	Yedi.	Zain.
8	Vosem.	Kahdeksän.	Wyth.	Oacht.	Ahtan.	Seckiz.	Bheth.
9	Deviat.	Yhdeksän.	Nan.	Nuegh.	Niun.	Dokus.	Teth.
10	Desiat.	Kymmenän.	Deg.	Doach.	Taihun.	On.	Yod.
11	Odenatset.	Yksitoista.	Undeg.	Undech.	Ainstaihun.	Onbier.	Yodaleph.
12	Dvenatset.	Kaksitoista.	Doudeg.	Ghadech.	Tvaitaihun.	Onicki.	Yodbeth.

No.	Arabian.	Persian.	Hindoo.	Chinese.	Japanese.	Sanscrit.	Duodenal
0	Siforon.			Bow.	Ley.		Zero, 0
1	Ahed.	Yika.	Ek Ache.	Yat.	Itchi.	Aika.	An, 1
2	Ishnan.	Du.	Do.	Ge.	Ni.	Dwau.	Do, 2
3	Saylaset.	Seh.	Ien.	Sam.	San.	Tri.	Tre, 3
4	Erbayet.	Chebaur.	Châr.	Tze.	Tchi.	Chatur.	For, 4
5	Jemset.	Pendj.	Pánoh.	Ngnu.	Go.	Pancha.	Pat, 5
6	Sittet.	Shesh.	Chha.	Luck.	Lock.	Shash.	Sex, 6
7	Saybet.	Helft.	Sáth.	Tchut.	Sytchi.	Saptan.	Ben, 7
8	Saymaniet.	Hesht.	Ath.	Pbat.	Hatchi.	Ashta.	Ott, 8
9	Tiset.	Nuh.	Nau.	Geo.	Ku.	Navan.	Nev, 9
10	Eshret.	Deh.	Das.	Shop.	Dgiu.	Dashan.	Dis, ♀
11	Ahedeshere.	Yikadeh.	Gyárah.	Shopyat.	Dgiuitchi.	Aikadashan.	Elv, ♉
12	Ishnaneshere.	Dudeh.	Bárah.	Shopgaa.	Dgiuni.	Dwandashan.	Ton, 10

The following figure shows the full size of a meton with its divisions.



The first 6 mesans are divided into metos, and the last into quarters of mesans. The ordinary shop-metre need not be divided finer than into quarters of mesans, for in so small divisions the metos can easily be approximated.

The metons and mesans would be the most convenient for expressing short measures in the mechanic arts.

DIVISION OF THE CIRCLE - The circle to be divided into 100 equal parts (*144* decimal).

Duodenal System		*Old System*
One circle	= 100 grads.	*360* degrees.
One grad	= 100 lents.	*2* degrees *30* minutes.
One lent	= 100 ponts.	*1* minute *2.5* seconds.
One pont	=	*0.43418* of a second.
One quadrant	= 30 grads.	*90* degrees.

One duodenal mile on the earth's surface corresponds with an angle of one lent.

One duodenal chain on the earth's surface corresponds with an angle of one pont.

The latitude and longitude to be divided as the circle.

The angular measures correspond with the linear measures on the earth's surface. The terms minute and second are omitted in the division of the circle, so as not to confound angles with time.

The circle can thus be divided into *2*, *3*, *4*, *6*, *8*, *9*, *12* or *16* parts, without leaving fractions of a degree or grad.

The quadrant of the circle, containing 30 grads (*36*), can be divided into 2, *3*, *4*, *6*, *9* or *12* parts without leaving fractions of a grad. These advantages with the duodenal division of the circle are of great importance in geometry, geography, trigonometry, astronomy and in navigation.

Either of the divisions corresponds with an even linear measure on the earth's surface.

DUODENAL DIVISION OF TIME - The division of time should conform to that of the circle.

The time from noon to noon, including one night and day, to be divided into twelve equal parts, called hours.

	Duodenal System	*Old System*
1.	One day = 10 hours.	*24* hours.
	One hour = 10 grads.	2 hours.
	One grad = 10 minutes.	*10* minutes.
	One minute = 10 lents.	*0.83333* of a minute.
	One lent = 10 seconds	*4.1666* seconds.
	One second = 10 ponts.	*0.3472* of a second.
2.	One day = 10 hours.	
	One hour = 100 minutes.	
	One minute = 100 seconds.	
3.	One day = 100 grads	
	One grad = 100 lents.	
	One lent = 100 ponts.	

Either of these three divisions can be used in practice. The first division includes the second and third.

If the duodenal division of time was introduced all over the world, some nations would probably use the second expression, and others the third, but the third division is the best, because the hands on the watch would show the number of grads.

In the notation of time, say 3 hours and 46 minutes, will appear 3.46 hours, or 34.6 grads, or 346 minutes.

5 hours, 36 minutes and 15 seconds will appear 5.36.15 hours, or 53.61.5 grads.

The conversion of angle into time, or time into angle, is only to move the point one place.

