is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve-base arithmetic. The lessons and examinations are free to those whose entrance applications are accepted. Remittance of $6, covering initiation fee ($3) and one year's dues ($3), must accompany applications.

The Duodecimal Bulletin is the official publication of the Duodecimal Society of America, Inc., 20 Carlton Place, Staten Island 4, New York. George S. Terry, Chairman of the Board of Directors. F. Emerson Andrews, President. Ralph H. Beard, Editor. Separate subscriptions $2.00 a year, 50¢ a copy.

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WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numbers universally used. *"Why even try to learn to use these heathenish scrawly symbols, with their stubborn propensity for error, in place of the beautiful clear numbers which our fathers have used for untold generations? Think of the needless waste. We would have to change all of our counting boards and abaci. X is X, isn't it? And whey do we need a symbol for nothing? You can't count it! No! Let us keep to our simple tried and true numerals, and let the barbarians scratch their heads, and rub themselves out. It will all come to 0 anyhow."*

Yet, although it took D years, the new notation became generally used, and man's thinking leapt forward like an arrow sped from a bow. The early years of the Renaissance marked a new stage in the use of symbols, with the advent of algebra, fractionals (decimals?), logarithms, analytical geometry, and the calculus. Can you imagine what it would be like to try to express the co-ordinates of points on a curve in Roman notation?

Mathematicians became conscious of a new dimension in symbolism, and the fundamental concepts of number were re-examined. Man awoke to the fact that different number bases could be used, and Simon Stevin stated in 1585 that the duodecimal base was to be preferred to the decimal.

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The new Arabic notation accomodated mathematical statement better, and facilitated ideation. All thinking accelerated when released from the drag of the cumbrous Roman notation.

The parallel seems tenable. The notation of the dozen base accomodates mathematical statement better, and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base embodies a concurrent analysis and definition of numbers that stimulates classification and generalization. Yet this is accomplished by such simple means that students in the primary grades easily learn to perform computations in duodecimals, and can tell why they are better. Literally, the decimal base is unsatis-factory because it has "not-enough-factors."

Then, shouldn't we change? No! No change should be made, and we urge no change. All the world uses decimals. But people of understanding should learn to use duodecimals to facilitate their thinking, and to ease the valuative processes of their minds. Duodecimals should be man's second mathematical language. They should be taught in all the schools. In any operation, that base should be used which is most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity. But no change should be made. Perhaps by the year 2000, or maybe by *1200*, which is *14* years later, duodecimals may be the more popular base. But then no change need be made, because people will already be using the better base.

When one is familiar with duodecimals, a number of accessory advantages become apparent. Percentage is a very useful tool, but many percentages come out in awkward figures because of the inflexibility of decimals. When based on the gross, twice as many ratios come out in even figures, and among them are some of those most used, as thirds, sixths, and twelfths, -eighths and sixteenths. There are advantages associated with time and the calendar. Monthly interest rates or charges are derived from annual rates, or the reverse, by simply moving the unit (decimal?) point. The price of a single item bears the same relation to the price of the dozen, and so does the inch to the foot.

The proper correlation of weights and measures has always been one of the world's serious problems. None of the present systems is completely satisfactory. The American and English standards are convenient to use since they are the final result of a long process of practical evolution in which many inconvenient measures have been adjusted or abandoned. The French decimal metric measures have the advantage of being set upon the same base as the number system, and are well systemized. But many of the units are awkward because of their arbitrary sizes, and because their decimal scale does not accomodate division into thirds and fourths readily.

The duodecimal system of weights and measures, based on the inch and yard, the pint and the pound, has the desirable elements of both systems, and few of their faults. This Do-Metric System retains the familiar units of the American and British standards in approximately their present size, and arranges them into an ordered metric system using the scale of twelve. This fits perfectly into the duodecimal notation, and the combination accomodates the inclusion of the units of time and of angular measure within the system, which hitherto has not been possible.

If "playing with numbers" has sometimes facinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have thought staid and established, and without new trails, then, whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is cordially invited.

The Duodecimal Bulletin

All figures in italics are duodecimal.

FINE MEASUREMENT BY MONOCHROMATIC LIGHT

by Ralph H. Beard

The visible portion of the light spectrum extends from the red to the violet, with respective wave lengths of about .0008 to .0004 millimeters, respectively. If we divide a beam of light by reflecting it from two glass plates, and bring these rays together again after they have traveled different distances, they may either neutralize or reinforce each other, depending upon the relative phases of their waves. If the beam is composed of light of a single color, the interference phenomena may take the form of alternate light and dark interference bands.

In his interferometer, Dr. A. A. Michelson used this effect to measure the difference in the lengths of the paths of the rays in wave lengths of monochromatic light by counting these interference bands. Michelson determined the length of the standard meter to be 1 553 164.13 wave lengths of the red ray (6438 A) of the cadmium spectrum, with an accuracy of one part in 20 million.

In a curious piece of bureaucratic legerdemain, the 1893 order of T. C. Mendenhall, Superintendent of the Coast and Geodetic Survey, officially defined the United States inch as being 100/3937 of the much younger meter. The length of the official U. S. inch is thus 39 450.33 wave lengths of the cadmium red ray.

Because it affords a convenient means for the comparison and reprocuction of length standards, and because of its greater freedom from variation, the practice of defining standards of length in terms of the cadmium red wave length has supplanted the older bar standards. Under the sponsorship of the American Standards Association, and of Dr. Lyman J. Briggs of the U. S. Bureau of Standards, the use of the ratio of 25.4 millimeters to the inch has become accepted manufacturing practice.

The past year has seen two revolutionary developments in this field of fine measurement. Mercury has long been under inquiry as a possible light source for this purpose. But mercury normally includes six isotopes, the finest source of monochromatic light among them being Mercury 198 which constitutes ten percent of the total, and is very difficult to separate. In April of this year, however, Drs. Wiens and Alvarez of the University of California bombarded gold in the cyclotron and secured Mercury 198 in such purity that only one atom in a million is another mercury isotope.

COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	X	Ɛ	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	136	Five ft. nine in.	5.9'
31	694	Three ft. two in.	3.2'
96	3Ɛ2	Two ft. eight in.	2.8'
19Ɛ	1000	Eleven ft. seven in.	Ɛ.7'

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2Ɛ*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
12 ) 30  + 5
12 ) 2   + 6
     0   + 2     Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *X*, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *X*.

