COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	Χ	Ɛ	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	*136*	Five ft. nine in.	*5.9'*
31	*694*	Three ft. two in.	*3.2'*
96	*3Ɛ2*	Two ft. eight in.	*2.8'*
19Ɛ	*1000*	Eleven ft. seven in.	*Ɛ.7'*

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2Ɛ*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
12 ) 30  + 5
12 ) 2   + 6
     0   + 2      Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *Χ*, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *Χ*.

Numerical Progression

1	One		
10	Do	*.1*	Edo
100	Gro	*.01*	Egro
1,000	Mo	*.001*	Emo
10,000	Do-mo	*.000,1*	Edo-mo
100,000	Gro-mo	*.000,01*	Egro-mo
1,000,000	Bi-mo	*.000,001*	Ebi-mo
1,000,000,000	Tri-mo	and so on.	

Multiplication Table

1	2	3	4	5	6	7	8	9	Χ	Ɛ
2	4	6	8	Χ	10	12	14	16	18	1Χ
3	6	9	10	13	16	19	20	23	26	29
4	8	10	14	18	20	24	28	30	34	38
5	Χ	13	18	21	26	2Ɛ	34	39	42	47
6	10	16	20	26	30	36	40	46	50	56
7	12	19	24	2Ɛ	36	41	48	53	5Χ	65
8	14	20	28	34	40	48	54	60	68	74
9	16	23	30	39	46	53	60	69	76	83
Χ	18	26	34	42	50	5Χ	68	76	84	92
Ɛ	1Χ	29	38	47	56	65	74	83	92	Χ1

The Duodecimal Bulletin

Volume 5 - No. 1
June, 1949

THE DUODECIMAL SOCIETY OF AMERICA

20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y.

THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve-base arithmetic. The lessons and examinations are free to those whose entrance applications are accepted. Remittance of $6, covering initiation fee ($3) and one year's dues ($3), must accompany applications.

The Duodecimal Bulletin is the official publication of the Duodecimal Society of America, Inc., 20 Carlton Place, Staten Island 4, New York. George S. Terry, Chairman of the Board of Directors. F. Emerson Andrews, President. Ralph H. Beard, Editor. Separate subscriptions $2.00 a year, 50¢ a copy.

~ ~ ~ ~ ~ ~ ~ ~

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numbers universally used. *"Why even try to learn to use these heathenish scrawly symbols, with their stubborn propensity for error, in place of the beautiful clear numbers which our fathers have used for untold generations? Think of the needless waste. We would have to change all of our counting boards and abaci. X is X, isn't it? And why do we need a symbol for nothing? You can't count it! No! Let us keep to our simple tried and true numerals, and let the barbarians scratch their heads, and rub themselves out. It will all come to 0 anyhow."*

Yet, although it took D years, the new notation became generally used, and man's thinking leapt forward like an arrow sped from a bow. The early years of the Renaissance marked a new stage in the use of symbols, with the advent of algebra, fractionals (decimals?), logarithms, analytical geometry, and the calculus. Can you imagine what it would be like to try to express the coordinates of points on a curve in Roman notation?

CONTENTS

~ ~ ~ ~ ~ ~ ~ ~

Mathematicians became conscious of a new dimension in symbolism, and the fundamental concepts of number were re-examined. Man awoke to the fact that different number bases could be used, and Simon Stevin stated in 1585 that the duodecimal base was to be preferred to the decimal.

The new Arabic notation accomodated mathematical statement better, and facilitated ideation. All thinking accelerated when released from the drag of the cumbrous Roman notation.

The parallel seems tenable. The notation of the dozen base accomodates mathematical statement better, and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base embodies a concurrent analysis and definition of numbers that stimulates classification and generalization. Yet this is accomplished by such simple means that students in the primary grades easily learn to perform computations in duodecimals, and can tell why they are better. Literally, the decimal base is unsatis-factory because it has "not-enough-factors."

Then, shouldn't we change? No! No change should be made, and we urge no change. All the world uses decimals. But people of understanding should learn to use duodecimals to facilitate their thinking, and to ease the valuative processes of their minds. Duodecimals should be man's second mathematical language. They should be taught in all the schools. In any operation, that base should be used which is most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity. But no change should be made. Perhaps by the year 2000, or maybe by *1200*, which is *14* years later, duodecimals may be the more popular base. But then no change need be made, because people will already be using the better base.

When one is familiar with duodecimals, a number of accessory advantages become apparent. Percentage is a very useful tool, but many percentages come out in awkward figures because of the inflexibility of decimals. When based on the gross, twice as many ratios come out in even figures, and among them are some of those most used, as thirds, sixths, and twelfths, - eighths and sixteenths. There are advantages associated with time and the calendar. Monthly interest rates or charges are derived from annual rates, or the reverse, by simply moving the unit (decimal?) point. The price of a single item bears the same relation to the price of the dozen, and so does the inch to the foot.

The proper correlation of weights and measures has always been one of the world's serious problems. None of the present systems is completely satisfactory. The American and English standards are convenient to use since they are the final result of a long process of practical evolution in which many inconvenient measures have been adjusted or abandoned. The French decimal metric measures have the advantage of being set upon the same base as the number system, and are well systemized. But many of the units are awkward because of their arbitrary sizes, and because their decimal scale does not accomodate division into thirds and fourths readily.

The duodecimal system of weights and measures, based on the inch and yard, the pint and the pound, has the desirable elements of both systems, and few of their faults. This Do-Metric System retains the familiar units of the American and British standards in approximately their present size, and arranges them into an ordered metric system using the scale of twelve. This fits perfectly into the duodecimal notation, and the combination accomodates the inclusion of the units of time and of angular measure within the system, which hitherto has not been possible.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have thought staid and established, and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is cordially invited.

The Duodecimal Bulletin

All figures in italics are duodecimal.

THE DUODECIMAL PERPETUAL CALENDAR

by Louis Paul d'Autremont

The index of time is the related motion of three bodies: the sun, the earth, and its moon. The travel of the earth in its orbit around the sun determines the year and the seasons. The travel of the moon about the earth is the basis of our concept of the month. The rotation of the earth on its axis clocks the day. Each of these motions varies slightly in its rate, and there is no exact natural period of the relative motions or positions of these bodies. They lack a common denominator.

Man needs a reliable frame of time reference to record and measure his works, and to relate them to natural phenomena. This time measure should be convenient, regular and accurate. The independent motions of the sun, moon, and earth make this a complex problem.

Early efforts in constructing a calendar were unsatisfactory because of inaccurate measure of the year. The present standard Gregorian calendar reflects a very accurate measurement of the sun cycle, but it is divided into twelve months of very uneven lengths.

The concept of the month is directly traceable to the moon, both in name and period. The Moslem calendar is based on twelve lunations, resulting in a year of 354 days. Other calendars intercalate extra days to adjust the length of twelve months to the year and season-cycle of the sun.