There is no necessity of *A.M.* and *P.M.* in the duodenal time.

Astronomers would surely use the third expression of time, which corresponds with the divisions of the circle.

<u>DUODENAL CLOCK-DIAL</u> - The following figure represents a duodenal clock-dial.

The hour-hand makes one turn in one night and day, the minute-hand goes round once per hour, and the second-hand once per minute.

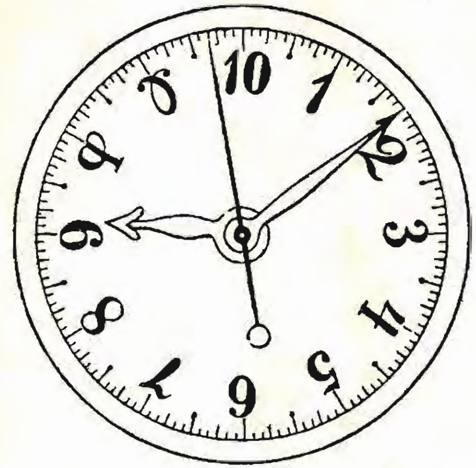
The hour-hand will point to 10 at noon, to 3 at *6* o'clock in the evening, to 6 at midnight, and to 9 at *6* o'clock in the morning.

The length of the pendulum vibrating duodenal seconds will be

$$l = 39.1 \times 0.3472^2 = 4.711 \text{ inches, or}$$
$$= 1.3 \text{ metons.}$$

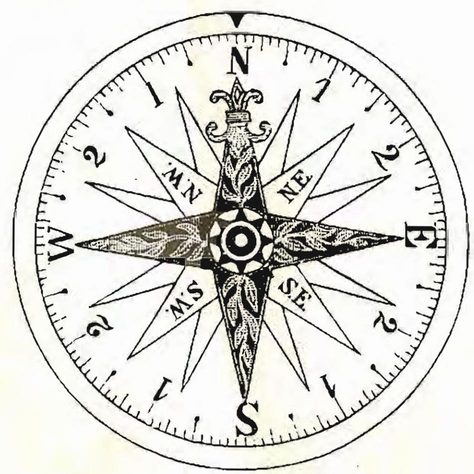
The duodenal metre will vibrate

$$n = \frac{6.254 \times 60}{\sqrt{43.944}} = 56.6 \text{ times per old minute.}$$
$$= 41.55 \text{ times per duodenal minute.}$$



<u>DUODENAL COMPASS</u> - The compass to be divided into grads like the circle, but numbered from North and South toward East and West, making 30 grads in each quadrant. The following figure represents a duodenal compass.

The hours 1 and 2, corresponding each with 10 grads, are marked on the dial in each quadrant.



The nomenclature will be nearly the same as for the old compass, only the expression of fractional points would be changed to grads; for example, South South-East, one-half South, would be called simply South ott East.

Our present compass is divided into *32* points, and each point into four quarters, making *32* divisions in each quadrant, which shows the natural tendency toward binary divisions; but it is accompanied with a clumsy nomenclature. A course of 3¼ points from North toward East is termed North-East by North, one-quarter East. The duodenal expression would be simply North an tre Est, meaning one hour and three grads from North toward East, without expression of fractions; and the course is given with greater precision than by the present nomenclature.

DUODENAL MEASUREMENT OF SURFACE - Small surfaces can be expressed in square metres, square metons or square mesans.

Duodenal System		Old System
One square chain	= 1 lot.	
10 chains square	= 1 acre.	
One square cable	= 1 acre.	*6.3925* acres.
One acre	= 100 lots.	*278075* square feet.
One lot	= 100 square metres.	*1931.1* square feet.
One square mile	= 100 acres.	*920.52* acres.
One square grad	= 10,000 square miles.	
One square grad	= 1,000,000 acres.	

DUODENAL MEASURE OF CAPACITY - The cubic metre to be the unit for capacity.

Duodenal System		Old System
One cubic metre	= 1 tun.	*49.113* cubic feet.
One tun	= 10 barrels.	*49.113* cubic feet.
One barrel	= 10 pecks.	*4.0927* cubic feet.
One peck	= 10 gallons.	*653.92* cubic inches.
One gallon	= 10 glasses.	*53.66* cubic inches.
One glass	= 10 spoons.	*4.47* cubic inches.

The duodenal gallon is one cubic meton, or about one quart. An ordinary quart bottle would contain one duodenal gallon.

Dry and wet measures of capacity should be measured by the same units. A cord of wood 10 cubic metres.

The volume of solids should be measured by the cube of the linear units.

DUODENAL MEASURE OF WEIGHT - The weight of one cubic metre of distilled water is assumed to be the unit of weight, and called one *ton*.

The duodenal ton will weigh about *3063.8* pounds, or *1.368* old tons.