Numerical Progression

1	One		
10	Do	.1	Edo
100	Gro	.01	Egro
1,000	Mo	.001	Emo
10,000	Do-mo	.000,1	Edo-mo
100,000	Gro-mo	.000,01	Egro-mo
1,000,000	Bi-mo	.000,001	Ebi-mo
1,000,000,000	Tri-mo	and so on.	

Multiplication Table

1	2	3	4	5	6	7	8	9	X	Ɛ
2	4	6	8	X	10	12	14	16	18	1X
3	6	9	10	13	16	19	20	23	26	29
4	8	10	14	18	20	24	28	30	34	38
5	X	13	18	21	26	2Ɛ	34	39	42	47
6	10	16	20	26	30	36	40	46	50	56
7	12	19	24	2Ɛ	36	41	48	53	5X	65
8	14	20	28	34	40	48	54	60	68	74
9	16	23	30	39	46	53	60	69	76	83
X	18	26	34	42	50	5X	68	76	84	92
Ɛ	1X	29	38	47	56	65	74	83	92	X1

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THE DUODECIMAL SOCIETY OF AMERICA

20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y.

The U. S. Bureau of Standards and the Eastman Kodak Co. have been co-operating in the development of a quartz tube for interferometer use, employing the vapor of Mercury 198, which produces a fine green spectroscopic line of a wave length of 5461 Angstroms, or *1.9495* cads.

The sharp wave form of this green ray is ideal for measurement purposes. It is estimated that it will be possible through this means to achieve measurements of a fineness of one billionth of a centimeter.

For the measurement of certain types of atomic radiation in nuclear operations, this standard is still not fine enough. The scientific world was startled this last June, by the announcement before a meeting of the American Physical Society of a new Gamma Ray Spectrometer, developed by a team of physicists at the California Institute of Technology, consisting of Drs. Du Mond, Watson and Lind.

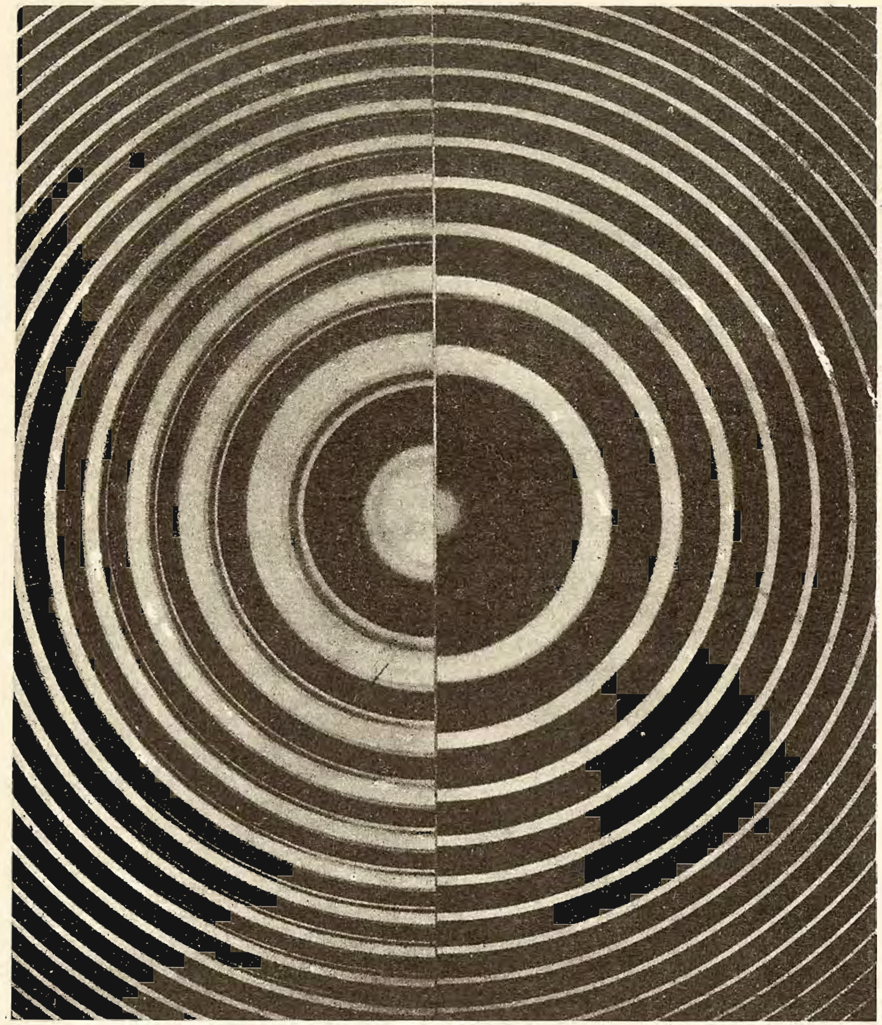
Gamma rays are measured in terms of X-units, which are one-thousandth of an Angstrom. The wave lengths of gamma rays vary between 8 and 500 X-units, and the new spectrometer will measure these rays with a precision within five thousandths of an X-unit.

In order to secure a clearer idea of the relative sizes of these fine linear standards, comparative scales, with the location of these units, are given.

	Decimeter	Centimeter	Millimeter			Micron			Millimicron	Angstrom			X-unit
Meter	.0	0	0	0	0	0	0	0	0	0	0	0	1
			(1.7)			(3.0)			(5.2)			(8.9)	

	Palm	Quan	Karl			Cad			Emocad			Ebimocad
Yard	*.0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*0*	*1*

While the above provides a notational comparison, the relative quantities of the decimal place system must be divided by successive powers of 1.728 for each group of three figures to obtain their comparable size in the duodecimal statement, as shown. Thus, the emocad is about 1¾ Angstroms, and the ebimocad and the X-unit are nearly equal. Exact values are as follows:



NATURAL AND ARTIFICIAL MERCURY, 5461 A

Circular interference fringes, formed by Fabry-Perot Interferometer illuminated with green light (5461 A) from electrodeless discharge mercury lamp: (left) 7 isotopes of natural mercury; (right) A single isotope, Mercury198, made by transmuting gold. Latter provides a standard of length superior to any in nature, with potential accuracy of one part in a billion. Photographed by William F. Meggers, Chief, Spectroscopy Section, National Bureau of Standards.

		Duodecimal	Decimal	
Emocad	=	*1.9324*	1.7722	Angstroms
Ebimocad	=	*1.0382*	1.0256	X-units
X-unit	=	*.Ɛ84Ɛ*	.9571	Ebimocads
Angstrom	=	*.6931*	.5643	Emocads

One of the important units in present day physical chemistry is the length of the carbon bond as found in the linkage of the molecules of the diamond. This was measured with extreme accuracy by Dr. D. P. Riley at the Cavendish Laboratory, Cambridge, as 1.5445 Angstroms. The do-metric equivalent is *1.1929* (or 1.1474) emocads.