The length of the week has no astronomical significance, and weeks of 5, 7, 8, and 10 days have been used. The week of 7 days became general at the beginning of the Christian era.

There have been many proposals to correct the unevenness of the months of the Gregorian calendar, and a number of ideas for the improvement of the calendar have been developed. The League of Nations inaugurated official study of this problem in 1923.

One of the most popular of the suggested calendar improvements is the idea of the perpetual calendar. In this arrangement, the particular dates of the year fall upon the same week day every year. The Gregorian calendar could be made perpetual simply by not giving December 31st a week day name, and the same with February 29th. Thus starting Sunday, January 1st, 1950, if December 31st of that year were not given a week day name, then January 1st of 1951 and all succeeding years would fall on Sunday, providing that February 29th were not given a weekday name in leap years. Any particular date thereafter would fall upon the same weekday year after year.

Perpetual calendars would not require annual replacement. The economy involved is surprising. In addition to their far greater convenience, and their facilitation of planning, perpetual calendars could be made part of more permanent structures, such as clocks, desks, wallets, vanity cases, etc.

Popular support has largely concentrated on two of the proposed perpetual calendars. The Fixed International Calendar divides the year into 13 months of 28 days, arranged in 4 weeks of 7 days each. While this calendar has the fault of awkward division into half and quarter year periods, or seasons, the dates of the year would be easily memorized, as the same date of each month falls upon the same day of the week. An amusing characteristic is that the thirteenth day of every month would fall on Friday, the 13th, in this thirteen month calendar.

By far the most popular proposal is the perpetual World Calendar. This advocates a 12-month year of 52 seven-day weeks. The quarters are of equal length and identical form, containing 13 weeks, 91 days, with months of 31, 30 and 30 days. Each quarter starts on Sunday and ends on Saturday. Each month has the same number of weekdays, (26,) the 31 day months including 5 Sundays. Intercalary days are Worldsday, (December 31st, or W December,) and Leapyearday, (June 31st, or W June.)

This calendar has been energetically promoted internationally by the World Calendar Association and its affiliates, under the devoted leadership of Elisabeth Achelis. It is receiving active and serious consideration by most nations of the world and by the UNESCO. There is a concerted drive for general adoption of the World Calendar by 31 December 1950. A Bill for its adoption by the United States is now pending in Congress, (S. 1415), and it will be brought before the General Assembly of the United Nations in the session starting September 1949, on a motion proposed by Cuba, and seconded by Mexico.

The manifest advantages of the World Calendar involve only minor changes from our standard Gregorian Calendar, and it is considered by many that general adoption will be achieved as planned. Duodecimally, it is far preferable to the 13-month proposal, and its support is advocated.

THE WORLD CALENDAR

FIRST QUARTER		
JANUARY	FEBRUARY	MARCH
SECOND QUARTER		
APRIL	MAY	JUNE **
THIRD QUARTER		
JULY	AUGUST	SEPTEMBER
FOURTH QUARTER		
OCTOBER	NOVEMBER	DECEMBER*

S	M	T	W	T	F	S
1	2	3	4	5	6	7
8	9	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

S	M	T	W	T	F	S
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

S	M	T	W	T	F	S
					1	2
3	4	5	6	7	8	9
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
						*W

* Worldsday, (a World Holiday), W or 31 December (365th day), follows 30 December every year.

** The Leapyear Day, (another World Holiday), W or 31 June follows 30 June in leap years.

Yet further improvements in the calendar are possible and desirable. These are of a type perhaps not now of pressing concern, but they should be considered. For instance the civil year starts on a date without astronomical significance, which is out of step with the seasons, and celebrates no acknowledged anniversary. It differs widely from the start of the astronomical year, which is the day of the vernal equinox.

There is a noticeable trend in industry toward a shorter work week, and this might enter into our considerations. Since weeks of 5, 6, 7, 8, and 10 days have already been used, it is possible that a calendar based on a 6-day week and designed for maximum factorability might find popular acceptance, - especially in that time when duodecimal numeration has come into general use. The Duodecimal Perpetual Calendar, first promulgated in 1912, attempts certain of these desired improvements not considered in the World Calendar.

The Duodecimal Calendar proposes that the year start with the spring equinox, as the astronomical year does. The first day of the year is Sunday. The year is divided into twelve months of 30 days each. Each week is of 6 days, starting Sunday and ending Saturday, with Thursday omitted. There are 60 normal weeks in the year, 5 in each month, 15 in each quarter.

Between the last month of the year, and the beginning of the new year, there is a week of 5 days. This week begins on Sunday and ends on Friday. For leap years the week has 6 days, ending on a Saturday. This week is by itself, and is not attached to any month. It could be called "New Year Week," and be a holiday week.

The Duodecimal Calendar is made perpetual by the omission of the Saturday of New Years Week in ordinary years. In leap years no omission is necessary.

The outstanding characteristic of this calendar is its complete regularity. Each month is like every other month, and so are the quarters. Because of the formal regularity, all dates are readily memorized. For this reason, too, intervals can be calculated with ease. All planning is greatly facilitated.

With March as the first month of the year, the months coincide with the signs of the zodiac. The correlation of dates with astronomical events is much simpler. Once again the months of September, October, November, and December are in their proper places, as the 7th, 8th, 9th, and 10th months of the year. The seasons fall into their proper groupings in entire accord with the months of the calendar.

The element of factorability has not been unduly stressed, as it is not well to attempt to predict the form that the work week will take. But, since all of the periods of the calendar are highly factorable quantities, much of the awkwardness of the current calendars is eliminated.

Greater detail on the Duodecimal Calendar, as well as on other proposed calendars will be found in three brochures published by the author: The Duodecimal Perpetual Calendar, 1926, The Calendar of the Future, 1931, and tables of the Calendars, 4th edition 1944.

The Duodecimal Calendar

1	2	3	4	5	1	2	3	4	5
1	MARCH	30	30	21	7	SEPTEMBER	30	210	17
2	APRIL	30	60	20	8	OCTOBER	30	240	17
3	MAY	30	90	20	9	NOVEMBER	30	270	16
4	JUNE	30	120	19	10	DECEMBER	30	300	16
5	JULY	30	150	19	11	JANUARY	30	330	15
6	AUGUST	30	180	18	12	FEBRUARY	30	360	14

SUN	MON	TUE	WED	FRI	SAT
1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30
1	2	3	4	5	6

NEW YEAR WEEK

5|365|

LEAP YEAR

1|366|

COLUMN 3. NUMBER OF DAYS IN A MONTH.
" 4. " " " SINCE THE FIRST OF THE YEAR.
" 5. THE DAY OF THE MONTH OF STANDARD CALENDAR CORRESPONDING TO FIRST DAY OF MONTH OF THIS CALENDAR.

INVENTED 1912 By Louis Paul d'Autremont

DATES

by Ralph H. Beard

Events that happen in regular order are normally well suited for representation by numerical series. Yet this is not true about dates. Our peculiar habits in designating the days of the year, and dates in general, introduce awkward complications in fitting normal numbering procedures to this application.