Duodenal System		Old System Avoirdupois
One ton	= 10 pud.	*3063.8* pounds avoirdupois.
One pud	= 10 vegts.	*255.3166* pounds avoirdupois.
One vegt	= 10 ponds.	*21.276* pounds avoirdupois.
One pond	= 10 ounces.	*1.773* pounds avoirdupois.
One ounce	= 10 drachms.	*2.3640* ounces avoirdupois.
One drachm	= 10 scruples.	*0.1969* ounces avoirdupois.
One scruple	= 10 grains.	*0.0164* ounces avoirdupois.
One grain	=	*0.598* grains Troy.

UNITS OF FORCE - Force can be measured by either one of the units of weight. The pond would be the most convenient unit in estimating power and work in machinery.

UNIT OF VELOCITY - Metons per second would be the most appropriate expression of velocity in machinery. A velocity of metons per second is the same as miles per hour.

UNIT OF TIME - The second is the best unit of time to be used in the operation of machinery and falling bodies.

UNIT OF POWER - A force of one pond moving with a velocity of one meton per second to be one unit of power, and called Effect. A power of one pond moving with a velocity of one meton per second would be = *1.605* foot-pounds per old second. This will make 30 duodenal effects per man-power, and 300 effects per horse-power.

UNIT OF SPACE - The unit of linear space in the operation of machinery should be the meton or metre.

UNIT OF WORK - The work of lifting one ton through a height of one metre is a proper unit for estimating heavy work; it is equal to *11375* foot-pounds. This unit should be termed metreton and be used in the estimate of work of heavy ordnance.

The work which a laborer can accomplish per day would be about 100 metretons, which unit ought to be called a Workmanday.

The unit of work corresponding to velocity and effects should be one pond lifted one meton, which is *0.5567* of a foot-pound.

UNIT OF MASS - The duodenal unit of mass would be the amount of matter in one cubic meton of distilled water, to be called one Matt, which is *53.668* cubic inches of water.

UNIT OF GRAVITY - The velocity which a falling body would attain at the end of the first duodenal second is g = 2.Ɣ33 metres per second, which would be the acceleratrix of gravity.

UNIT OF TEMPERATURE - The thermometer scale should be divided into 100 duodenal parts *(144)* between the freezing and boiling points of distilled water at the level of the sea in latitude 16 grads (45°).

One duodenal grad = *1.25°* Fahrenheit scale.

One duodenal grad = *0.69°* Centigrade.

UNIT OF HEAT - The heat required to raise the temperature of one pond of distilled water from φ° to Ɣ° to be one unit of heat, which answers to *1713* foot-pounds of work.

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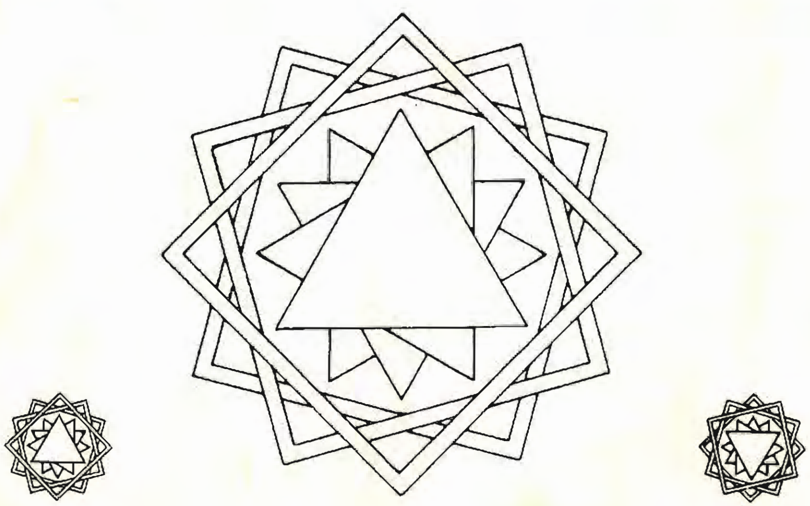
A DUODECIMAL INSIGNE
by H.K. Humphrey

On several occasions, it has occurred to me to explore the possibilities of developing a duodecimal symbol or emblem that might better express the divisibility of the dozen by three (into squares,) and by four (into triangles,) than by any emphasis on the numerical symbols. None of the results has been entirely satisfactory, but one of the better designs is illustrated below.



NUMERALS OF POWERS and POWERS OF NUMERALS
by George S. Terry

A. If we add the numerals in a given power of any number N on any base thereby getting N, and continue the process on N, and so on, we arrive at a recurring sequence. For example:- on base twelve with squares

If N = *13* we have $13^2 = 169$ (with *1 + 6 + 9 = 14*), $14^2 = 194$, $12^2 = 144$, $9^2 = 69$, 13^2 = etc. the sequence being *(9.13.14.12)*.

B. If we add a given power of the numerals in any number N on any base thereby getting N and continue the process on N and so on, we arrive at a recurring sequence. For example on base twelve with squares

If N = *18* we have $1^2 + 8^2 = 55$. $5^2 + 5^2 = 42$. $4^2 + 2^2 = 18$ the sequence being *(18.55.42)*.