ANNUAL MEETING

The Society's Fifth Annual Meeting was held on Thursday, January 27th, 1949, at the Gramercy Park Hotel, Lexington Avenue and 21st Street, New York City.

The meeting was called to order by President Andrews at 8:30 P.M. After the transaction of routine business, including the election of new directors, the three principal speakers were introduced.

Mr. Velizar Godjevatz spoke on "The New Musical Notation," developing the musical theories presented in the October Bulletin. Mr. Kingsland Camp spoke on "The Duodecimal Slide Rule"; some of his remarks on that subject appear elsewhere in this issue. Mr. Frederick Condit spoke on "The Duodecimal System" with particular relation to rhythmic and psychological factors.

The meeting was then thrown open for general discussion, and refreshments were served. Although officially adjourned at 10:30, it continued in informal discussion groups until well past midnight.

A more complete report, together with Committee appointments and other official business, will be carried in the next Bulletin.

ELEMENTS OF PROPOSED DUODECIMAL SLIDE RULE

by H. K. Humphrey

The thoughtful consideration which Mr. Camp has put into his proposal for the duodecimal slide rule is perfectly obvious, and I shall not spend any time on the phases of the proposal on which we agree, except to offer sincere congratulations on a job well done. Instead, attention will be given to the few ideas of mine which differ from his.

Before commenting on Mr. Camp's proposal, I should first convey some idea of the degree of my familiarity with the operation and characteristics of slide rules. I first used a slide rule in about 1903, bought my first one in 1907, and have used them with considerable regularity ever since. The rules used include all the popular types, and the work done on them has been of more than usually varied character. The most intense use was a period of some three months during which my whole activity, eight hours a day, was slide rule computation. But there have been very few weeks in all these years in which some slide rule computation was not made, including this morning - in a bank!

There are four points in Mr. Camp's proposal which I think I might want to change. But, before discussing them, let me say that I do want one of these rules, when, as, and if produced, at any price that can conceivably be considered reasonable - and I am well aware that they cannot be produced in a quantity which could take advantage of mass production economies.

The idea of a slide rule with the division line between slide and frame bare of scales is startling, and had never occurred to me - I am a little ashamed of myself at this point. The advantage of placing each scale right along the same line as its reciprocal is so great that it must receive very serious consideration before it can be discarded. In fact, if there is any possible way to secure this advantage and still keep the C and D scales along one division, and the A and B scales along another, it should be done. But after serious thought, I believe that I would rather sacrifice the juxtaposition of scale and reciprocal than to see the scales named taken away from the positions which they have so long held.

This is not mere clinging to tradition, or rationalizing a habit. It is based fundamentally upon a disagreement with Mr. Camp's statement: "We habitually use the indicator in setting the slide and reading our answers anyway." Many good operators, including myself (blushes) avoid the use of the cross hair whenever it can be done. In my own case, this was studiously developed for two reasons, the gain of speed, and even more important, the gain of accuracy. Whenever either of the numbers to be set together is represented by a line on its scale, and when the number against which the answer is to be read is represented by a line, it is more accurate to avoid the cross hair and use the line. Further, when line is to be set against line, it is more accurate to avoid the cross hair. In addition, time is saved.

Lest there be some doubt, try setting two lines under the cross hair, then move it away and examine the setting critically; then set the cross hair at some value in a space, away from any line, and set against it a line on the other scale, again removing the cross hair and subjecting the setting to close scrutiny. Admitted, I have not done this for a long time, but I did it enough times long ago to convince myself that it was worth while to learn to use the rule without the cross hair. Now I find myself frequently moving the runner out of the way, and use the cross hair in only two cases: (1) setting together two values, neither of which is represented by a line, and (2) using a result in a continued computation without actually reading its amount. After careful review in the last few weeks, I am still convinced that the method which I taught myself has value, and I am naturally loath to see its use made impossible.

Elimination of A or B scale

The rule which I have used most, is an old - until recently I thought, an original - Polyphase Duplex which has an A scale but no B scale. For very many computations, this lack is no handicap, and for some which require B but not A, the lack can be compensated by reversing the function of the scales. But my own work furnished enough examples of the need for both in the same computation that I have kept this rule's predecessor convenient to hand all these 35 years! And while it can hardly be said that its use is frequent, it is enough to justify having it handy when needed, or at least, wanted, for the time saving it gives.

Folding at π or at $\sqrt{10}$

While we are considering the slide rule, beginning with first principles, let us inquire the real purpose of the folded scale. It is to prevent answers from "coming off the scale." It has no other use. There is the *talking point* that multiplication or division by π may be done without setting the slide. But as Mr. Camp points out, (p. 8) this can all too easily introduce the error $\pi^2/10$, which is too small to be discovered, and so will slip through. No teacher will say this does not happen! I even had a text book in which it appeared, all nicely printed, in a

worked example. So, for all my familiarity with my slide rule, I will not multiply by π using the *talking point* scale.

Further, folding at $\sqrt{10}$ automatically folds also at the central index of the A and B scales, so that for these the real advantage, that answers can not come off the scale, applies as well as for the full length scales. Not so, of course, when the fold comes at π. So, while I practically never have to "reverse the slide" when using the full length scales (and then only when I go to sleep), I frequently have to do that when using the A and B scales. Years ago I urged Keuffel and Esser to make this correction in their Polyphase Duplex rules, but the selling point of multiplication by π was too strong. Since we need no selling points, I strongly urge folding at $\sqrt{10}$.

Magnifying Indicators

Mr. Camp has put his finger upon one of the serious troubles with magnifying indicators, the disappearance of a nearby part of the scales, which can lead to real trouble, and his scheme for avoiding it is clever, indeed. Nevertheless, I would prefer to have the plain runner. First, it has never seemed useful to me to magnify a scale on which errors can be detected with the naked eye, and this includes practically all slide rules. I am reminded that I picked out my third rule (about 1910) by spending an afternoon examining slightly over 100 rules. All but three were thrown out without recourse to a lens, and a small eye loupe disclosed errors in these. Second, if a lens is ever wanted, the ordinary jeweler's loupe is so cheap and so convenient; rough setting can be done with the naked eye, and the loupe can be used for the final adjustment and reading. And above all, when it isn't wanted, it is not in the way.