In planning a duodecimal dating practice we have certain suggestions to offer, which, if they be found acceptable, will inaugurate an adequate and satisfactory arrangement.

American practice differs from that of most of the rest of the world in the way that dates are stated. We state a date as September 2, 1947. In other countries this is generally written 21 September 1947. We find the latter form preferable. We have used it for some time, mainly for its economy of punctuation.

But when we state a date entirely in numbers, as 21-9-1947, we are exactly inverting what is our standard numerical practice. The order in which we write the figures of our numbers is in descending magnitudes from left to right. To state a date in full accordance with this principle, we would write 1947 921. And the following day would be 1947 922. Corresponding duodecimal statements would be *1163 919* and *1163 91X*. In decimal statements of this form, some months will require the use of two places for the designation of the month. But it must be noted that two places must always be used to state the day of the month. For the early days of the month, a zero must occupy the second column; the 6th of September would be written *1163 906*.

However, we depart from our standard numbering practice in the way in which we number the months. January is the first month and we number it "one", December is the twelfth month and we number it "twelve", or "do". There is no "zero" month, nor "zero" day. This is probably a relic of the days when Roman numerals were used. This practice is ill suited to modern number systems, but it would be extremely difficult to change. For duodecimal use, the alternative is to adopt a single symbol for representation of the "do", and use it to represent the last month of the year. Suppose we use "*D*". Then on the first day of the next month, (January, or "*1*",) we must remember to carry one over into the next column to advance the year. As an illustration of this, the day after *1164 D27* would be *1165 101*. This is about as near to standard numeration as is possible under the circumstances, and the practice is recommended.

In line with this dating practice, it would be possible to express simply a definite instant in time. For instance the minette after noon of the day mentioned last above would be stated as *1165 101.601*, and the time fractional could be extended to designate as fine a division of time as might be desired.

~ ~ ~ ~ ~ ~ ~ ~

THE APPEAL OF DUODECIMALS

by Frederick Condit

In these days of high pressure campaigns for social reforms, debates about labor conditions and peace problems, struggles to accomodate ourselves to the changing values of the dollar, and so on, - even temporary escape from all controversial questions and drafts on one's purse would seem to be a blessed relief. To be able to bury oneself in a subject which does not require immediate solution, - one which requires only as much concentration as one wishes to give, yet yields rich rewards in opening unlimited fields for exploration within the scope of everyone from elementary to highly technical levels, - is an inviting prospect. If it appeals to you, then go to the technical section of the nearest public library and look up the literature on the Duodecimal System.

Here is a subject that is as close to the interest of everyone as a man's vest, and yet very few even know of its existence. If you ask a group of one hundred people if they have ever heard of the duodecimal system, perhaps a dozen will confuse it with the Dewey Decimal System, which is used for classifying books in our libraries. One of the entire hundred may know of it, having heard that efforts to put it into use were made in Sweden during the reign of King Charles XII.

There is a group with headquarters in New York, the Duodecimal Society of America, which is interested in spreading information as to the advantages in using twelve instead of ten as the basis of our number system, and in collating the records of the work that already has been and is being done on the subject. In its publication, The Duodecimal Bulletin, will be found material covering the historical background of the origin and development of our present number system, and the parallel development of the use of twelve as the basis of grouping, measuring, etc. These alternate methods cause us double work in our everyday computations in converting from one scale to the other. We buy our wares in the twelve system, and pay for our purchase in the ten system. There is no necessity for this complex duplication.

In addition to interested average people, the membership of the Society includes mathematicians, teachers, and engineers, who have had, and are having, a field day working out the mathematical tables of duodecimals to facilitate their work. One of the enthusiastic members has had a computing machine modified to perform operations in duodecimals. The design of a duodecimal slide rule is being elaborated by an active committee. A new duodecimal musical notation has been proposed, which would eliminate all accidentals, assigning instead a single definite pitch to each line and space of the staff. There being twelve tones and half-tones in our "octave," and the musical staff having five lines, four internal spaces, two external spaces, and an interstave line, all of this falls naturally into the duodecimal notation.

The period of transition from the general use of the decimal base to the duodecimal will present many problems, especially to the teachers. They are the ones who are now faced with the problem of making the study of arithmetic interesting. Their successors will reap the reward of their contributions to the change. They have always realized that there must be some serious lack in the subject to cause the aversion which most children show toward it. Many methods of arousing interest have been tried, such as card tricks, emphasizing the so-called magic of the numbers nine and eleven, or cutting a small piece of paper so that a piano can be passed through it. All of these tricks help for a time, but when the novelty wears off, the drudgery is still present.

The writer never taught school and cannot speak with authority on problems of education. In fact the whole subject of number systems is no more than academic to him, and his interest in figures is mainly limited to efforts to keep those of the red variety out of his business. However, pending the arrival of the numerous contributions which may be expected from the educators after they become aware of this most interesting field, it will not be out of order to assemble a few obvious facts bearing on why arithmetic is *non grata* to children, and what, if anything, can be done about it.

To help out in this discussion let us call on the musician. He is not ordinarily considered a person of science. Music as an art is great, but as a science it is a poor step-child. The physicist looks down on the science of music because its scale does not follow a regular mathematical pattern. Granted, but it should be realized that our ancestors acquired their ideas of pitch and of the musical scale long before Sir Isaac Newton watched the apple drop and made his contribution to the science of physics. Music may be a step-child, but its basis is enough of a science to make apparent the inherent fault of the ten-system, and that is that it has no rhythm, or if you prefer, that it has an unnatural rhythm.

Rhythm is everywhere in nature, - in the seasons, in astronomy, chemistry, physics, and (except for the ten system) in mathematics. It finds a response in humans, even those who have no ear for melody or harmony. Music is said to be the universal language, but the element which contributes most to its universal appeal, is rhythm. With one recallable exception, no composer has used a ten-base rhythm. Tschaikowsky set the second movement of his sixth, Pathetic, symphony in five-four time. In spite of the fact that this movement has two excellent themes, no one ever asks to have it repeated. The rhythm is so disturbing that even the orchestra conductors have difficulty with it. When popular orchestras play it, the five-four time is changed to one of the natural rhythms.

The rhythm of the ten-system can hardly be called disturbing when music is not involved. Instead, it is of negative value, as that it cannot be set to music without providing for hiatuses, such as occur in hymns. For this reason, learning the numbers of the ten system is like memorizing prose, and that, for a child, is real work. No one who has witnessed the agony a child goes through in reciting a speech in prose, after succeeding magnificently with a poem, will need to be convinced of this. It lacks the points of support which are available through the use of rhythm, as expressed in music, marching, dancing, etc., which perform a real function in the association of ideas.

Let us hear from the department of psychology for a time. Much is being heard of the importance of educating the "whole child." The more "pegs" a child has on which information can be hung, the more easily he can absorb it and the longer he can retain it. It is found that the co-ordination of physical activities including singing, marching, and dancing, with verbal and visual instruction, furnishes such pegs.