I give the sequences for squares and cubes on various even bases.

A.	Squares.	Base twelve	*(1)* *(Ɛ)* *(9.13.14.12)*
		Base ten	(1) (9) (13.16)
		Base eight	(1) (7) (2.4)
		Base six	(1) (5) (14)
	Cubes.	Base twelve	*(1)(19)(1𝒳)(1Ɛ) (28)(29)(2𝒳)* *(8.15.16.11) (13.18.21.14)*
		Base ten	(1)(8)(17)(18) (26)(27) (19.28)
		Base eight	(1)(6)(15)(16) (17.26)
		Base six	(1)(13)(15)(23) (11.12)
B.	Squares.	Base twelve	*(1)(25)(𝒳5)(5.21) (68.84) (18.55.42)* *(8.54.35.2𝒳.88.𝒳8.118.56.51.52)*
		Base ten	(1) (4.16.37.58.89.145.42.20)
		Base eight	(1)(24)(64) (4.20) (15.32) (5.31.12)
		Base six	(1) (5.41.25.45.105.42.32.21)
	Cubes	Base twelve	*(1) (577) (668) (𝒳83)(11𝒳𝒳)* *(8.368.52Ɛ.𝒳20.700.247.2𝒳7.947.7𝒳8.10𝒳7.940.561.246.200)*
		Base ten	(1)(153)(370)(371)(407) (136.244) (919.1459) (55.250.133) (160.217.352)
		Base eight	(1)(134)(205)(463)(660)(661) (662.670.1057.725.734)
		Base six	(1)(243)(514)(1055) (13.44.332.142.201)

NOTES AND COMMENT

by H. C. Robert, Jr.

Mr. Terry's Case A can be simplified if after we sum the digits of $(N)^n$ we sum the digits of the sum, etc., until we obtain a single digit sum. Thus $13^2 = 169$ $1 + 6 + 9 = 14$ $1 + 4 = 5$. The single digit sum is always the remainder when $(N)^n$ is divided by Ɛ. When $(N)^n$ is divisible by Ɛ, the remainder may be called either 0 or Ɛ.

The same is true for any base, B, if we use (B-1) instead of Ɛ. Thus for squares - bases XII to VII - our sequences are:

N.	XII	XI	X	IX	VIII	VII
1	1	1	1	1	1	1
2	4,5,3,9	6	4,7	8	2,4	4
3	4,5,3,9	1	9	1	2,4	3
4	4,5,3,9	6	4,7	8	2,4	4
5	4,5,3,9	5	4,7	1	2,4	1
6	4,5,3,9	6	9	8	1	0(6)
7	4,5,3,9	1	4,7	1	0(7)	1
8	4,5,3,9	6	1	0(8)	1	
9	4,5,3,9	1	0(9)	1		
𝒳	1	0(𝒳)	1			
Ɛ	0(Ɛ)	1				
10	1					

Base XII				
Terry's Sequence	9	13	14	12
		-Ɛ	-Ɛ	-Ɛ
Robert's Sequence	9	4	5	3

For cubes:

N	XII	XI	X	IX
1	1	1	1	1
2	2,8,6,7	2,8	8	8
3	3,5,4,9	3,7	9	3
4	3,5,4,9	4	1	8
5	3,5,4,9	5	8	5
6	2,8,6,7	6	9	8
7	2,8,6,7	3,7	1	7
8	2,8,6,7	2,8	8	0(8)
9	3,5,4,9	9	0(9)	1
𝒳	𝒳	0(𝒳)	1	
Ɛ	0(Ɛ)	1		
10	1			

Base XII				
Terry's Sequence	8	15	16	11
		-Ɛ	-Ɛ	-Ɛ
Robert's Sequence	8	6	7	2

Note 1. For base XII the two sequences can be added:

2867
9354
ƐƐƐƐ

Note 2. The *9354* sequence is just the reverse of the *4539* sequence for squares.

Note 3. For squares - when N = (B - 2) then our sequence stops at 1, and for N = (B - 1) our sequence stops at (B - 1).

For cubes - when N = (B - 1) the sequence stops at (B - 1) and for N = (B - 2) the sequence stops at (B - 2). There may be other relationships of interest but no very clear pattern.

We might expect for Base X that we would have a cycle of 4 to correspond to the cycle of 5 for Base XII; that is, a cycle of $\frac{B-2}{2}$. But this is not correct; for Base X the cycle appears to be 3.

For Base XII the double cycle, that is, *2(5)* = *𝒳* gives a repetition of the sequences. Thus the sequences of N^2 = sequences of N^{10}.

And for base X we have sequences N^2 = sequences N^8
$N^3 = N^9$, etc.

Thus the exponents for Case A show the unusual characteristic of cycling *5* and *𝒳*, which are factors of Base X, for Base XII, and then for Base X, the exponent cycles at 3 and 6 which are factors of Base XII.

I don't understand it. If enough other bases are investigated we may find an answer. For Base XI there are cycles of 2 and 4. (Base Eight has same cycles as Base Ten - No help here.)