This all sounds like carping criticism, to which a fine job such as Mr. Camp has done should not be subjected. I do not mean it so, but time to smooth out this presentation of my points does not exist. Nor can I take longer to say what I should like about those ideas of Mr. Camp's that are so right: the discussion of scales and marking; the extension of all scales to a numerable point, even though it may not be possible actually to set to that point; the discussion of subdivisions; the convention for the representation of final and initial *0*'s and of initial *9*'s and *Ɛ*'s (which may be used whether or not one is referring to the slide rule); the insistence that the useful length be exactly one foot. With all these I enthusiastically agree.

On the last, one comment is to be made. I had a student, once, who covered up his lack of a scale by using what he called the "slide rule inch" - actually, of course, 2.50 cm., taken from the log scale of his so-called ten-inch slide rule.

DUODECIMAL SLIDE RULE - FURTHER REMARKS

by Kingsland Camp

The importance of Mr. Humphrey's letter is that it sets forth a thoughtful point of view; before we make our decision and undertake the expense of actually constructing an instrument, we should have as many such views as possible to weigh and choose between. All readers are urged to submit any views and suggestions of their own also, and preferably soon. Our Society wants to construct the best duodecimal slide rule possible for the present era.

It will be helpful to define in advance the proper nature of a slide rule: is it primarily an instrument for precision, or is it primarily for obtaining approximate results of as great variety, with as much promptness and confidence as possible? I myself would lean strongly to the latter definition. Precision is desirable, but a high degree of it is not possible in any event and the other qualities are chiefly sought.

Now Mr. Humphrey's first point of difference with my suggested arrangement brings out the fact that some users, possibly many of them, work chiefly with numbers exactly represented by scale markings. It would be a helpful bit of information, to know what is the proportion of such users. My own belief - and I may be mistaken of course - is that the great majority of us have far more frequent occasion to estimate between lines, so that we fall into the habit (as remarked in my paper and quoted by Mr. Humphrey) of using the indicator all the time even though avoiding it would be more accurate for those quantities that happen to be exactly represented on the scales. If the inaccuracy introduced by the indicator on those occasions is no worse than is involved anyhow when interpolating, then the advantage of the B and C scales on the slide bearing respectively against the A and D scales on the frame, is altogether inconsiderable: discarding this feature would not affect the degree of accuracy ordinarily attained.

It would be particularly helpful to have the views of other members of our Society on Mr. Humphrey's second point: is it important or desirable to have a two-cycle (A) scale on the frame of the instrument, as well as a similar (B) scale on the slide, which is found on my proposed model? Working usefulness is of course the important thing: for simple squares or square roots, the loglog scales serve for a wide range of numbers, while the separation of the marks for a given quantity on the paired scales represents the square on a full-length scale basis, as remarked on page *8* of my paper. If the A scale is really needed, we can probably make room for it by taking out the DIF (folded reciprocal) scale on the frame of my proposed model.

But is it not better to regard a two-cycle scale in the same light with other special scales, such as those for sines and tangents? Looked at this way, its chief proper function is that of enabling us to set the indicator quickly to the proper point of a full-length scale for the square root of an assigned quantity; thereafter, as should be standard practice, to carry out actual calculations on the full-length scales. And if, as in my proposed model, the two-cycle B scale on the slide is paired with a reciprocal (inverted) scale, the square root of a product is represented by the separation between the marks on the B scale and on its inversion, and the square root of a quotient by the separation between the marks on either the B scale or its inversion. Either way, the need for an A scale is not very apparent; omitting it affords room for a peculiarly versatile and powerful set of standard and folded scales, all with inversions, and all of full length, on both the slide and the frame.

My model did include one scale, however, that my own reflection suggests is of very doubtful value; the inverted (reciprocal) side of the proposed decimal B scale on the slide. This is useless in connection with duodecimal calculations and is particularly convenient only for the very infrequent problem of furnishing a series of roots of a given decimal quantity to which the slide is set. A more useful substitute might be either a K (cubes) scale or else an auxiliary full-length sine scale. The latter, with the aid of the trigonometric scales already on the frame, would be helpful with simpler problems in spherical trigonometry.

Mr. Humphrey's suggestion that scales be folded at $\sqrt{10}$, I would like to submit to engineers; in one or more of their fields it may well be that folding at π is much more than a "talking point."

Perhaps my paper should not have touched upon the subject of magnifying indicators, as I agree with Mr. Humphrey that users with ordinarily good eyesight hardly need them. I was not thinking of proposing a magnifier as standard equipment; it was impossible to refrain from suggesting what seemed a better design for them.

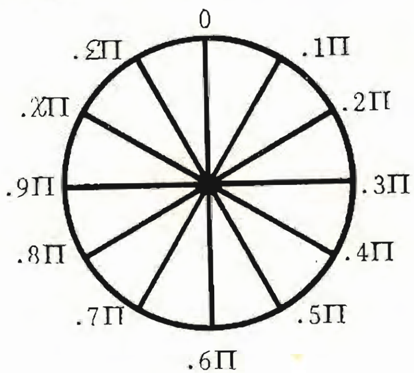
In conclusion, I want to thank Mr. Humphrey heartily for the points he has raised so clearly. It is especially pleasant to have his assurance that we don't need "selling points," and his testimony on the ridiculous "slide rule inch," and the actual published misuse of the CIF scale, nicely printed right in a textbook! *If* we decide to fold at π, I hope my plan of pairing with the CF scale will eliminate such blunders!

A BETTER RATIO FOR PI

by Dallas H. Lien

That the value of Pi should be fixed by law to the round number 3 was once proposed by a member of a State Legislature. Needless to say, the bill was defeated.

I wouldn't propose such a radical change as that, but it would be better if π were defined differently: namely, as the ratio of the circumference to the radius. For the time I will refer to this irrational ratio as Π, the Greek letter pi major. Its value is 2π, and it represents the number of radians in a circle. Therefore when it is multiplied by the angle of a duodecimal circle, the result is the angle in radians.



Angle in Radians

$$\Pi = \frac{\text{Circumference}}{\text{Radius}} = 2\pi$$

Duodecimal angles would have a close relationship to those of radian measure.

(Mr. Lien might call this transcendental, "Kappi". - Ed.)

SQUARE SUMS OF N CONSECUTIVE SQUARES WHEN N IS NOT SQUARE

by Harry C. Robert, Jr.