The musician has pointed out at least one serious fault of the ten-system, and the psychologist has given us a hint as to what use can be made of replacing the negative quality of the ten-system by the positive quality of the twelve-system.

The twelve-system is rhythm in essence. It contains the masculine form, the martial four-four march time enjoyed by boys, and the feminine three-four waltz time preferred by girls. One of the favorite meters of composers of band music is twelve-eight time, or four-four time with each quarter beat containing three eighth notes. Nearly every child will have marched to this rhythm before reaching first grade.

Ditties can be arranged which will contain the numbers of the twelve system. They can be sung and marched to. A group of three stanzas, each containing four lines, with twelve notes to a line, will provide a note for each number of the GRO, now expressed as 144. A child starting a study which uses a rhythm familiar to him together with physical activities which he enjoys, would feel as much at home as in his mother's pantry. The job would be undertaken with interest and enthusiasm. All the timidity now felt by young entrants into the strange world of arithmetic would be absent. Instead, it will have been turned into a game.

When he is ready to start graphs, the staves of his music, with notes corresponding to his numbers will furnish him a ready made graph, and this graph will be useful not only for learning, but will be a tool which he can carry through life, using it in every computation. Each child, knowing that all other children use the same that he does, will talk about it freely, as about other things shared in common.

Many other advantages, not mentioned here, are to be had by changing to the twelve system. The problem of getting the idea accepted and started should be a prime interest to the educators of the young. They are hereby invited to turn their imaginations loose in this most interesting field of research.

ON THE GRADUATION OF SCALES

by William Shaw Crosby

The slide rule is one of the greatest arithmetical short cuts ever invented. Since the most obvious purpose and effect of duodecimal arithmetic is to simplify the paperwork of figuring, with which the slide-rule expert has learnt to dispense altogether, he might object that for him a change to dozens would be superfluous. He would be wrong. The slide rule is one application of the graduated scale, and uncial graduations can be more nicely adapted to any interval, and more logically patterned, than the divisions now in use, yet easy to read.

The fewness of the divisors of ten is as much a handicap in graduating scales as in any other field of applied decimal arithmetic. Often an interval to be subdivided is too small to hold ten graduations. Then the division could be into fifths. But if the interval is not quite big enough to contain fifths, it cannot be split into useful, easily recognized quarters, nor into thirds--these fractions are too hard to deal with in decimal arithmetic--; instead, the subdivision can only be into coarse halves. Note the awkward change at point 2 of the decimal slide-rule scale, Figure *1*.

Duodecimal arithmetic is more adaptable. Where twelfths[1] of a scale interval would be too crowded it permits us to graduate in sixths, quarters, thirds or halves, whichever bests fits the case.[2]

But when there are as many as five different ways of dividing intervals it is important to prevent the mistakes that could result from confusing one sort of

1. Must I apologize to members of a Duodecimal Society for using this word? By "twelve" I mean the number that you get by doubling six or by adding five to seven. To most of us the good old word means primarily that--the number --rather than the verbal rendering of the symbol 12. "Do?" "Gro?" "Mo?" NO!

2. Camp, in Dd. Bul. Vol. 4, No. 2, p. *10*, terms graduation by sixths or thirds "confusing" and proposes to use only twelfths, quarters and halves as recognized divisions on the slide rule. I agree that in particular cases he may be right, but a general refusal to use thirds and sixths seems to me a needless sacrifice of one of the advantages of duodecimal arithmetic. Although tradition has limited our practice in the use of scales marked with other than binary or decimal graduations, we should yet be able to stretch our imagination to make us the equals of someone whose experience will not have been so limited, and who can take thirds in his stride. Actually there is something to be said in favor of preferring a division into sixths or thirds to a division into quarters. When we must visually estimate between the lines, let the instrument maker perform the comparatively tricky trisections and leave the easy halvings for our eyes!

division with another. A standard and easily legible *pattern* should distinguish each mode of graduation--so that, for example, a division into quarters will look unmistakably different from a division into sixths, and the eye will need no second glance to assure itself of the value of each graduation.

Logic and system can easily be carried too far in making such patterns. The most notable example of this undesirable extreme is furnished by the ordinary desk ruler marked off into inches and binary fractions of an inch. Figure *2* shows part of such a ruler graduated to 1/32″.

Repeat divisions into halves is the simplest method of graduation and makes possible a very logical ranking of the different fractions according to simplicity. That is, the line marking the half inch is longer than the lines at the quarters, the quarters are accentuated in comparison with the eighths, and so forth. But note that when this process has been carried as far as to 16ths or 32nds the graduations become confused. In the forest of lines within each inch one is at a loss to pick out some fraction like 27/32″ without first doing a little quick mental arithmetic, such as 24/32″ = 3/4″, and then actually counting divisions from a reference point like 3/4″ that can be recognized at a glance. Binary fractions smaller than eighths are troublesome to use not merely because our arithmetic is not based on 2, but especially because their pattern of graduation is so repetitious that it is easy to get lost in it.

Except for the inch, most of our scale intervals are divided to accord with the decimal system, which is to say that they are graduated either in halves, fifths, or tenths.

Halves are unmistakable and need no distinguishing pattern. Fifths are more difficult. Five marks are about as many as can be counted at a glance--too many in some cases. A pattern would help us recognize that the interval is divided into five and not some other number of parts, and instantly and positively identify each mark, but five is a prime number and no logical pattern suggests itself. So fifths are usually marked by graduations of even length.

Occasionally an "illogical" pattern is used. Figure *3* shows a type of scale introduced by the Ford Motor Company, on which inches are divided into numbered tenths, each tenth is subdivided into fifths, and the second and third

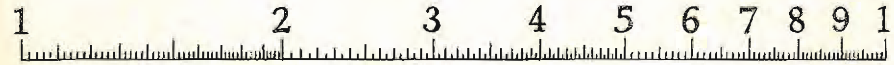

Fig. I

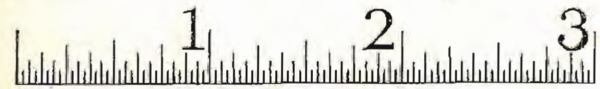

Fig. 2

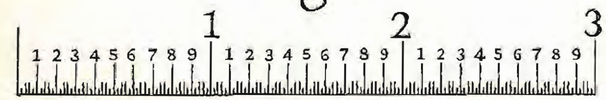

Fig. 3

Fig. 4

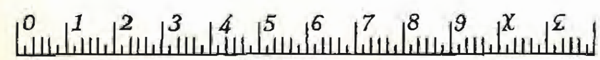

Fig. 5

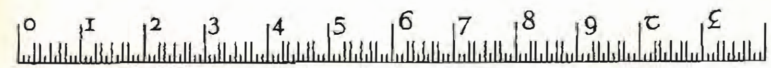

Fig. 6

Fig. 7

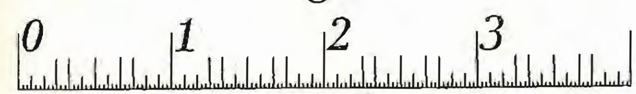

Fig. 8

Fig. 9

Fig. X

fifth of each tenth are accentuated. The pattern is arbitrary, of course -- .04″ and .06″ are not highly important scale divisions in the sense that 1/4″ or 1/8″ were in figure *2* -- but it is *distinctive*. The user is less likely to confuse 1/5 of a tenth with 2/5, or 2/5 with 3/5, than he would be if all these small divisions were of the same length, and he could hardly fall into the absent-minded mistake of supposing that any of the marks within each tenth represented a half or a quarter of the interval. The eye is quickly set free for the task of estimating hundredths.

