The
Duodecimal Bulletin

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THE DUODECIMAL SOCIETY OF AMERICA

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THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve-base arithmetic. The lessons and examinations are free to those whose entrance applications are accepted. Remittance of $6, covering initiation fee ($3) and one year's dues ($3), must accompany applications.

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The Duodecimal Bulletin

All figures in italics are duodecimal.

THE NEW MUSICAL NOTATION

by Velizar Godjevatz

The musical theory stands on the supposition of seven tones of a "natural order," and our musical pedagogy is a mixture of false theology reinforced by military drill. We have tolerated an irrational way of designating the tones that we have worked with for the last 250 years, until now we have arrived at a stage where there are more exceptions than rules. But it was not always so.

In times when accidentals were really exceptions, up to the sixteenth century, everything was clear, and a highly elaborate technique was evolved. The number of musical signs (notes), names, and tones was equal, and I look upon this equality as the preliminary condition for all fruitful undertaking. It is true that even then the semitones were graphically represented as whole tones, and this leads us to the conclusion that music has never possessed a true notation.

But with the advent of the 12-tone tempered system, things became worse. We know that 7 is a prime number, and that 12 is divisible by 2, 3, 4, and 6. It is impossible to adequately express anything of a twelve system in a seven system notation. To cite only a few musical idioms, there is no adequate expression in the whole-tone scale for the much used and abused diminished chord of the seventh, or for the augmented triad, which are simply the division of a region in 6, 4, and 3 equal parts. A notation and a theory which continues to ignore for two and a half centuries the existence of nearly half of the elements of our musical system, the materials of which our musical works are built up, has lost every right of existence.

It should be borne in mind that our present musical notation was conceived for another musical system in use centuries ago. When Werckmeister definitely established our 12-tone equidistant system, and Bach drew the necessary conclusions in publishing his "Well Tempered Clavichord" in 1722, the notation was not changed to accomodate the practical results obtained by embracing this new system.

But this was not the only drawback in music. It did not even follow the general acceptance of Arabic numerals, although Leonardo

Fibonacci had published in 1202 his "Liber Abaci," in which for the first time Arabic (or rather, Hindu) numerals were introduced to the West. We still designate musical intervals in Roman numerals, with the disastrous result that everyone who has anything to do with written music is compelled to adopt an illogical and tiresome way of reckoning.

The error is that we do not have a unit of measure, - although it is quite easy to establish one for an equidistant system, - and that we use ordinal numbers for measuring distances. The Roman numerals have no conception and symbol for zero, and we cannot measure distances to be expressed in numbers without it. Zero not only expresses the absence of units in the position it occupies, but it also, as an ordinal number, indicates the initial point, and denotes a direction in a set of numbers.

Another illogical stock-in-trade that a musician has to deal with, is the inexact graphical representation of intervals on the stave. The graphical distances do not correspond to the intervals, hence the need for using clefs, in order to determine the position of the so-called diatonic half-tones (semitones). Our intuition is constantly challenged by the inexact graphical representation, and the number of unnecessary mental operations in determining the exact position of a tone is so great, that it takes years for even intelligent persons to master this king of cryptography.

It is impossible to reason on false figures. They prevent rationalizations which might shorten our work and perfect our technique. Let us replace these pre-logical trials and errors by a real method. Instead of musical catechisms with ready-made questions and answers (written by persons immune to the reality of facts), let us bring common sense to the knowledge of musical fundamentals, and try to draw the necessary conclusions for all particular cases by ourselves.

To sum up briefly, the problem of musical notation is the most important problem in music today. On its satisfactory solution depends the progress of music, despite the fact that the majority of musicians desperately seek to prove that it is not possible, and not even desirable.

The smallest frequency by which we hear and perceive a sound is 16 vibrations a second, and the greatest is about 16,384. Between these limits lies the Audition Range.

Starting upwards from any sound, producing a series of sounds as close to each other as possible, we arrive at a sound which seems like the initial sound; and continuing, arrive at another like sound, and so on. The same is true in the opposite direction. For the interval which separates two consecutive similar

sounds, I propose the name Homosonance, as more suitable than the term Octave.

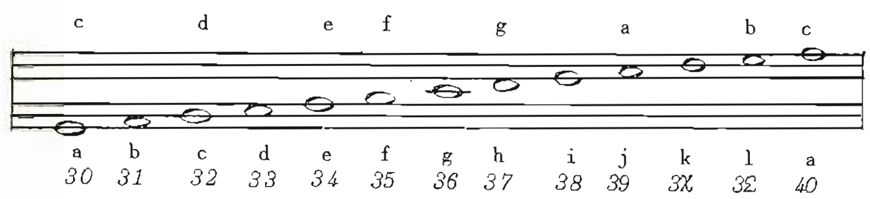
Starting from the lowest sound (16), we can establish its ten homosonances, and we shall call the range between two consecutive homosonances, a Region. The term Region will be used also to locate any tone between these limits; - for instance, as lying in the 3rd Region.