In the October 1947 Duodecimal Bulletin, (Vol. 3, No. 3, page 17) a complete solution of the problem of square sums of N consecutive squares for the case of N, a square, was presented. When N contains non-square factors the problem becomes more difficult. The problem of the sum of N consecutive squares equal to a square may be stated -

$$\text{Sum} = N\left[a^2 + \frac{N^2 - 1}{10}\right] = b^2 \qquad \text{Eq. } (1)$$

where (a) is the root of the mid-square if N is odd or is the mean of the roots of the two mid-squares if N is even. By assuming values for N, Eq. *(1)* may be reduced to the form -

$$c^2 - De^2 = \pm H \qquad \text{Eq. } (2)$$

where (c) and (e) are unknown integers, one of which is a function of (a) in Eq. *(1)* and the other is a function of (b). H and D are known integers with D having no square factors for the case of non-square N now under consideration. If N is even, (a) in Eq. *(1)* is not integral, so for this case, in reducing Eq. *(1)* to Eq. *(2)*, either (c) or (e) will be equal to (2a), an odd integer.

The complete solution of Eq. *(2)* by Lagrange is considered one of his outstanding contributions to Number Theory. Space will not permit a presentation of that solution here. Chrystal's Algebra, Book II, a reference available in most libraries, gives a clear explanation of the application of Lagrange's method which those who are interested in the complete theory may supplement with Dickson's History of the Theory of Numbers, Vol. II. However, where the size of the initial solution is moderate, certain very elementary methods may be used.

The necessary and sufficient conditions for Eq. *(2)* to have integral solutions are too complicated to present here. For this particular problem it has been found that simpler conditions, while not entirely sufficient, are quite effective. After screening possible values of N to eliminate those that will not produce square endings for the Eq. *(1)* expression for the sum and also eliminating those for which the quantity within the brackets is not divisible by the non-square factors of N, as explained in the previous paper, the use of a single additional condition is desirable. For Eq. *(2)* to have *rational* solutions it is necessary that D be a quadratic residue of the non-square factors of H. The effectiveness of these conditions is shown by the fact that only one value of N less than 600 meets them without producing integral solutions. This value of N is 5X2 which has rational fractional solutions but no integral ones.

Eq. *(2)* with non-square D, if it has any integral solutions, has an infinite number of such solutions. These solutions occur in cycles with one or more sets of solutions, each set being represented by one solution in each cycle. The maximum number of sets of such solutions is a function of the factors of H. There appears to be no relation between the different sets of solutions, but the infinite number of solutions in any one set are related to each other by various recurrence forms, all of which may be derived from the smallest positive integral solutions of the "Pellian" equation -

$$P^2 - DQ^2 = 1 \qquad \text{Eq. } (3)$$

which has integral solutions for all non-square values of D. These solutions may be found most easily by developing the continued fraction for $\sqrt{D}$. (See Note 1.) If P_o and Q_o are the "fundamental" solution, that is, they are the smallest positive integers satisfying Eq. *(3)*, the recurrence forms are -

$$P_k = \frac{1}{2}\left[(P_o + Q_o\sqrt{D})^k + (P_o - Q_o\sqrt{D})^k\right]$$

$$Q_k = \frac{1}{2\sqrt{D}}\left[(P_o + Q_o\sqrt{D})^k - (P_o - Q_o\sqrt{D})^k\right]$$

where (k) is any integer. The fact that P_k and Q_k are also solutions of Eq. *(3)* and that this is true for all values of (k) may be shown by substituting these values into Eq. *(3)* -

$$\frac{1}{4}\left[(P_o + Q_o\sqrt{D})^k + (P_o - Q_o\sqrt{D})^k\right]^2$$

$$-\frac{D}{4D}\left[(P_o + Q_o\sqrt{D})^k - (P_o - Q_o\sqrt{D})^k\right]^2 = 1$$

$$\frac{1}{4}\Big[(P_o + Q_o\sqrt{D})^{2k} + 2(P_o + Q_o\sqrt{D})^k (P_o - Q_o\sqrt{D})^k$$

$$+ (P_o - Q_o\sqrt{D})^{2k} - (P_o + Q_o\sqrt{D})^{2k}$$

$$+ 2(P_o + Q_o\sqrt{D})^k (P_o - Q_o\sqrt{D})^k - (P_o - Q_o\sqrt{D})^{2k}\Big] = 1$$

$$(P_o + Q_o\sqrt{D})^k (P_o - Q_o\sqrt{D})^k = 1$$

$$\left[(P_o + Q_o\sqrt{D})(P_o - Q_o\sqrt{D})\right]^k = 1$$

$$(P_o^2 - DQ_o^2)^k = 1 \quad \text{and since} \quad P_o^2 - DQ_o^2 = 1, \ (1)^k = 1$$

thus all values of P_k and Q_k are solutions and also there are an infinite number of solutions. (See Note 2.)

If (c_o) and (e_o) are any solution of Eq. *(2)*, then all other solutions belonging to the same set are found by -

$$c_k = P_k c_o \pm Q_k D e_o$$

$$e_k = Q_k c_o \pm P_k e_o$$

with (k) extending over all integers. A simpler recurrence may be used when any three consecutive solutions of either unknown are found in any set -

$$c_{k+2} = 2P_o c_k - c_{k-2}$$

$$e_{k+2} = 2P_o e_k - e_{k-2}$$

To find solutions of Eq. *(2)* therefore it is necessary only to find the smallest solution in each set, find the smallest solution of Eq. *(3)* and apply the recurrence forms. To find the smallest solutions in each set for Eq. *(2)* it is necessary to check possible values of the unknowns within the limits -

When H is positive:
$$0 \leqq e_o \leqq \sqrt{\frac{H(P_o - 1)}{2D}}$$

$$\sqrt{H} \leqq c_o \leqq \sqrt{\frac{H(P_o + 1)}{2}}$$

When H is negative:

$$\sqrt{\frac{H}{D}} \leq e_o \leq \sqrt{\frac{H(P_o + 1)}{2D}}$$

$$0 \leq c_o \leq \sqrt{\frac{H(P_o - 1)}{2}}$$

As an example, consider the case of N = 42

$$\text{Sum} = 42\left[a^2 + \frac{42^2 - 1}{10}\right] = b^2 \quad \text{and placing} \quad b = \frac{5c}{2} \quad \text{we obtain -}$$

$c^2 - 2(2a)^2 = \text{Ɛ6X}$ and since all of our conditions are met, we proceed by solving Eq. (3) for D = 2, $P_o = 3$ and $Q_o = 2$, being the fundamental solution of $P^2 - 2Q^2 = 1$. Now the upper limit of $(2a)_o$ to which we must check is, $\sqrt{\frac{H(P_o - 1}{2D}} = \sqrt{\frac{\text{Ɛ6X}\cdot 2}{2\cdot 2}} = \sqrt{595} < 25$

and since $(2a)_o$ must be odd, it is only necessary to check the odd numbers through 23. This is easily done with a difference table -