Now all of this is rather to be expected from a completely ordered system such as our number system of integers. Actually we need not confine ourselves to operations such as raising N to the nth power. Any function of N such as;

$$7N_o^3 - 3N_0^2 + 2N_o - 6 = abc \text{ with } a + b + c = N,$$

will give similar results. The summing of the digits is a consistent mathematical operation and therefore we expect our ordered system to remain ordered under any methematical operations. The generality of this does not necessarily make it

For various powers of N on Base XII we have the following:

N	η=2	η=3	η=4	η=5	η=6	η=7	η=8	η=9	η=X	η=Ɛ	η=10
1	1	1	1	1	1	1	1	1	1	1	1
2	4,5,3,9	2,8,6,7	5,9	X	9	2,7,6,8	3,5,4,9	2,6	1	2	4,5,3,9
3	4,5,3,9	3,5,4,9	3,4	1	3	3,9,4,5	3,5,4,9	3,4	1	3	4,5,3,9
4	4,5,3,9	3,5,4,9	3,4	1	4	3,9,4,5	3,5,4,9	3,4	1	4	4,5,3,9
5	4,5,3,9	3,5,4,9	5,9	1	5	3,9,4,5	3,5,4,9	5,9	1	5	4,5,3,9
6	4,5,3,9	2,8,6,7	5,9	X	5	2,7,6,8	3,5,4,9	2,6	1	6	4,5,3,9
7	4,5,3,9	2,8,6,7	3,4	X	4	2,7,6,8	3,5,4,9	7,8	1	7	4,5,3,9
8	4,5,3,9	2,8,6,7	3,4	X	3	2,7,6,8	3,5,4,9	7,8	1	8	4,5,3,9
9	4,5,3,9	3,5,4,9	5,9	1	9	3,9,4,5	3,5,4,9	5,9	1	9	4,5,3,9
X	1	X	1	X	1	X	1	X	1	X	1
Ɛ	0(Ɛ)	0(Ɛ)	0(Ɛ)	0(Ɛ)	0(Ɛ)	0(Ɛ)	0(Ɛ)	0(Ɛ)	0(Ɛ)	0(Ɛ)	0(Ɛ)
10	1	1	1	1	1	1	1	1	1	1	1

Note similarity of sequences for η=*2,7,10*, that is η=5κ+2, and for η=*3,8*, that is η=5κ+3. The sequences for η=5κ+*3* are reversed from those for η=5κ+2. Also η=5κ+*1* and 5κ+*4* and 5κ show that the form 5κ+ρ gives the pattern, – but why? Particularly, – why *(5)!??*

trivial although some mathematicians so consider it. I consider it rather that the problem is so general that its complexity makes it discouraging to investigate completely.

Now what happens if, instead of a normal mathematical operation we introduce an arbitrary operation? Such as –

Case C: If N_1^2 has only one or two digits $N_2 = N_1^2$, but if N_1^2 has more than two digits, $N_2 = az$ where a is the first and z the last digit of N_1^2. That is:- Base XII

$N_1 = 2 \quad N_1^2 = 4$
$N_2 = 4 \quad N_2^2 = 14$
$N_3 = 14 \quad N_3^2 = 194$ and $N_4 = 14 = N_3$

Or $N_1 = 3 \quad N_1^2 = 9$
$N_2 = 9 \quad N_2^2 = 69$
$N_3 = 69 \quad N_3^2 = 3969$
$N_4 = 39 \quad N_4^2 = 1209$
$N_5 = 19 \quad N_5^2 = 309$
$N_6 = 39$ so our cycle is *(39,19)*

or $N_1 = 29 \quad N_1^2 = 769$
$N_2 = 79 \quad N_2^2 = 5009$
$N_3 = 59 \quad N_3^2 = 2909$
$N_4 = 29$ so our cycle is *(29,79,59)*

So for this screwy operation on Base XII we have:-

For N = 6K ± *1* our operation either arrives at $N_p = 11 \quad N_p^2 = 121 \quad N_{p+1} = 11 = N_p$ or we arrive at the sequence *(31,91,61)*

For N = 6K ± 2 we get *(14)* or *(54,24)*

N = 6K ± *3* we get *(39,19)* or *(29,79,59)*

N = 6K we get *(10)* or *(30,90,60)*

The same operation for Base Ten gives:-

For N = 2K we get (10) or (26,66,46)

N = 5K we get (25,65,45)

other odd N we get (11) or (81,31,91)

While this arbitrary dropping of one or two digits is not a consistent mathematical operation we still have an ordered system in that numbers of certain types consistently produce the same repeating sequence.