A region can be divided into an even or odd number of intervals, which can be equal or unequal. The System in general use divides the region into a dozen equal (equidistant) intervals. It is interesting to note that our system is not the only equidistant divisional system. The system Slendro from Java has 5 tones, and the Siamese has 7 tones.

The Notation Godjevatz

In our musical system we have twelve tones. They can be written on and between the lines of a stave of seven lines. For ease of reading the central line can be ommitted.

It would be advisable to employ for our dodecaphonic musical system the duodecimal number system. Every tone could easily be designated by one digit only, and this would be its relative pitch. Each region can be designated by a number of two digits, and this would state its exact or Absolute Pitch. The tones for the 3rd Region are written below, with their location on the new stave, their common names, their numbers, and their new names.



The notes on the first and last line (exterior lines) represent homosonances. Staves may be superposed by combining the lower line of the upper stave and the upper line of the lower stave. The common line of the combined staves should be heavier for ease of distinction. For illustration, we show an extract from Chopin's Etude op. 25 no. 9 as it is in the original edition, and as written in the Notation Godjevatz.

The mental burden which fetters every musician nowadays can be relieved, and himself freed from all unnecessary work in order to concentrate on more advanced problems, only by adopting an adequate musical notation. In other words, using a symbolism by which all transitory reasoning could be done almost mechanically, and so this time-saving device be of the highest value to creative work proper.

It is not an exaggeration to say that the solution of certain tasks and problems, as well as the development of new possibilities, depends solely on the logical order of the constitutive elements and the proper choice of the nomenclature.

Our task is to revise all the fundamental ideas of music, and this must be done by the introduction of a new notation. This notation should offer a more perfect tool in the hands of the musician, offering him a wider outlook to new possibilities, for it is a time-saving device which eliminates many unnecessary mental and material operations.

At the same time, it should be accessible to every intelligent person, just as the reading and writing of one's mother tongue. Its foundations and general ideas should not be in contradiction to other fields of human knowledge, so that the logic we use for everything else would be equally applicable to music.

The advantages of the proposed notation are:

1. All of the tones of our musical system are enumerated. Their number is twelve.

2. Every tone is designated by its proper name, which cannot be applied to another tone.
3. Every tone is represented by a single graphic sign, which has its proper place on the stave.
4. Every tone is written in the same manner, independently of the region where it belongs.
5. The graphical distances correspond exactly to the intervals.

The adoption of the duodecimal musical notation would create, besides the improved geometrical representation of the musical process, another way of dealing with tones, a sort of musical arithmetic. The mastery of instruments by professionals, as well as amateurs, would be highly augmented, and the learning time would be considerably reduced. This would encourage more people to learn to play an instrument, people who otherwise would not expend the time and effort required to master a good reading-knowledge of music, a condition of prime necessity.

Let me add that recent development of new musical systems of 24, 36, and 48 tones to the region can have a real start only by the adoption of a musical notation on the dozen base, and not seven as it stands now. While a few supplementary signs would be necessary, the duodecimal notation is excellently adapted for facility in these new fields.

EXPANSIBLE AND REVERSIBLE SQUARES

by Robert Gutmann

The paper by Charles Q. De France in the Bulletin of February, 1947, suggested to me that the following list of squares, which behave similarly to his Expansible Integers to a limited extent, might prove interesting. These constructions are valid for the decimal as well as the duodecimal base; in fact they hold true for any base larger than nine.

$12^2 = 144$	$102^2 = 10404$	$1002^2 = 1004004$	$10002^2 = 100040004$
$21^2 = 441$	$201^2 = 40401$	$2001^2 = 4004001$	$20001^2 = 400040001$
$13^2 = 169$	$103^2 = 10609$	$1003^2 = 1006009$	$10003^2 = 100060009$
$31^2 = 961$	$301^2 = 90601$	$3001^2 = 9006001$	$30001^2 = 900060001$
$112^2 = 12544$	$122^2 = 14884$	$10102^2 = 102050404$	
$211^2 = 44521$	$221^2 = 48841$	$20101^2 = 404050201$	
$113^2 = 12769$		$10103^2 = 102070609$	
$311^2 = 96721$		$30101^2 = 906070201$	
$11^2 = 121$	$111^2 = 12321$	$1111^2 = 1234321$	
$101^2 = 10201$		$10101^2 = 102030201$	

WHAT IS π?