$(2a)_o$	c_o^2		$(2a)_o$	c_o^2	
1	Ɛ70		13	1284	
	14			X8	
3	Ɛ84		15	1370	
	28			100	
5	ƐƐ0		17	1470	
	40			114	
7	1030	$= 36^2$	19	1584	
	54			128	
9	1084		1Ɛ	16Ɛ0	
	68			140	
Ɛ	1130		21	1830	$= 46^2$
	80			154	
11	11Ɛ0		23	1984	
	94				
13	1284	$= 3\text{X}^2$			

thus we have three solutions within our limits, each being the smallest in its set. Calling the set for which $(2a)_o = 7$ is the smallest solution, Set I., we may apply the recurrence form,

$$(2a)_{1,2} = Q_o c_o \pm P_o (2a)_o = 2\cdot 36 \pm 3\cdot 7 = 53 \text{ and } 89$$

Thus the first three Set I solutions are $(2a)_{o,1,2} = 7$, 53 and 89 and using the second recurrence form,

$$(2a)_4 = 2P_o(89) - 7 = 6\cdot 89 - 7 = 43\text{Ɛ}$$

and $(2a)_6 = 6\cdot 43\text{Ɛ} - 89 = 2129$, etc., to any desired limit. And since the next solution smaller than (+7) is (-7) we may extend the alternate solutions in this set by -

$$(2a)_3 = 6\cdot 53 - (-7) = 276 + 7 = 281$$

and $$(2a)_5 = 6\cdot 281 - 53 = 1373, \text{ etc.}$$

For a second example, consider N = 61

$$\text{Sum} = 61\left[a^2 + \frac{61^2 - 1}{10}\right] = b^2 \quad \text{and placing } b = 61c, \text{ we obtain -}$$

$a^2 - 61c^2 = -310$ so with D = 61, $P^2 - 61Q^2 = 1$ has the fundamental solution: $P_o = 920201$ and $Q_o = 10\text{X}620$ and the limits to be checked are: $3 \leq c_o \leq 1635$ and $0 \leq a_o \leq 11034$

To check these values with the methods used for N = 42 is a sizable undertaking. For such cases, the number of values of the unknowns to be checked may be greatly reduced by the methods given in the previous paper (October 1947 Duodecimal Bulletin) where for the case of N = 61 it was shown that (a) must be of the two forms

$$(610k \pm 8\text{X}) \quad \text{or} \quad (610k \pm 33\text{X}).$$

In some cases the foregoing methods will not serve, as for example, the case of N = 321.

$$\text{Sum} = 321\left[a^2 + \frac{321^2 - 1}{10}\right] = b^2 \quad \text{and placing} \quad b = 321c, \text{ we}$$

obtain - $$a^2 - 321c^2 = -\text{X0X4} \qquad \text{Eq. (4)}$$

with D = 321, the smallest solution of $P^2 - 321Q^2 = 1$, is -

$$P_o = 19\text{X}\ 36\text{X}963\ 505726\ 880001$$
$$Q_o = 10\ 32\text{X}217\ 3\text{XX}7\text{X}2\ 54\text{X}400$$

which leads to the following values of the upper limits to be checked - upper limit $(c)_o$ = 5X675 000000 (approx.)
upper limit $(a)_o$ = X36700 000000 (approx.)

and although there are an infinite number of solutions to this problem, there are at most only four values of (a) smaller than the 10 digit upper limit which will satisfy Eq. (4). Obviously the methods used for N = 42 or N = 61 are entirely inadequate for this case. Using the Lagrange reduction chain, two of the possible maximum of four fundamental solutions for N = 321 have been found -

$$a_o = 27\ 873856 \qquad a_o = 452336\ \text{X}30696$$

using the recurrence form on the second of these values gives -

a_1 = 2 0X00X3 X52126

a_2 = 141 930367 464636 79X427 370492 2Ɛ3296

and a_4 in this set has 45 figures.

Unquestionably the case of N = 321 is an ideal problem for those who have a special fondness for magnificent numbers.

This paper concludes the report on the first phase of the investigation of the problem of square sums of N consecutive squares. This phase has consisted of the employment of a number of devices, some of them quite elementary in character, for the determination of values of N for which the problem is possible and the finding of one or more solutions for every possible value of N not greater than 600. As finally presented some of the approach to the problem and some of the devices used appear to be an advance over the methods originally used by George S. Terry in his preliminary investigation. Such improvements however, in large measure, perhaps entirely, have been possible because Mr. Terry's preliminary work accomplished most of the necessary eliminations and also provided one or more solutions for all but a very few of the values of N up to 600.

Dozenal base arithmetic has been used throughout the investigation by both Mr. Terry and the writer. Its superiority over base ten for such problems has even exceeded our expectations. The principal feature of this superiority is the helpful simplicity of the square endings in the dozen system, which feature will hold for any other problem which requires the solution of numerous quadratic congruences. The simpler rules of factorisation have also been very helpful.

Whether or not this investigation has accomplished more than to demonstrate the advantages of using the dozenal base in calculating the solutions of certain types of problems is yet to be determined. Out of the work there has developed a general solution of the more general case of this problem, that is, the sum of the squares of N consecutive terms of an Arithmetical Progression equal to a square. With this development, the suggestion in a previous paper that N does not contain odd powers of factors of the form, $(10k \pm 5)$, can readily be proven. Also from the general solution there can be derived an infinite number of forms for N for any special case, as for example, in the case now being considered, that of consecutive squares, $N = 3n^2 - 1$, has solutions for every integral value of (n). Another development is a special case solution for values of $N = 2k^2$ and $N = 6k^2$ whereby the recurrences appear to be derived without use of the Pellian equation, $P^2 - DQ^2 = 1$, which seems to offer a new and interesting approach to certain problems although further investigation may prove it to be of trivial importance. In addition to these two items several other interesting and enticing leads for future investigation have been encountered.

In conclusion some of the solutions for N not greater than 600 are presented: (See Note 3.)