But now let's take the opposite case:-

Case D: If N_1^2 has an odd number of digits N_2 = the middle digit of N_1^2 and if N_1^2 has an even number of digits N_2 = the two central digits. Now for

Base XII we have no sequences of more than one number. There are four end results *(0) (1) (40) (90)* with most numbers reducing to *(0)* - I can find no pattern. But for

Base X we have

$2^2 = 4 \quad 4^2 = 16 \quad 16^2 = 256 \quad 5^2 = 25 \quad 25^2 = 625$

or a cycle of (2,4,16,5,25)

$3^2 = 9 \quad 9^2 = 81 \quad 81^2 = 6561 \quad 56^2 = 3136$

$13^2 = 169 \quad 6^2 = 36 \quad 36^2 = 1296 \quad 29^2 = 841$

(2,4,16,5,25)

the same cycle - but

$7^2 = 49 \quad 49^2 = 2401 \quad 40^2 = 1600 \quad 60^2 = 3600 \quad 60^2 = 3600$

(60)

There may be other values for Base X. The pattern for Case D is beyond me.

(Other comment will be welcomed. Ed.)

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CORRECTION: Please correct the following proof errors in the last issue (Vol. 5 - No. 2:)

PAGE	*READS*	*SHOULD READ*
26	$-\frac{N}{12}$	$+\frac{N}{12}$
29	*1.0005755*	1.00057778
	1200th	1200th
	1.059461	1.059461
	.702ℰ	*.702𝒳*
	.8ℰ16	*.8𝒳16*
	.93𝒳8	*.93ℰ8*
30	*.002ℰ*	*.002𝒳*

THE ANNUAL MEETING

The sixth Annual Meeting of the Society was held at the Gramercy Park Hotel in New York City on Wednesday, January 18, 1950.

After President Andrews had extended a warm welcome to the members present, Secretary Beard reviewed the events of the year, which included a gain of five new members, the third reprinting of Mr. Andrews' "An Excursion In Numbers," and publication of our offer to supply free sets of introductory duodecimal material to mathematics pupils of teachers colleges.

The publicity high-lights of the year were the comment on the proposal of a duodecimal perpetual calendar of Louis Paul d'Autremont, and on the new musical notation of Velizar Godjevatz, which included the highly prized letter from George Bernard Shaw.

The budget for 1949 approximated $1750. The total receipts of the year, although they included donations of $1150 from 20 of our devoted members, fell somewhat short of covering our expenses, so that we enter 1950 with a smaller balance than for 1949.

The Nominating Committee recommended that the Directors of the Class of 1950 be renominated as the Class of 1953, and proposed that the Nominating Committee for 1951 consist of Frederick S. Condit, Chairman, Arthur Le Pori, and John E. Whiteside. The nominations were approved, and President Andrews announced the election of the following officers of the society for the ensuing year.

Chairman of the Board,	F. Emerson Andrews
President	H. C. Robert, Jr.
Vice Presidents	Paul E. Friedemann
	Dr. Nathan Lazar
Treasurer	H. K. Humphrey
Secretary	Ralph H. Beard
Editor of Bulletin	George S. Terry

The formal business of the meeting was thus concluded, and Dr. Lazar made the talk of the evening, "From the Abacus to the Adding Machine." Dr. Lazar displayed abaci which illustrated the gradual development from the primitive ten-button type to those in use today, demonstrating their use and operation, and showed that certain of these easily accomodated duodecimal computations. The discussion which followed was largely concerned with the device found in Central and South America, thought to be an abacus of special form, and with the possible characteristics of an abacus designed in the best form for use with the 60-base. Kingsland Camp reviewed the year's progress in the development of the duodecimal slide rule, and Mr. Andrews introduced President Robert to the meeting. Mr. Robert's committee appointments for the year will be published as soon as they become available.

The meeting was formally closed, at 2230 P.M., but everyone remained to continue the enjoyable refreshments of food and duodecimal conversations.

THE MAIL BAG

Repercussions to duodecimal ideas continue to repercuss. The September issue of Mechanical Engineering carried an excellent review of Louis Paul D'Autremont's proposal of a duodecimal perpetual calendar. George Bernard Shaw - as mentioned elsewhere in this issue - has highly commended Velizar Godjevatz' duodecimal musical notation.

William Shaw Crosby, while praising the work of Godjevatz and Trenchard More on musical notation, takes certain exceptions as to the temperament of musical scales. We hope that his reaction takes the form of a nice fat paper on musical theory from the duodecimal approach.

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Paul and Cam Adams have been doing some numerical sketching of the kinetic units for the Do-Metric System of weights and measures. Dallas Lien has independently been similarly preoccupied. This is no mean problem, - the selection of the best systemic expression for the acceleration by gravity, force, work, energy, and power. But it is terrifically absorbing. You pick up a pencil and some quadrille ruled paper, - and before you know it, a month has slipped by. Presently we will be able to exhibit some of the fruits of these creative labors.

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The department of vital duodecimal statistics will please record another son. The happy and proud parents are Leon and Yvette L'Heureux of Quebec. This is their third son, and we gather that he is somewhat lively for a statistic.