by George S. Terry

The value and evaluation of π appear to have perennial interest. The following alternative method to Mr. Scifres' neat exposition in Vol. 3. No. 1. of the Bulletin of Feb. 1947 is offered because derived from simple fractions, having no apparent connection with the circle.

$$\pi = 6\left[\frac{1}{2} + \frac{1}{2\cdot 3\cdot 2^3} + \frac{1\cdot 3}{2\cdot 4\cdot 5\cdot 2^5} + \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 7\cdot 2^7} + \frac{1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 6\cdot 8\cdot 9\cdot 2^9} + \cdots\right]$$

The second term may be obtained by multiplying the first by $\frac{1^2}{5^2-1}$; the third by multiplying the second by $\frac{3^2}{9^2-1}$; the fourth from the third by $\frac{5^2}{11^2-1}$; and in general, the (n+1)th from the nth by $\frac{(2n-1)^2}{(4n+1)^2-1}$.

Since the multiplier approaches more and more closely to a square, this method may perhaps be called "squaring the circle."

The terms necessary to give one dozen places are:-

3.			
.16			
.0203	7249	7249	7
.0037	486X	3518	7
.0007	4716		
.0001	4386	5888	9
	31Ɛ4	X559	3
	0777	5916	Ɛ
	016Ɛ	45X5	4
	0040	04Ɛ6	2
	000X	3X3X	6
	0002	2ƐX2	1
		5Ɛ46	Ɛ
		13X7	X
		0369	6
		0098	1
		0022	5
		0006	1
		0001	5
			3
			1
3.1848	0949	3Ɛ92	

A DUODECIMAL SLIDE RULE

by Kingsland Camp

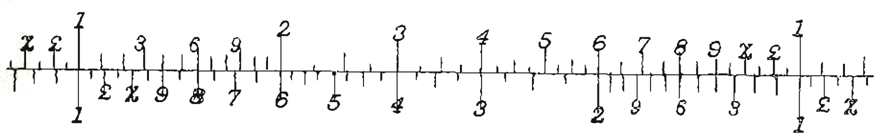
A slide rule should be designed for maximum utility. In order of simplicity, there are three general uses for this tool:

Reference, by setting of the indicator only: This use is simplest but so far least important or frequent since even a few small pages of four-figure tables will furnish reciprocals, squares, cubes, roots, sines, tangents, and logarithms as quickly and more accurately than can any slide rule of convenient size.

Two-factor calculations, by setting both indicator and slide: Simple products or quotients, involving only two numbers. Probably many users venture beyond this stage only if one factor is a sine, tangent, or square, making use of the proper scale obvious.

Compound Calculations: The possibilities of this are only dimly felt by the ordinary user, but are the stock in trade of the slide rule "wizard." The secret of his skill is complete awareness of the fixed algebraical relation of each scale on the slide with each other such scale, and also the similar relations that hold between the scales on the frame. Illustrations will appear hereinafter; but it may confidently be laid down as a principle, that the more simply and logically the various scales on the instrument are related to each other, the wider would be the circle of those who make use of these possibilities of compound calculation, and the more frequent and confident would be their use even by experts.

If our Society is to produce a slide rule of maximum usefulness and efficiency, let us put traditional arrangements on the defensive at once and ask, What has been the necessity, since the time of the invention of the hair-line indicator, of always having the B and C scales of the slide adjacent respectively to the A and D scales of the frame? We habitually use the indicator in setting the slide and reading our answers anyway. Suppose we lay down a first simplifying principle of arrangement: *every* scale whose reciprocal (or inverted) scale is on the same member of the instrument, should lie immediately alongside that inverted scale, with the latter lettered in red and its markings extending in the opposite direction. Thus for illustration (except that red ink is not used in our Bulletin):



Of course similar pairing of decimal scales with their reciprocals would show no such frequency of lines common to the paired scales. More detail in the above sketch would show at least all the following pairs of numbers with reciprocals as single straight lines:

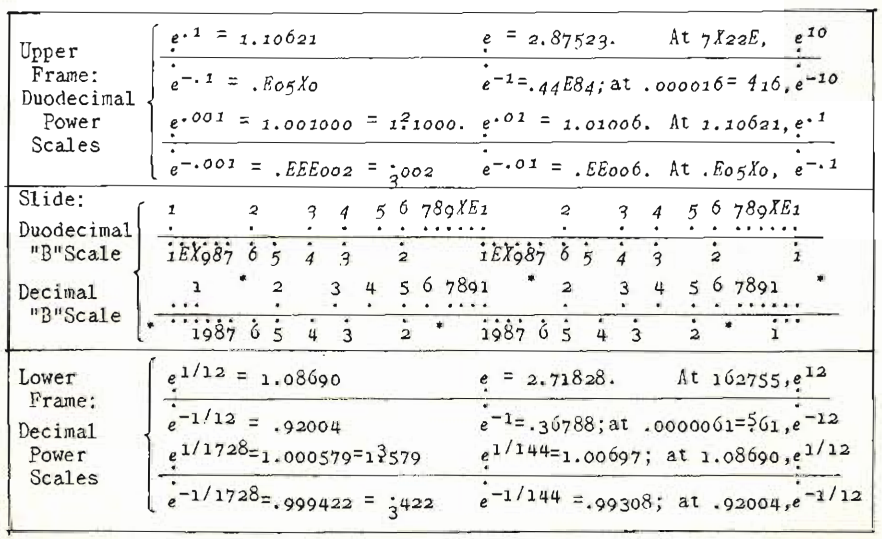
14	*16*	*20*	*23*	*28*	*30*	*40*	*46*	*54*	*60*	*80*	*90*
90	*80*	*60*	*54*	*46*	*40*	*30*	*28*	*23*	*20*	*16*	*14*

With such pairing, separate designation of the inverted scale (DI, CI, CIF, LL0, etc.) becomes unnecessary as the significance of the reciprocal scale is clear. Especially clear would be the nature of the CIF scale; one wonders if even the experts sometimes mistake it for a folded CI scale instead of the inverted CF scale that it is. π as it happens is so close to the square root of ten (and little farther from that of twelve) that an answer found on such a misapprehension could easily pass undetected.

Perhaps also a certain stunt of the "lightning" slide rule worker would by this arrangement be a little easier to grasp and use: the separation between different marks on a scale and on its inverted scale may be treated as representing the *product* of the quantities indicated if the inverted scale mark is to the left; otherwise as if it represented the *reciprocal* of the product, in the course of any calculation in which we do not care about the product for itself.

Proposed Arrangement in General.

I. The Conversion and Power (or "Loglog") Side of Instrument.



This outline of course is schematic and not proportioned, its height being perhaps twice the width but its width only about one third the length, of the actual instrument proposed, which is to measure twelve inches from the left to the right index of its C or D scales.

The arrangement should serve the present important era of transition: as the upper duodecimal power scale parallels the upper decimal one throughout, conversion from either base to the other is effected simply by reading from the indicator; the lower duodecimal power scale parallels the lower decimal one similarly. Thus, including the reciprocal power scales paired with the others, any indicator setting converts four different numbers on the duodecimal power scales into their equivalents on the decimal scales, and vice versa.

Such conversion is also possible on the slide for the restricted range *0.1* to *10* (1/12 to 12), regarding its central index as unity on both bases, but the chief use of the slide would be for computation. It will be possible to find any nth power or root (along with its reciprocal) within the scope of the duodecimal power scales, simply by setting a unity point of the duodecimal B scale to the given quantity and, opposite n on that B scale, reading the required result on whichever duodecimal power scale it falls. The decimal B scale is not quite so versatile: it will serve to find an nth power or root (and reciprocal thereof) within either one of the decimal power scales; but if the result is on the other decimal power scale from the given quantity, the latter is set opposite an upper B-scale reference mark (asterisks at 144 in diagram) if the given quantity is on the upper decimal power scale; vice versa if the given quantity is on the lower decimal power scale.

It will be easy to master the trick just above with very little practice, however the description may sound; but it deserves passing mention that, just as a device for finding decimal powers and roots, this instrument will be more helpful with interest and discount problems, certainly not an unimportant field, than almost any slide rule now extant, simply because it pairs power scales with their reciprocals: frequently in such problems we require the reciprocal rather than the power or root itself. A minor incidental advantage: with the slide in a position of rest, as it were (its central unity index set to the mark for e on either power scale as in the illustration), hyperbolic sines and cosines are fairly easily improvised - in the absence of tables, of course - as half the difference or sum of e^x and e^{-x}, both then available at one reading on the power scales by setting indicator to x on the slide.

II. The Duodecimal Scales on Reverse Side.

The arrangement of the "loglog" side of the instrument, detailed just above, is almost inevitable for our purposes and probably cannot be very much improved on. But the scales to appear on the other side, and their arrangement, should be reviewed by a practical-minded committee acquainted with several of the more important fields of use of the slide rule. The following arrangement for this reverse side is my own present preference, subject to revision if good reasons are advanced.