1. Complete solution for square sums of N consecutive squares where $N = 20k^2$ and N not greater than 600.

Values of $(2a)$, $(2a)$ being the sum of the two mid-terms

N = 20	0 cycle	1	7	13	
	1 cycle	53	35	21	
N = 80	0 cycle	15	29	33	3Ɛ
	1 cycle	147	107	Ɛ5	X1
N = 420	0 cycle	25Ɛ	295		
	1 cycle	461	413		
N = 600	0 cycle	7	35	301	33Ɛ
	1 cycle	1489	1373	753	6X9

All other solutions can be obtained from the form:

$$(2a)_{k+2} = X(2a)_k - (2a)_{k-2}$$

and the above lists give the necessary four cycles since the -0 and -1 cycles have the same values as the corresponding positive numbered cycles but with negative signs.

Thus for N = 20

-1 cycle	$(2a)_{-1}$	= -53
-0 cycle	$(2a)_{-0}$	= -1
0 cycle	$(2a)_0$	= +1
1 cycle	$(2a)_1$	= +53

so $(2a)_2 = 61$, $(2a)_3 = 447$, $(2a)_4 = 509$, etc.

Similarly for N = 80, N = 420 and N = 600.

2. Values of $(2a)$ for other cases of even N.

N = 220	0 cycle	245	283	
	1 cycle	144Ɛ	1279	
N = 2	0 cycle	1		
	1 cycle	7		
N = 22	0 cycle	3		
	1 cycle	63		
N = 42	0 cycle	7	13	21
	1 cycle	53	3Ɛ	29

N = 62	0 cycle	5	377	471	
	1 cycle	?	388Ɛ	2Ɛ51	
N = X2	0 cycle	83	165	1X3	
	1 cycle	31Ɛ7	1659	134Ɛ	
N = 102	0 cycle	7			
	other solutions 6615 and 7827				
N = 142	0 cycle	25Ɛ	2X9		
	1 cycle	1Ɛ13	1801		
N = 162	0 cycle	24Ɛ	857	985	3897
N = 182	0 cycle	9			
	other solutions: 269; 2Ɛ3; 1353; 1569				
N = 242	0 cycle	35	53		
	other solutions: 287; 309; 493; 541; 1489				
N = 262	0 cycle	Ɛ	236Ɛ		
N = 322	0 cycle	1623	17Ɛ9		
N = 362	0 cycle	11	2Ɛ7		
	other solutions: 2425; 7337; 7Ɛ51				
N = 3X2	0 cycle	423	743	805	1534Ɛ
N = 402	0 cycle	X1	107		
	other solutions: 407; 455; 95Ɛ; X41; 2141; 2367				
N = 482	0 cycle	13			
	1 cycle	102313			
N = 4X2	0 cycle	939	X0Ɛ	X579	
N = 502	0 cycle	15	5Ɛ	25Ɛ	295
	other solutions: 69Ɛ; 7X1; 867; 995; 1635				
N = 562	0 cycle	Ɛ7	1911	3177	4852Ɛ

For the following values of N the cycle limits have not been computed:

N = 74 solutions: 333; 389;
N = 134 solutions: 145; 38Ɛ; 1ƐX5; 518Ɛ; 8Ɛ2Ɛ;
N = 254 solutions: 63Ɛ; 67Ɛ; Ɛ81; 1487; 2515; 2675;
N = 274 solutions: 239; 563; 1403; 1ƐX3; 5823;
N = 3Ɛ4 solutions: X11; 15Ɛ9; 1859; 3031; 176Ɛ1;
N = 5Ɛ4 solutions: 51Ɛ; X575; 18155;

3. Solutions, values of (a) for odd N.

N = 61 a = 33X; 16ƐX;
N = 81 a = 53;

N = 141 a = 52; 40352;
N = 181 a = 2333;
N = 221 a = 1160;
N = 241 a = 303Ɛ;
N = 2X1 a = 24; 35784;
N = 321 a = 27 873856; 452336 X30696; 2 0X00X3 X52126;
N = 341 a = 1127;
N = 401 a = 1Ɛ; 80X21; X765Ɛ;
N = 461 a = 262; 406; 15836;
N = 481 a = 319;
N = 561 a = 118;
N = 29 a = 1; 10; 1Ɛ; 37; 94; 145; 183; 434; 779; 11X9;
N = 129 a = 57; 455; X23; 3951;
N = 189 a = 14; 488; 11Ɛ4;
N = 209 a = 82; 516; 3052;
N = 289 a = 6Ɛ2; 3446;
N = 389 a = 158; 1X79 582590;
N = 529 a = 5Ɛ; 6087; X85Ɛ;
N = Ɛ a = 1; 1Ɛ; 37; 325; 5Ɛ7; 53X5; 9Ɛ01;
N = 1Ɛ a = 16; 24; 624; 956;
N = 3Ɛ a = 2; 3XX; 52X;
N = 4Ɛ a = 43; 57; 7X1; X23;
N = 8Ɛ a = 3; 13733; 16Ɛ43;
N = 13Ɛ a = 4; 27X4; ƐƐ8378; 13ƐƐ3 935X38;
N = 17Ɛ a = 52; 626; 70X;
N = 20Ɛ a = 5; 1Ɛ5; 225; 2931; 3141; 17691; 19Ɛ6Ɛ;
N = 21Ɛ a = 48; 14X8; 12520; 141ƐX;
N = 24Ɛ a = 6799; 74X5;
N = 27Ɛ a = 5X80; 6634;
N = 29Ɛ a = 118; 282; 2Ɛ8; 533X; 22Ɛ42;
N = 2ƐƐ a = 6; 5Ɛ96;
N = 30Ɛ a = 141; 9X2Ɛ;
N = 33Ɛ a = 4918; 5274;
N = 34Ɛ a = 53; 6949; 7511; 156Ɛ7;

N = 38Ɛ *a* = 166; 183; 309; 343; 2993; 3099;

N = 40Ɛ *a* = 7; 87 X26X57; 94 9624ƐƐ;

N = 41Ɛ *a* = 1X4X;

N = 53Ɛ *a* = 8; 2 295488; 2 496488;

Note 1: *Fundamental solutions of the Pellian Equation for certain values of D are as follows:*

$D = d^2 - 2$	$P_o = d^2 - 1$	$Q_o = d$
$D = d^2 - 1$	$P_o = d$	$Q_o = 1$
$D = d^2 + 1$	$P_o = 2d^2 + 1$	$Q_o = 2d$
$D = d^2 + 2$	$P_o = d^2 + 1$	$Q_o = d$

Note 2: *The values of P_k and Q_k are also the only solutions of the Pellian Equation.*

Note 3: *The list of solutions includes one or more solutions for every non-square N not greater than 600. Solutions are listed according to cycles only where cycle limits have been determined. The arrangement of the solutions for $N = 20k^2$ was suggested by George S. Terry and most the the solutions for other values of N are also his work.*

DONATIONS

Members and friends of our Society pay a glowing tribute to the appeal of duodecimals in their contributions of time, services, and money to our work. This year, we have received financial contributions from more of our people than ever before, and the total will exceed $800. Many of us are of moderate means, and this evidence of a desire to help in the support of the Society is deeply appreciated.