Tenths are almost always graduated as fifths of halves, as in the scale of centimeters, Figure *4*. This doesn't form much of a pattern. The five-tenths' mark is stressed but the wide spans to either side are just as patternless as the ordinary way of showing fifths, and indeed can too easily be read as fifths of a basic unit half as large.

One never sees the division into tenths made in the other possible order: first into five parts, then each fifth into halves. It would be just as sound arithmetically, but we are not accustomed to it; it is not standard practice, and a first encounter with it would be confusing. Duodecimal graduations can also be approached in more than one way. What shall we establish as standard practice? If an interval has to be marked into sixths, for instance, shall we accustom ourselves to stressing the thirds, or the half?

My suggestion is "stress both." Stress both equally. The result (Figure *5*, a scale of Do-Metric "quans"), with its clumps of three long marks in the middle, is at first sight funny-looking. But it is not more so than the Ford scale of Figure *3*, which as we have seen has a cluster in the middle for two very good reasons: to diminish the chance of mistaking one mark for its neighbor and to make a *distinctive pattern* that will be peculiar to a division into fifths. These reasons apply with equal force to this pattern of sixths. Furthermore, the elongated divisions in the pattern of sixths *do* stand for something important in their own right: the useful fractions *1/3*, *1/2*, and *2/3*.

Figure *6*, a scale of "centiells,"[3] shows a distinctive pattern for a graduation into twelfths. Its two clusters of two accentuated marks, paired on either side of the lone accentuated center-line, stamp it instantly as something different from a pattern of sixths, and each of the stressed divisions represents a frequently needed fraction: *1/4*, *1/3*, *1/2*, *2/3*, or *3/4*.

When twelfths or sixths would be packed too close for easy reading, an interval will have to be divided into quarters, thirds, or even into halves. Danger of confusing these should be slight. Quarters can be distinguished by stressing the middle division, as is done at present, and it would hardly be possible to mistake thirds for halves.

It should be possible to compound the pattern of twelfths with any of the other patterns and still have a scale that can easily be read. Figure *7* is a scale of "egrogages"[4] graduated to *.06* egrogage; Figure *8*, "tums"[5] to *.04* tum; and figure *9*, inches to *.03* inch. With a little practice any of them can be read much more quickly than the scale of Figure *2* with a *lot* of practice.

Finally, all the uncial patterns I have discussed are exhibited in a single logarithmic scale, Figure *X*. Although this is intended more as a *tour de force* than as an example of a practical slide rule scale, I believe that compared with the decimal scale of Figure *1* it fulfils the promise of my first paragraph: its graduations are "more nicely adapted to any interval, and more logically patterned, yet easy to read."

---

3. A "centiell" is a unit of a system proposed by the writer in Dd. Bul. Vol.1 No.2 pp.*X*, *20*. It is defined as exactly 8 4/19 millimeters, and it is approximately double the distance a freely-falling body drops from rest in the first *0.00 001* day of its fall. In this system the writer has used the decimal metric prefixes with duodecimal meanings, so that a centiell is *1/100* of an "ell" of approximately 46 35/64 inches.

This system is not to be confused with a proposal by Prof. Carl Forssell in the Swedish Magazine *Norden* (cited in *Newsweek* Nov. 5, 1945 p. 29), which involves an "ell" of 50 inches decimally divided into "centi-ells," "milli-ells," etc.--a scheme which seems to combine the disadvantages of the English and metric systems.

4. The "egrogage" is a centimeter in deep disguise. See Dd.Bul.Vol.1 No.2 p.*23*, in which Handy advocates a measure system based on the standard railroad gage slightly readjusted to make *1/100* "gage" equal precisely 1 centimeter.

5. The "tum" is a unit of the "twecimal" measures proposed by Norland (see Dd.Bul.Vol.3 No.3 p.*3*), and would be the distance traversed in free space by a ray of light in *0.000 000 000 000 01* day--well, in practically no time at all. Lien independently proposed (in Dd.Bul. Vol.1 No. 2 pp.*21* - 22) a unit based on the speed of radiation: the "mark", equal to Norland's "link" of *10* tums, although inaccurately computed. It should not be forgotten that in the lore of dozenry the name "mark" stands also for Pitman's *10*- shilling unit of currency.

MATHEMATICAL RECREATIONS

D. M. Brown, Editor

One of the aims of the editor of this department is to provide problems which will give practice in manipulation of numbers expressed in various bases. As one becomes familiar with the mechanics of such manipulations, he begins to see just what a number system is, an artificial device. Although such a device is necessary, practice in the use of a variety of bases enables one to more fully appreciate the advantages of particular bases, especially the dozenal base. So ideas any of you readers have for new kinds of problems will be welcome.

In the last (Dec. 1948) issue of the Bulletin, we posed problems involving number $(xyzw)_B$ in which x, y, z, and w, are digits, and B is the base, so that:

$$(xyzw)_B = xB^3 + yB^2 + zB + w.$$

The solutions to the problems are as follows:

1. $(xy)_{10} + (xy)_{12} = (xy)_{32}$ Ans. $x = 0$, $y = 0$. (A trivial case)

Ye Editor apologizes; he still thinks decimally, and the problem should have read:

$(xy)_{10} + (xy)_{12} = (xy)_{28}$ Ans. $x = 1$, $y = 8$.

2. $(xyz)_4 + (xyz)_{\mathcal{E}} = (xyz)_{10}$ Ans. $x = 1$, $y = 2$, $z = 1$.

Harry Robert submitted several similar problems, some with more than one solution. He says that:

1. $(abc)_{4n} + (abc)_{5n} = (abc)_{7n}$ has one solution, and that
2. $(abc)_{5n} + (abc)_{6n} = (abc)_{8n}$ has 9 solutions for each

value of n. Can you find them?