Since calculations are more accurate with full-length than with two-cycle ("A" and "B") or three-cycle ("K") scales, since practically all useful powers and roots can be found with the "loglog" scales anyway, and since there is already a "B" scale on the other side of the slide, not to mention the possibility of representing x^2 or its reciprocal by the separation of the marks for x on any scale with its inversion, it seems practicable to dispense with multi-cycle scales as far as possible and to equip this side of the instrument thus:

Upper half of frame: D scale paired with reciprocal (DI) scale
DF scale paired with reciprocal (DIF) scale
Slide: C paired with reciprocal (CI) scale
CF paired with reciprocal (CIF) scale
Lower half of frame: Full-length sine (S) scale
Full-length tangent (T) scale
Full-length small-angles (ST) scale
Also the L (Log_{10}) scale

On most present models, the trigonometric scales appear on the slide, possibly for use in conjunction with the power scales in some field of mathematics with sine or tangent exponents; but on this instrument such a use would raise the given number, not to the sine or tangent power, but to the square of such a power: a result probably wanted comparatively seldom.

This arrangement affords ample means of working product, square, cube, reciprocal and quotient problems with full-length scales. Also it enables solving in one setting of the slide, any plane right-triangle problem* for which the least angle A exceeds $.01E^c$ (about 4°50'). For with side $c>b>a$, we have

$$\frac{c^{-1}}{\sin A} = \frac{b^{-1}}{\tan A} = \frac{a^{-1}}{1}$$

* *Except for the case that is probably least frequent: if c, b are the given parts it is necessary first to find angle A or B.*

reading the sides on whichever reciprocal (CI or CIF) scale of slide contains c, b in the order given. Among other uses this setting enables instant solution of the important vector relation, $c = \sqrt{a^2+b^2}$, transformation from rectangular to polar coordinates, and vice versa. It should be committed to memory by all users of this rule.

Of course for the least angle less than about $.01Ɛ^c$ the middle side is too nearly equal to the hypotenuse for the slide rule order of accuracy. Nevertheless a duodecimal devotee cannot help remarking at this point on the simplicity and efficiency of our proposed ST scale, essentially a "folded" scale with the factor 2π: no subsidiary seconds point is needed as for present-day circular measure; all sines, angles, or tangents less than $.01Ɛ^c$ are indicated merely by properly pointing off ciphers. On this scale, the circumference-factor 2π = 6.2832 or *6.3494*, coincides with the unity index of the frame.

Details.

In the interest of easy reading, the series of parallel marks that constitute a scale should one and all represent commensurable numbers. Their sequence should not be confusingly disturbed or interrupted by a special mark of the same kind for e on the loglog scales or for π on any scales. Such quantities should be indicated otherwise: probably by a red dot suitably placed between the regular marks, with the proper symbol just over or under it. (On the sketch of the loglog scales earlier in this essay, several powers of e with their arithmetical values were shown, in order to suggest the proper scope of the loglog side of the rule. Only e itself is intended to be actually indicated on the instrument, and at that only by a dot as just suggested.)

No scale should terminate abruptly at an incommensurable point but, even if (on the frame) the indicator cannot be pushed so far, the scale itself should extend to include some easily read or recognized number just beyond. And it would be convenient, if mechanically possible, for scales on the slide to run practically its whole length.

If practicable, the series of marks for subdivisions of a given order should steadily decrease in length as they become crowded and are discontinued towards the larger digits of a scale. (I endeavored to suggest this in my crude drawing of scale-and-reciprocal scale above.) It would be desirable also if the printed reference numbers could be made to follow such a trend of the marks by size or position or both.

The important subdivisions will be units, halves, fourths, and twelfths; no confusing sixths or thirds should appear as such on the scales. When the twelfths become crowded, the outstanding fourths and halves should continue; where fourths crowd, the halves should continue; where these must disappear, only the units should survive to become twelfths in the next cycle.

A compact convention for denoting series of zeroes, of 9's with decimals, or of Ɛ's with duodecimals, is rapidly becoming necessary not only in connection with stellar distances, atomic dimensions and forces, and national debts, but quite evidently also at the extremes of the loglog scales where the exact index point for a long number is sometimes not immediately clear. As illustrated in several places in the foregoing diagram of the conversion and loglog scales, I tentatively suggest a dot below a displaced numeral for indicating omitted ciphers following a decimal point (*unless* such dot and numeral end the number); a dot above a displaced numeral to indicate omitted 9's or Ɛ's. Thus, whether decimals or duodecimals, $1^{\underset{\cdot}{2}}1 = 1.001$; $^{\underset{\cdot}{3}}1 = .0001$; $2^{\underset{\cdot}{4}} = 20,000$; but $\overset{\cdot}{_3}4 = .9994$ with decimals while $\overset{\cdot}{_3}4 = .ƐƐƐ4$ with duodecimals.

A remark on magnifying indicators may not be amiss. These are usually simple cylindrical lenses, cheap to everyone but the purchaser, and they enlarge the axis of their field but obscure the margin in a way that is often confusing on portions of the scales. A more useful and quite feasible design of moulded glass would taper towards level at the edges but towards a cylindrical hump in the middle, with cross-section resembling the normal curve of error, $y = e^{-x^2}$. This would magnify the axis of its field but minify a little parallel zone on either side and thus avoid obscuring any marks or numbering below it.