In these early years of the Society's history, when our membership is still small, it would not be possible to accomplish all that we have without these gifts. We wish to express the gratitude of the Society to the following donors.

Paul and Camilla Adams
Ralph and Lesbia Beard
Arthur Coldewe
Louis Paul d'Autremont
Paul E. Friedemann
Dudley A. George
Jamison Handy, Jr.
H. K. Humphrey
Robert and Mary Lloyd
Lewis Carl Seelbach
Elaine Swanson
George S. Terry

MATHEMATICAL RECREATIONS

by Donald M. Brown, Editor

Most of us have difficulty in writing all the digits with the same facility. If one of the dozenal digits is troublesome to you, maybe practice is what you need. Multiply the troublesome digit by Ɛ. Then multiply the result by *123456789Ɛ*. You will find that considerable practice in writing the chosen digit is provided by this exercise.

. . . . .

Let $(ab)_c$ denote a two digit number to the base c; let $(pqr)_s$ denote a three digit number to the base s; etc.

Solve the following equations for x, y, and z.

1. $(xy)_{10} + (xy)_{12} = (xy)_{32}$

2. $(xyz)_4 + (xyz)_{\mathcal{E}} = (xyz)_{10}$

. . . . .

If you can produce a really hard problem of this type, send it to us for use in the Recreations section.

. . . . .

Here's another cryptogram contributed by Mary Lloyd.

```
             A R C
P R A Y ) D O U B T I T
          D Y T T A
          ---------
            R A C O I
            I U D I R
            ---------
              C T Y P T
              Y D C H C
              ---------
                O P A Y
```

REVERSIBLE NUMBERS

by Ralph H. Beard

In the recent book, Mathematics, Our Great Heritage, there is quoted from W. W. Rouse Ball's "Mathematical Recreations," the statement that:

> 8712 and 9801 are the only four-figure numbers which are integral multiples of their "reversals."

$$8712 = 4(2178) \qquad 9801 = 9(1089)$$

This calls to mind Mr. Terry's talk at our last Annual Meeting, on Reversible Numbers. For the decimal base, such numbers must have the factors 9 and 11^2. The smallest is the product of these factors, 1089. Similar constructions are available for any number base.

Stated in general terms the requisite factors are (Base - 1), and $(\text{Base} + 1)^2$. Constructions for several bases follow.

Decimal		Duodecimal		Septimal	
1089	9801	*10XƐ*	*ƐX01*	1056	6501
2178	8712	*219X*	*X912*	2145	5412
3267	7623	*3289*	*9823*	3234	4323
4356	6534	*4378*	*8734*		
5445	5445	*5467*	*7645*		
		6556	*6556*		

These numbers are a rather glamorized illustration of the bookkeepers' old trick in tracing differences. If the difference is divisible by 9, a transposition of figures has been involved in the error. The differences between any of the above figures is, of course, divisible by (Base - 1).

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THE MAIL BAG

Lewis Carl Seelbach, Chairman of the Committee on Bibliography, has been having himself a wonderful time, writing to libraries all over the world, asking for a summary of their duodecimal bibliography. His request is phrased in English, and repeated in Esperanto. He has gotten an excellent response, and much valuable information.

So far, the prize response has been a wealth of material from the New York Public Library, indicating a most welcome exercise of individual initiative in independent research by Paul North Rice, Chief of the Reference Department. He cites the books listed under Duodecimals, and then lists a dozen other items "collected from our catalogues, and partly from chance finding."

Among the supplemental items is listed "Mathematische Unterhaltungen and Spiele," by W. Ahrens, Leipzig, 1901, and the comment: "On page 24 it states that Simon Stevin (1584-1620) proposed the replacing of the decimal system by the duodecimal. Cf. his 'l'Arithmetique,' Leiden, 1585, p. 6."

Besides being the earliest reference to duodecimals that we know of, this citation is particularly valuable because Simon Stevin is credited with being the author of the use of the unit (decimal) point. We have long thought that both he and Leonardo Fibonacci must inevitably have considered the use of other bases than ten, and this confirmation is exciting. Now for Leonardo, whose "Liber Abacci," is dated 1202. Also, since Stevin was not alone in proposing the use of the unit point and fractionals, there is the need for scrutiny of the works of his contemporaries.

An orchid to Paul North Rice, and to Lewis Carl Seelbach, with our plea for more of the same.

. . .

We have had an interesting letter from Dallas H. Lien, discussing some of the units of fine measurement in current use in the research laboratories. We hope to prevail on Mr. Lien to give us a paper on this important phase of the weights and measures problem.

. . .

Another treat in store is an article by Louis Paul d'Autremont on The Duodecimal Calendar. Mr. d'Autremont is one of our members, and we were embarrassed to be this long in discovering his extensive work, and interesting publications on this subject. Then we read of an earlier Louis Paul d'Autremont, in "The Queen's Asylum," and wondered whether our calendar had slipped.

. . .

With this number of the Bulletin, Donald M. Brown assumes the chair of our department on Mathematical Recreations, at the urgent request of its former editor, Mary Lloyd. His problems for this issue have a sort of itch that won't let you alone. Goodness knows the sins of time-thieving we constantly commit, but we keep reaching for that pencil.

In welcoming him in his new role, we wish to solicit contributions from everyone, member or not, who finds an exceptionally hard nut to crack. These problems limber up our duodecimal muscles, and keep us mathematically spry. So, make a note of his address (519 Soule Boulevard, Ann Arbor, Mich.) and route your try for the mathematical recreations section straight to him.

. . .

The current discussion of the details of the duodecimal slide rule is important, and exciting as well. Some of the features suggested are splendid innovations. It is becoming apparent that we will have the slide rule this year, and that it will most probably be of the log-log type. As the comments clearly show, however, the details are not finally determined, and it is earnestly desired that each one of us who has any suggestion to offer, or any comment or criticism, should not fail to write us about it. We are pioneering new paths, and we don't want to miss any of the springs or natural resources of the area. Be as informal as you like, but scratch it down and send it in.

. . .

Wot's oppened? We aint gettin' no pomes from nobody!

Ye Ed.

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