George S. Terry suggests the type

$$[(xy)_a]^2 + [(xy)_b]^2 = [(xy)_c]^2$$

Try the following:

3. $[(xy)_3]^2 + [(xy)_{\chi}]^2 + [(xy)_{\mathcal{E}}]^2$
4. $[(xy)_2]^2 + [(xy)_3]^2 + [(xy)_4]^2$
5. $[(xy)_9]^2 + [(xy)_{15}]^2 + [(xy)_{20}]^2 = [(xy)_{2\chi}]^2$

You may find the preceding too simple for your taste. Here is one which you may find a bit more challenging! Below is given a coded magic square containing 21 two-digit numbers. The sum along the two diagonals, along each row, and each column, is the three-digit number AAT. Each of the twelve letters represents a different dozenal digit. If the letters are arranged in order of decoded magnitude, they will spell out what I think of those who didn't tackle this problem!

ZC	AT	YU	YA	ZR		
YY	ZU	ZM	AR	YO		
ZA	YP	YL	ZE	ZY		
ZP	ZO	AE	YT	YZ		
YM	YC	ZT	ZZ	ZL	=	AAT

~ ~ ~ ~ ~ ~ ~ ~

NUMBERS AND THE OPEN MIND

by Paul Van Buskirk

The word "Arabian" calls up thoughts of Ali Baba and the Forty Thieves and that delightful world in which Aladdin could obtain his slightest wish by merely rubbing his lamp and repeating the magic words "Open-Sesame".

But, these same Arabians carried another lamp, the lamp of learning, for centuries after it fell from Greek hands. They not only preserved the old light, but prevailed on the lamp to shed new light, not by the magic words "Open-Sesame", but by the use of the method of "Open Mentality". The studious Arabs applied their mind to a minor branch of Greek mathematics and produced that major branch which we know as Algebra.

What is meant by the method of open mentality? If I hold my watch up and ask you the function of the long hand you will reply that it measures the sixty minutes of the hour. But, many men of open mentality have observed that it also divides the hour into twelve fractions and that each of these can be divided into twelve parts. If the watch be then regarded as the right hand driving unit in a series of recording dials, as in the familiar gas meter, we have a device that registers in terms of dozens and dozens of dozens. In order to set down in figures the reading from such a meter, in which 10 must be used to mean twelve, we must open our minds to the idea that "ten" can be a "digit", if it has a separate symbol. That is not hard to do since we often see "X" used for ten in Roman numerals. It is a little harder to accept the idea that eleven can be a "digit" since we unconsciously associate the word "digit" with the fingers.

You need not feel chagrined that you have never before known that there could be a system of numbers other than the decimal. The Romans, for centuries, used the uncial or twelve-part system for fractional numbers, even using a single name for ten-twelfths and another for eleven-twelfths, without ever arriving at a true twelve system of numbers, as far as their written records show. If Caesar did not see it, why should you?

The first man to leave a written record of the twelve system was the 17th century French scholar, Blaise Pascal. Other French scholars explored the system which his open mentality had revealed, and urged its adoption by the new French Government when the old monarchy was overthrown. Had their counsel prevailed, the whole world would now be counting by dozens, measuring in feet, inches and twelfths of inches, and using volume and weight units related to each other in terms of twelve as the decimal system units are related in terms of ten.

We cannot have the thrill of discovery that rewarded Pascal for his work, but we can have the thrill of exploring this system and working out its application to our own modern problems of figuring. Pioneering in figures is fun. You can have the stimulus of company by joining the Duodecimal Society of America, from whom you can get publications bringing you up to date on what has been done, how to go about the number work, and what particular fields other members are exploring.

Talk delivered at the Engineering Society of Detroit Speakers Club, January 25, 1949, by Paul VanBuskirk

THE ANNUAL MEETING

The fifth Annual Meeting of the society, held January 27th, 1949, at the Gramercy Park Hotel, in New York, proved to be even more interesting and enjoyable than its predecessors. Those interested in music and the dance, who had come to hear Mr. Godjevatz talk, lent color and gayety to the occasion.

The formal business of the meeting was briskly handled. President Andrews welcomed the members and friends of the society, and reviewed the outstanding features of the year's work: the work of Mr. Seelbach in extending the duodecimal bibliography, which was marked by the discovery of Simon Stevin's advocacy of duodecimals in his "l'Arithmetique" of 1585; the proposal of a new musical notation on the duodecimal base by Velizar Godjevatz, of which we would hear more in the course of the evening; and the publication of the index of the first four volumes of the Bulletin in the issue just coming off the press.

The financial summary for 1948 showed an initial balance of $625.37, normal revenues of $301.75, and donations of $892.00, for a total of $1,819.12. Expenses were $1,248.31, leaving a balance of $570.81 to start the new year. Membership figures continue their gradual increase with a gain of 15 for the year, for a total of 55. A better index than the number of members for measuring the growing interest in duodecimals, is that our present printing of 1,000 copies of each issue of the Duodecimal Bulletin has become insufficient.

The Nominating Committee suggested the re-election of the two directors whose terms were expiring, and the addition of three new directors to the Board, - H. K. Humphrey, Nathan Lazar, and Kingsland Camp, for the classes of 1950, 1951, and 1952 respectively. For the new Nominating Committee, the names of Paul Van Buskirk, Chairman, Paul Adams, and H. F. Stevens were proposed. These nominees were unanimously elected, and President Andrews announced the committee appointments for 1949, as shown in the attached list.

The Secretary informed the meeting that our active group of members in Baltimore had formed a local chapter, and elected officers. He submitted the formal application of that group for recognition as the Baltimore Chapter of the Society, with a copy of their proposed by-laws. Enthusiastically, the application was approved and the

Baltimore Chapter recognized. The by-laws were referred to Vice-President Friedemann for review and approval.

The establishment of the first branch chapter is an event of outstanding importance in the history of the Society. It is hoped that the initiative of our Baltimore members will stimulate member groups in other large cities to similar action.

The necessary business of the meeting having been covered, President Andrews introduced Velizar Godjevatz, who gave an interesting exposition of the advantages of his new duodecimal musical notation. The simplicity and lucid rationality of his method is attractive. It opens up new ground for musical development, in that it easily accomodates the notation of the finer musical intervals, those less than half-tones.

Equally important is the fact that the "Notation Godjevatz" is completely reasonable. The present notation is another of those conventional disciplines which hamper the free ideation of the mind with the constant necessity to translate a thought into an irregular and complex transcription. In performance, playing such a transcription involves the reverse of the same complex process. The new notation is simpler and more natural, which will make sight reading easier to learn, and more enjoyable in escape from restraint.

A talk by Kingsland Camp on The Duodecimal Slide Rule followed. Mr. Camp covered the current stage of the plans for the design of the rule, and explored the preference of those in the audience who were familiar with slide rules, as to folding the scale at π, or at the square root of the base. A show of hands indicated a surprising evenly-divided opinion. More ample statement of these details will be found elsewhere in this Bulletin.

Next was a talk by Frederick Condit, who approached the advantages of duodecimals from an entirely novel angle. Using a special form of motor-driven metronome, Mr. Condit proved, beyond a doubt in mind of anyone present, that one could count much more rapidly in duodecimals than in decimals. The reason for this surprising facility was demonstrated to lie in the natural introduction of a convenient rhythm into the duodecimal counting process. Decimal counting does not lend itself to a similar subdivision into groups because of the awkwardness of the five-factor.