The L (Log_{10}) scale has possibilities of peculiar usefulness on a duodecimal instrument, not at all shared by its counterpart on the decimal base. On the rule we propose, this is to have the useful length of one foot, and usefully subdivided duodecimally throughout. If the end pieces of the frame hold the instrument sufficiently above a chart or drawing laid over a smooth surface such as a desk or drafting-table top, for the sliding indicator to just clear that surface, a mark or pointer on the indicator will measure the separation of any two points (within a foot apart of course) with accuracy of probably *.01* inch and a minimum of eyestrain. This incidental useful feature should not be overlooked in the final design of our instrument.

CIRCULATING DIVIDENDS

by Harry C. Robert, Jr.

Revolving numbers, sometimes called cyclic or circulatory numbers or periodic fractions, have for many years been a subject of considerable interest and have been extensively investigated by mathematicians of every degree from amateur to master. There is another class of numbers which appear to be fundamentally related to revolving numbers but as far as can be learned from a casual search of the literature have never been investigated. These numbers are sequences of integers which are always divisible by some prime, p, regardless of which integer is used to start the sequence. For example:- *418Ɛ* is devisible by *25*, and also *18Ɛ4*, *8Ɛ41* and *Ɛ418* are divisible by *25*. For the purposes of this preliminary paper such sequences will be referred to as,- "circulating dividends."

The mathematical basis for the performance of these numbers is simple. Let $N=(abcd..)$ be a number of K integers, and let $M=(bcd...a)$ be a number composed of the same K integers used in the same order but starting with b instead of a, that is, we have simply moved the first integer, a, from the left end of the sequence to the right end. Expressing this operation algebraically-

$$M = 10\left[N - (10)^{k-1}(a)\right] + (a)$$
$$M = 10N - a(10^k-1) \qquad Eq.(1)$$

It is obvious from Eq.(1) that if any factor of (10^k-1) is a devisor of N, it must also be a divisor of M. Since this operation is entirely independent of the value of the integer a, we may repeat the operation by moving b, c, etc., each in turn from the left end of the sequence to the right end and each number thus formed will be divisible by any factors of (10^k-1) which are also factors of the original number, N.

The divisors of (10^k-1) are all primes having a length of period which is a factor of K. A list of prime divisors for small values of K follows; others may be found by referring to The Dozen System:

k	Prime Divisors of (10^k-1)	k	Prime Divisors of (10^k-1)
1	Ɛ	7	Ɛ, 46Ɛ, 2X3Ɛ
2	Ɛ, 11	8	Ɛ, 11, 5, 25, 75, 175
3	Ɛ, 111	9	Ɛ, 111, 31, 3X891
4	Ɛ, 11, 5, 25	X	Ɛ, 11, 11111, Ɛ0Ɛ1
5	Ɛ, 11111	Ɛ	Ɛ, X1, 1Ɛ, 754ƐƐƐ41
6	Ɛ, 11, 111, 7, 17	10	Ɛ, 11, 5, 25, 111, 7, 17, ƐƐ01

Now any number which has an odd prime factor other than *3* is the basis of a circulating dividend. All that is required is the addition of sufficient zeros to make the sequence total the *k* figures required for the particular divisor. For example:- *175* must have a *k* of *8* or multiples thereof. So we write *0000 0175*, and *1750*, *1 7500*, *17 5000*, *175 0000*, *1750 0000*, *7500 0001*, and *5000 0017* are all divisible by *175*. As another example, we can by looking at the above table opposite the value of *k* equal to *Ɛ*, write *4ƐƐ Ɛ410 0075* and know that this number is divisible by *754Ɛ 2Ɛ41*.

Obviously this device can be used to simplify certain types of problems. Suppose we wish to know how many numbers less than *10000* and divisible by *25* contain the figure *4*. Of course we might write down the *4Ɛ7* multiples of *25* from *25* to *ƐƐƐƐ* and then tally the desired result. But this is not necessary with our new knowledge concerning the divisibility of sequences. Instead we need only write the *4Ɛ* multiples of *25* from *4017* to *4Ɛ99*. The sequences for four of these, *441Ɛ*, *4469*, *4492* and *44Ɛ7*, are duplicated so that one entry of each is eliminated leaving *47* sequences which will produce all of the numbers under consideration. Of these sequences, one, *4444*, produces only one number; *Ɛ* sequences of the form *4040*, *4141*, etc., produce only two numbers each, totaling *1X* numbers; while the remaining *37* sequences produce their full quota of four numbers each, totaling *124* numbers, making a grand total of *143* numbers having the desired characteristic.