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We have had a gratifying response to our offer of free introductory sets of duodecimal literature to the mathematics students of teachers colleges. There could be no more promising burgeoning than this interest of the teachers of tomorrow's pupils in duodecimals. We would like to find a similar effective approach to the budding engineers, and physicists.

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The number of our members slowly grows. It seems to be characteristic of the field of our interest that we grow not so much by making converts, but by discovering those who have already been intrigued by duodecimals and drawing them into our circle. For this reason it is important that each one of us reports the name and address of every interested person for entry on our lists.

Ye Ed.

SEVERAL SHORT CUTS

by Paul and Camilla Adams

Most of the mathematical short cuts are applicable under any number base if put into proper relative form. A few of them that are duodecimally interesting will be outlined.

1. To square a number ending in 6.

   Let a = the digit in the dozens column.
   Then, $10a + 6$ = the number.
   And, $100a^2 + 100a + 30$ = its square = $100a(a+1) + 30$

   Hence, if we multiply the digit in the dozens place by the next higher number, and annex 30 to the product, we will have the square of the number.

   Thus $86^2 = 8 \times 9$, and 30, $= 6030$

   Another form of the same short cut applies to a number ending in 1/2. To square a number ending in 1/2, multiply by the next higher number and add 1/4.

   Hence, $(8\ 1/2)^2 = 8 \times 9$, and $1/4$, $= 60\ 1/4$

   This can be written in the same form as the preceding, and becomes:

   $8.6^2 = 8 \times 9$, and $1/4$, $= 60.30$

2. To square a number near 60, find its excess or defect, apply this to 30, and attach the square of the increment.

   Thus $67^2 = (60 + 7)^2 = (30 + 7 = 37)$ and $(7^2 = 41) = 3741$
   $57^2 = (60 - 5)^2 = (30 - 5 = 27)$ and $(5^2 = 21) = 2721$

   Because, $(60 \pm a)^2 = 3000 \pm 100a + a^2$
   $= 100(30 \pm a) + a^2$

3. To multiply numbers near 100 by supplement or complement.

   Example: Multiply $10\mathcal{X} \times 105$

   The supplements are $\mathcal{X}$ and 5. Add either supplement to the other number and take their sum as the first part of the answer. Thus, $105 + \mathcal{X} = 113$. Then annex the product of the supplements as the last two figures of the answer. $\mathcal{X} \times 5 = 42$ Answer: 11342

Example: Multiply ℰ7 × ℰ1

The complements are 5 and ℰ. Deduct either complement from the other number for the first part of the answer. Thus, ℰ7 - ℰ = 𝒳 . The product of the complements gives the last two figures of the answer.

5 × ℰ = 47 Answer: 𝒳847

4. Checking by Digit Sums.

It has long been known that the same operations performed on the sum of the digits as on the numbers themselves will provide the same result, thus providing a check on accuracy. This has been called the Rule of Nines for decimals. Since it is true for any base, it is more properly the Rule of Digit Sums. The symbol ≐ will mean "the sum of the digits equals" in the following examples.

Addition.	35𝒳4	≐	1𝒳	≐	ℰ		
	69ℰ1	≐	23	≐	5		
	78𝒳3	≐	24	≐	6		
	4917	≐	19	≐	𝒳		
	1𝒳993		82		28		
	≐		≐		≐		
	28	≐	𝒳	≐	𝒳		
Subtraction.	4𝒳89	≐	27	≐	9		
	-4137	≐	13	≐	4		
	952	≐	14	≐	5		
Multiplication	6𝒳7	≐	1ℰ	≐	10	≐	1
	× 1ℰ4	≐	14	≐			5
	2364						
	6385						
	6𝒳7						
	1146ℰ4	≐	23	≐			5

Division. Consider the dividend as being the product of the quotient and the divisor. Proceed as in multiplication.

Errors of inversion will escape detection by this check. For this very reason, when a bookkeeper has a "difference" in his decimal figures, he first divides it by 9 to check for inversion. For duodecimal figures, the division would be by ℰ.

ADDITIONS TO DUODECIMAL BIBLIOGRAPHY

by Lewis Carl Seelbach

Berckenkamp, Ioannem Albertum
Leges Numerandi Universales quibus Numeratio Decadica Leibnizi Dyadica nec non Reliqua Numerationis Genera Partim Plura, Partim Pauciora Numerorum Signa Continentia Aptantur.
Lemgoviae. 1747.
Demonstration of Bases 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 24, and 30. Uses literal symbols above Base 15.

Conant, Levi Leonard
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"Mr. G's plan would teach people to count duodecimally with two new digits: eight nine deck ell ten; and this by itself would recommend it, as duodecimal arithmetic is a coming reform."

UNITS OF FINE MEASUREMENT

by Dallas H. Lien

What should be the criterion for units of fine measurements? Practical experience in the realm of optics and spectroscopy has eliminated all but the best unit for that purpose. The micron is the nearest unit comparable in size to the length of light waves, but it is seldom used in such work. The same would hold true of the cad.