Mr. Condit progressed from this basis to the exposition of the advantage to the school child, in having the rhythm-

sense include in its development the numbers that he uses increasingly every day. The child mind accepts easily those things that fit naturally into familiar patterns, and when he finds a rhythm in numbers to accord with his marching, singing, dancing and pulse, he will find his numbers pleasant to learn, easy to use, and soon familiar. We can thus avoid the approach that demands of our children another thing that has to be learned by rote, a natural thing that has to be accepted in awkward form as being right because we say it is right.

Refreshments and general discussion followed Mr. Condit's talk. Among the comments noted was that scripts for dancing used a three-dimensional duodecimal notation for the delineation of motion, of posture, and gesture. The discussions were at least as refreshing as the excellent refreshments. There was so much of interest in the independent thinking expressed, so much of talent and charm in the personalities present, that all were reluctant to have the meeting end.

Officers and Committees for 1949

Chairman of the Board: George S. Terry,
507 Main Street, Hingham, Mass.
President: F. Emerson Andrews, 34 Oak St., Tenafly, N.J.
Vice-President: Paul E. Friedemann,
904 Mifflin Avenue, Pittsburgh 21, Pa.
Secretary: Ralph H. Beard,
20 Carlton Place, Staten Island 4, N.Y.
Treasurer: H. K. Humphrey, 520 Ash St., Winnetka, Ill.
Members of the Board:

Class of 1950: Ralph H. Beard, H. K. Humphrey, George S. Terry
Class of 1951: F. Emerson Andrews, William S. Crosby, Nathan Lazar
Class of 1952: Kingsland Camp, Paul E. Friedemann, Harry C. Robert, Jr.

Committee on Awards: George S. Terry, Chairman,
Ralph H. Beard, Harry C. Robert, Jr.

To decide whether a 1950 Award is to be made, and if so, to recommend for that award a person of outstanding achievement in mathematical research, as related to duodecimals.

Committee on Bibliography: Lewis Carl Seelbach, 163 Davidson Avenue, Buffalo 15, N.Y.
To complete the now well advanced annotated bibliography.

Committee on Finance: George S. Terry, Chairman, Ralph H. Beard, H. K. Humphrey.
To review the Society's budget, act upon major expenditure proposals, and audit the report of the treasurer.

Committee on Mathematical Recreations: Donald M. Brown, Chairman, 519 Soule Boulevard, Ann Arbor, Mich., Mary B. Lloyd, George S. Terry.
To prepare material in this field for the Bulletin, to be a center for member correspondence, to collect and devise material for illustration and visual aids.

Committee on Mathematical Research: Harry C. Robert, Jr., Chairman, 1683 Johnson Road, N.E., Atlanta 6, Ga., Donald M. Brown, William S. Crosby, Dr. Nathan Lazar, Eugene M. Scifres.
To conduct original research, to check Bulletin submissions for accuracy and validity, and to be a central advisory committee on application of duodecimals to various fields of mathematical research.

Committee on Membership: Mary B. Lloyd, Chairman, 2304 Forest Park Avenue, Baltimore 7, Md., Paul E. Friedemann, Yvette L'Heureux.
To promote new memberships, to handle correspondence and encourage qualification for advanced standing, and to endeavor to help members into useful work and acquaintanceships.

Nominating Committee: Paul Van Buskirk, Chairman, 1524 Clairmount Avenue, Detroit 6, Mich., Paul Adams, H. F. Stevens.
To submit to the secretary before December 1st, nominations for directors for new terms, to fill vacancies on the Board, and for the new Nominating Committee for 1950.

Committee on Public Information: Paul Van Buskirk, Chairman, F. Emerson Andrews, George Carreras, Frederick S. Condit.
To spread information on the duodecimal system through correspondence, meetings, discussion groups, preparation of news materials, publication and encouragement of writing of special articles, work through schools and colleges, and other appropriate channels, and relations with distionaries and encyclopedias.

Committee on Weights and Measures: Ralph H. Beard, Chairman, Paul Adams, Dallas H. Lien.
To promote discussion and use of the Do-Metric System, to investigate further proposals in weights and measures, including problems of navigation and avigation, and to be an advisory board in this field.

THE MAIL BAG

Louis Paul D'Autremont, author of the Duodecimal Perpetual Calendar, is named for an earlier Louis Paul d'Autremont, who came to America in 1792. They lived for a while at Asylum, Pa., a settlement originally established for French royalist refugees, as interestingly recorded in Mildred Jordan's recent novel, "Asylum for the Queen." His father, Hubert, was guillotined in the French revolution. It is from the brother of this Louis, Alexander Hubert d'Autremont, that our Louis Paul d'Autremont is descended.

He has been keenly interested in duodecimals and in calendar reform for many years, as is attested by the date (1912) of his proposal for the Duodecimal Perpetual Calendar. He is one of the older members of the World Calendar Association, and ably supports their proposed revision, The World Calendar, as the best of the current proposals. The adoption of the Duodecimal Perpetual Calendar could not well be considered under present conditions.

~ ~ ~ ~

Stewart J. Ogilvy, Editor of World Government News, writes: "If UN is going to set a standard 'A', I don't see why they don't adopt the duodecimal notation suggested in the Bulletin."

They certainly should. The Notation Godjevatz marks a distinct advance in technique, and provides advantages not otherwise available. It will be interesting to note the reception accorded this new musical notation as the composers and musicians become familiar with it.

There is another element involved, - an approach from quite a different angle. The Do-Metric System bases its measures of time on the duodecimal subdivision of the day. The *.000 001* day is called the Vic, and there are, in decimal terms, 34.56 Vics per second. Part of the argument about standard pitch is involved with International Pitch with A_4 given a frequency of 435 vibrations per second, and American Standard Pitch with the same A_4 given a frequency of 440 per second. If A_4 were given a frequency of 439 vibrations per second, then the frequency for C♯, is one vibration per Vic.

~ ~ ~ ~

Paul Van Buskirk comments: "I wish to thank Dallas H. Lien for his article on the circle which appeared in the December Bulletin. I struggled with that problem during the depression, but never solved it. Circular measure is very poorly handled in present engineering practice. The gradual accumulation of evidence as to the superiority of

Base-Twelve on more and more fronts, will ultimately become incontrovertible and conclusive.

"May I suggest that 'double pi' be named 'dopi', so that we can have one of the seven dwarfs on our side. It could be symbolized by simply doubling the horizontal stroke in the present symbol for pi: ₶."

This refers to Dallas Lien's suggestion of "A Better Ratio for Pi" in the December Bulletin. The use of a double π in radian measure is excellent, and brings that form of circular measure into exact conformance with the duodecimal measure of time and angle. Mr. Van Buskirk's suggested "dopi" involves a confusing application of the prefix "do", which should refer exclusively to the dozen. The use of the prefix "du", as in "duor", might avoid this fault, the name of the double π becoming "dupi".