As an extension of the foregoing problem, suppose we also wish to know how many of those *143* numbers which are not divisible by *5* are composed entirely of even integers, including zero. From our list of *47* sequences we find that only two will meet this requirement, and the required numbers are-

408X	*4X00*
8X4	*X004*
8X40	*4X*
X408	*4X0*

Possibly the solution of such problems is trivial and certainly the use of the device illustrated above is limited to the case of a relatively small number of primes for which the value of *k* is reasonably small. However any device which makes it possible to study the characteristics of the *4Ɛ7* multiples of *25* less than *10000* by studying only the *164* sequences which produce all of those multiples may have many other possibilities if we explore the field completely.

Preliminary investigations of circulating dividends reveal several interesting fields for further research. For example, we may take two four figure numbers such as *1237* and *4593* where each is divisible by *25* and form an *8* figure sequence that is also divisible by *25*. Thus

1237 4593	*4593 1237*
2374 5931	*5931 2374*
3745 9312	*9312 3745*
7459 3123	*3123 7459*

note that only the first and fifth numbers are such that the two groups of four figures composing the two halves of the number are themselves divisible by *25*. Now if we take the second number of this sequence, *2374 5931*, and add *25*, we will form a new *8* figure circulating dividend-

2374 5956	*5956 2374*
3745 9562	*9562 3745*
7459 5623	*5623 7459*
4595 6237	*6237 4595*

and while all of these are divisible by *25*, in no case are any of the groups of four divisible by *25*.

To form a *10* figure circulating dividend for *25*, we may take any of the above *8* figure numbers and insert a four figure sequence that is also divisible by *25*. Thus, using the second number above-

3745 4166 9562	*4166 9562 3745*	*9562 3745 4166*
7454 1669 5623	*1669 5623 7454*	*5623 7454 1669*
4541 6695 6237	*6695 6237 4541*	*6237 4541 6695*
5416 6956 2374	*6956 2374 5416*	*2374 5416 6956*

and only in the first, fifth and ninth numbers is one of the groups of four divisible by *25*, and this is always the *4166* which was inserted.

Now take the second of the above *10* place numbers and transpose the first two groups of four and we get- *1669 7454 5623*, a new *10* place sequence divisible by *25*. But it is not necessary that we transpose four figures at a time. We may transpose one, two or three figures as well as four, provided only that the first figures in the groups to be transposed are four or eight places apart. This is determined by the period of the divisor. Thus in our last circulating dividend we may transpose the third figure, *6*, with the next to last figure, *2*, and *1629 7454 5663* will be another circulating dividend for *25*. If we add *101* to the last number we get *1629 7454 5764* which is a circulating dividend for both *25* and *7*. We are therefore not restricted to prime divisors. It is only necessary that divisors be relatively prime to our number base.

The foregoing should suffice to introduce the interesting subject of circulating dividends. The device is applicable to any number base. Whether or not this subject will pay dividends in the realm of recreational mathematics or prove to be of more fundamental importance can not be predicted at this time. Certainly the subject of circulating dividends provides an extensive and interesting field that can be explored by almost anyone who is interested in the mystery of numbers using only the simpler devices of arithmetic for the investigation.

TWELVE DIRECTIONS

In the revival of learning in Europe that took place under Charlemagne about the year 800, an effort was made to change the four directions to twelve, with also the creation of names for the twelve Winds that would come from these directions. It would have been a convenient division for the duodecimal count, but it did not survive.

The names of the directions and winds, as given in Eginhard's *Life of Charlemagne*, II, 29, are these:

Subsolanus	East Wind
Eurus	East-South Wind
Euroauster	South-East Wind
Auster	South Wind
Austro-Afric	South-West Wind
Afric	West-South Wind
Zephyr	West Wind
Corus	West-North Wind
Circius	North-West Wind
Septentrion	North Wind
Aquilon	North-East Wind
Vulturnus	East-North Wind

F.E.A.

PYTHAGOREAN TRIANGLES WITH EQUAL PERIMETERS

by Harry C. Robert, Jr.