For most purposes four figures are used to express a wavelength of a spectroscopic line, and the angstrom unit accomodates this quite well, in that no decimal point is required when stating any visible line by four figures. Where such work is done, there is often much oral discussion of the various lines, and it would be awkward to state them as a fraction. If, however, we use the third basic subdivision (10^{-9} yds.), the visible spectrum falls between *11Ɛ8* units and *23Ɛ5* units and permits easy discussion of the spectrum. As has been shown of measurements in general, four-place accuracy in duodecimals is considerably greater than four-place accuracy in decimals.

The second basic subdivision (10^{-6} yds.) would be practical when working with infra-red rays and ultra high frequency radio waves, although with less accuracy required in common discussions of U.H.F. radio wavelengths, the first basic subdivision (karl) might become more common, as the millimeter and centimeter have done already.

Going the other way from the visible spectrum, the third basic subdivision unit falls in about the middle of the X-ray spectrum, and would not be practical for use with X rays. This has also been proved by long use of the X unit which is slightly shorter than the shortest gamma rays. Cosmic rays are of more interest now than ever before, so we can go a step further and select a name for practical use with such rays.

It is a problem to decide just how to name the units. The metric system has used several systems. The micron is short for micro-meter. The angstrom was named in honor of A.J.Angstrom, who was famous for his atlas of the solar spectrum, and the X unit was adopted because it's a practical unit for work with X rays. The following table might suggest a name for each subdivision:

Basic subdivn.	*Yds.*	*Radiation nearest in wavelength*	*Names proposed*
0	*1*	VHF radio	yard
1	10^{-3}	UHF radio	karl
2	10^{-6}	light	cad or ultrau
3	10^{-9}	medium X rays	rentrau
4	10^{-10}	hard gamma rays	gamrau
5	10^{-13}	hard cosmic rays	cosrau

The word Ultrau is derived from ultraviolet ray unit, Rentrau from Roentgen ray unit, Gamrau from gamma ray unit, and Cosrau from cosmic ray unit.

We have come to a point where some sort of groundwork in units must be laid before we can go on with the tables of conversion factors on any great scale, and these names are suggested as a step in that direction.

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THE MAD MATHEMATICIAN

by Harry C. Robert, Jr.

On leaving the asylum, the mad mathematician, (hereafter designated MM) was advised by his psychiatrist to eschew odd friends, odd pursuits and odd thoughts. Conscientiously and logically the MM took this advice to include odd numbers. And although he had been taught that zero was "even", if anything, he had always had a guilty feeling about putting down something for nothing so for the good of his soul the MM gave up zeros too, at least on fast days.

As might be expected of any MM, our friend then went in search of infinity, taking a number, squaring it, squaring the result, and continuing this process which should lead eventually to infinity. Of course he would have nothing to do with zeros or odd numbers so these were thrown away as fast as they appeared at each step of the operation. This is what he found, using base twelve:-

$2^2 = 4$ $4^2 = (1)4$ $4^2 = (1)4$ 4 is the end of the line.

$6^2 = (30)$ we only have nothing left.

$8^2 = (5)4$ $4^2 = (1)4$ we have been here before.

$\mathcal{X}^2 = 84$ $84^2 = (595)4$ and 4 comes up again.

The next number our MM can investigate is 22.

$22^2 = 484$ $484^2 = (1)\mathcal{X}(055)4$ $\mathcal{X}4^2 = 8\mathcal{X}(9)4$

$8\mathcal{X}4^2 = 6662(9)4$ $66624^2 = (3)6(9)8(9)64(55)4$

$68644^2 = (3905119)6(9)4$ $64^2 = (3)4(1)4$ $44^2 = (1)6(9)4$

so that we now cycle *64,44, 64,44, etc.*

And so checking all two digit numbers, where both digits are even, the MM found that *6, 26, 66, 86* all disappear.

22,42,44,4𝒳,64,88,𝒳2,𝒳4 and *𝒳𝒳* all lead to the sequence *64 — 44.*

And all others reduce to *4.*

Then he tried base ten:-

8,26,42,46,62,64,66 reduce to a three member sequence 46 - 26 - 66.

All others reduce to 6.

Again the largest number the MM found occurred when he operated on:-

$22^2 = 484$ $484^2 = 2(3)42(5)6$ $2426^2 = (5)88(5)4(7)6$

$8846^2 = (7)82(5171)6$ $826^2 = 6822(7)6$

$68226^2 = 46(5)4(7)8(707)6$ $46486^2 = 2(1)6(09)48(19)6$

$26486^2 = (70150)8(19)6$ $86^2 = (739)6$ $6^2 = (3)6$

The sixth operation on 22 for bases ten and twelve gives the largest number our MM could use so he decided that for

Base Ten	∞ =	68226
Base Twelve	∞ =	*68644*

which are nearly proportional to the fourth powers of the Base.

Investigating Base Eight, life becomes more complicated including a six member sequence 44-242-624-4662-24-62.

Since I came across the MM's notes I have been impressed by the limited world of numbers we would have if it were limited to even numbers.