~ ~ ~ ~ ~ ~ ~ ~

THE DUODECIMAL SLIDE RULE

Comments and Suggestions

Eugene M. Scifres.

I have read both articles on the Slide Rule in the Bulletin with great interest. On many points I agree perfectly, but on some details I wish to suggest changes from the tentative plan. The idea of having a reciprocal scale along each log-log scale is especially good. In fact, I wonder why that hasn't been applied to slide rules before now. On numerous occasions I have had to take the negative power of some number, and, with an arrangement like this it would have been reduced to one operation.

It is well, as was said, to put the traditional arrangements of a slide rule on the defensive. Even if this is done though, many of the arrangements will have a pretty good defense. They are there because through years of change it has been found that the greatest number of people in many different lines of work (many of which have nothing in common) can use the slide rule best the way it is.

Of the several rules I have, the best is a K&E #4081-3 Log-log Duplex Decitrig. The general arrangement of this rule is very satisfactory and, while some changes are necessary, I believe we should follow its general pattern as most universally adapted to many different types of problems.

On several points I agree with Mr. Humphrey. I avoid the use of the cross-hair whenever possible, and I will not have a magnifier on the indicator. It upsets my judgment for interpolation. But I do not agree that folding the scale at π is just a talking point. Quite frequently in my work I start a problem on the CF or DF scale, divide by π by dropping down to the bottom scale, go through any necessary proportions, and refer to the log-log scale to get the natural logarithm of the result.

As to details, the total length of the log-log scales should be 4 feet instead of 2 feet as shown in the October Bulletin, and should be referred to the D scale instead of the A scale. The increased length will assist in avoiding visual error. You will notice that the log-log scale in the range from $e^{0.001}$ to $e^{0.01}$ uses exponents small enough to say that e^y is approximately equal to $1 + y$. So for values of e^y less that *1.01*, we can use the D scale, insert the correct number of zeros and add one. For values less than *1.001* there will be no detectable difference. Example:

$$e^{0.00345} = \text{approximately } 1.00345$$
$$e^{0.00000678} = 1.00000678$$

For this reason we can use just two feet of scale to the "left" of e and, by eliminating one foot on the lower end, we can add one foot on the upper end, which will come in handy in finding powers or roots of large numbers.

The proposal that all of the trig scales be "full length" meets with my whole hearted approval. When the sine and tangent scales are both referred to the same scale, it becomes quite easy to solve any problem involving vectors at right angles to each other. The idea of graduating the length of the division lines in proportion to their spacing is indeed good, but may prove to involve expense out of proportion to the value gained. We should endeavor to hold the cost of the rule within reason.

Chairman Kingsland Camp

Discussion at the Annual Meeting, and experiments with a crude outline model rule, have brought this subject to a somewhat more definite stage. The following matters seem not entirely clear as yet to some practiced slide rule users among us; their views, of course, we most particularly desire.

I. Pairing of Scales with Reciprocals

This proposed innovation especially calls for general understanding and wider comment. It entails what seems to some of us a disadvantage, that no scales will then appear along edges where slide and frame bear against each other. On account of parallax - the small distance between the crosshair of the indicator and the surface of the rule - such numbers as are exactly indicated on the scales can be more accurately set against each other without using the crosshair at all. For most of us, probably, only a small minority of calculations involve such numbers; we far more often work with longer numbers that require interpolation, and therefore the crosshair, when setting the scales against each other.

Suppose that we pair scales with their reciprocals, the latter to be lettered in red as is the usage with the present form of separate inverted scales. This arrangement will have the following advantages:

1. Space is saved, with a gain in clarity. Inverted scales need not be separately designated as their functions will be obvious from their red lettering and the scales to which they are attached. The nature of the so-called CIF scale, especially, will be less likely to be mistaken than on current models.

2. It will promote among all users the habit (now, probably, confined mostly to those called "wizards") of *always* conceiving the separation of two marks on any scale as representing the *quotient* of the indicated quantities; and the separation between a mark on a scale and one on its inverted scale as representing the *product* of the numbers (or the reciprocal of such product, in readily recognizable and easily handled cases).

3. Square or cube scales, when paired with their reciprocals, save further space by dispensing with the need for similar scales on the opposite member of the instrument. For, just as with the full-length scales, quotients and products are represented by the separation of the appropriate scale marks; this separation, transferred to a full-length scale, represents the square root or cube root of such quotient or product. After such transfer, any further steps of the calculation may usually be performed and the answer found on a full-length scale.

Multicycle scales, by the way, have another feature worth mentioning: any unity point may be taken as the cardinal one in an operation of this nature, thus in effect treating them as folded scales.

II. Arrangement of Scales

This follows my original essay "A Duodecimal Slide Rule" (Vol. 4, No. 2, page 7 of our Bulletin), except that a cube scale with reciprocal is now added on the slide, and the inverted decimal square scale (dec'l BI) is omitted from

the slide as it would seldom be useful. It may be good practice hereafter to engrave on the instrument and use generally really descriptive names, such as the square, cube, log (of the full-length factor scale) rather than the cryptic traditional symbols. In the outline below, all scales are duodecimal except where otherwise noted.

Position	Descriptive Name	Details	Traditional Symbols	Scales
POWERS SIDE				
Upper Frame:	Powers Scales	$e^{.1}$ to e^{10} with reciprocals $e^{.001}$ to $e^{.1}$ with reciprocals	LL	4
Slide:	Square	Two cycle scale with reciprocal	B, BI	2
	Cube	Three cycle scale, reciprocal	K, KI	2
(On Edge)	Dec'l Square	Two cycle decimal scale	(Dec'l) B	1
Lower Frame:	Dec'l Powers	$e^{1/12}$ to e^{12} with reciprocals $e^{1/1728}$ to $e^{1/12}$ with reciprocals	(Dec'l) LL	4
OTHER SIDE				
Upper Frame:	Factors	Full-length scale with recirocals	D, DI	2
	Folded Factors	Same as last line, but folded	DF, DIF	2
Slide:	Factors	Full-length scale with reciprocal	C, CI	2
	Folded Factors	Same as line above but folded	CF, CIF	2
Lower Frame:	Sine	Full-length sine scale	S	1
	Tan	Full-length tangent scale	T	1
	Small-Angles	Full-length factor scale, folded at 1 ÷ 2π	ST	1
	Log	Foot-long scale, 40 subdivisions to the inch. For measuring	L	1
		Total number of graduated scales		21

In decimal notation, a total of 25 scales. *19* (21) seems to be the greatest total number of scales on any currently manufactured instrument; its illustration at least looks more crowded than the instrument proposed above probably will, because of the pairing of so many scales on the latter. Nevertheless it would probably be a real improvement in nearly all polyphase rules, to widen them somewhat and separate scales more generously, whether paired or not. To a user, it sometimes seems that if additional scales are wanted, the manufacturers merely squeeze them into an existing model, producing a crowded instrument rather hard for some of us to get used to.