A recent publication of the American Mathematical Society posed the problem of finding a general solution for Pythagorean triangles having equal perimeters. Although listed as an elementary problem, this proposition is an interesting one. Stated algebraically, this problem requires a solution of the following system of equations-

$$A_1^2 + B_1^2 = C_1^2$$

$$A_2^2 + B_2^2 = C_2^2$$

$$A_1 + B_1 + C_1 = A_2 + B_2 + C_2$$

Starting with the usual generators for Pythagorean triangles:

$$A = m^2 - n^2 \qquad B = 2mn \qquad C = m^2 + n^2.$$

the perimeter will be: $p = 2m(m + n)$

and the half-perimeter will be: $s = m(m + n)$

By placing $m = u\sqrt{k}$ and $(m + n) = v\sqrt{k}$

we obtain $s = kuv$

and there will be as many triangles with a half-perimeter of (s) as there are different arrangements of the factors of (s) in the form: $s = k_1u_1v_1 = k_2u_2v_2 = k_3u_3v_3 = k_4u_4v_4 = k_5u_5v_5$, etc., where all values of (k) are either 1 or k contains only non-square odd factors and if we wish to require that all sides of our triangle be positive, the following inequality must hold-

$$2u_i > v_i > u_i$$

Our general solution then is:

$$A = 2kuv - kv^2$$
$$B = 2kuv - 2ku^2$$
$$C = -2kuv + kv^2 + 2ku^2$$
$$\text{Perimeter} = p = 2kuv$$

Thus our problem is reduced to that of finding integers that satisfy the relationships: $k_1u_1v_1 = k_2u_2v_2$, $2u_1 > v_1 > u_1$ and $2u_1 > v_2 > u_2$, k_1 and k_2 equal 1 or contain only non-square odd and if $k_1 = k_2$, $u_1 \neq u_2$ and $v_1 \neq v_2$. The smallest solution appears to be:

$s = 1\cdot5\cdot6 = 5\cdot2\cdot3 = 26$

$A_1 = 20$	$A_2 = 13$
$B_1 = X$	$B_2 = 18$
$C_1 = 22$	$C_2 = 21$
$p_1 = 50$ =	$p_2 = 50 = 2s = 2\cdot26$

The smallest solution for three triangles appears to be:

$s = 1\cdot6\cdot X = 3\cdot4\cdot5 = 5\cdot3\cdot4 = 50$

$A_1 = 18$	$A_2 = 39$	$A_3 = 34$
$B_1 = 40$	$B_2 = 20$	$B_3 = 26$
$C_1 = 44$	$C_2 = 43$	$C_3 = 42$
$p_1 = X0$ =	$p_2 = X0$ =	$p_3 = X0 = 2s = 2\cdot50$

We can restrict our solutions to primitive triangles by requiring that $k = 1$, and placing rather complicated restrictions on u and v. It is simpler to resort to a new set of parameters for this purpose. Solutions in two or more primitive triangles are found only when-

$$s = abcd$$

where a, b, c, and d are relatively prime and all are odd, except a, which may be either odd or even. It is also necessary that $a \geq 2$. To eliminate any negative sides in the triangles, it is necessary that: $\frac{2c}{b} > \frac{d}{a} > \frac{b}{c}$ and $b > c$.

From $s = abcd$, we obtain two sets of generators:

$m_1 = ab \qquad m_2 = ac$

$n_1 = (cd - ab) \qquad n_2 = (bd - ac)$

and our general solution becomes:

$A_1 = 2abcd - (cd)^2$	$A_2 = 2abcd - (bd)^2$
$B_1 = 2abcd \quad -2(ab)^2$	$B_2 = 2abcd \quad -2(ac)^2$
$C_1 = -2abcd + (cd)^2 + 2(ab)^2$	$C_2 = -2abcd + (bd)^2 + 2(ac)^2$
$p_1 = 2abcd$ =	$p_2 = 2abcd$

The smallest solution for this case appears to be:

$a = 2 \quad b = 11 \quad c = Ɛ \quad d = 3$ and $s = 5Ɛ6$

$A_1 = 443$	$A_2 = 143$
$B_1 = 264$	$B_2 = 524$
$C_1 = 505$	$C_2 = 545$
$p_1 = ƐƐ0$ =	$p_2 = ƐƐ0 = 2s = 2\cdot5Ɛ6$

There also are three non-primitive triangles with the same perimeter which are found from the factorisation of: $s = 3\cdot11\cdot1X = 11\cdot6\cdot Ɛ = ƐƐ\cdot2\cdot3$, using the general solution for non-primitive cases.

Similar methods may be used to extend this solution to the case of three or more primitive triangles with the same perimeter. The smallest solution for the case of three triangles, appears to be:

$A_1 = 18249$	$A_2 = 11Ɛ35$	$A_3 = 46X9$
$B_1 = 1894$	$B_2 = X840$	$B_3 = 16654$
$C_1 = 18335$	$C_2 = 156X1$	$C_3 = 17115$

$p_1 = 36256 = p_2 = 36256 = p_3 = 36256 = 2s = 2(Ɛ\cdot15\cdot13\cdot11\cdot1)$

If we wish to consider only triangles whose perimeter is a square, we consider only $s = \frac{t^2}{2}$ and if we limit this to primitive triangles, then $a = \frac{a_1^2}{2}$ and b, c and d are all odd squares. The four factors of s must be relatively prime and the inequality for the respective magnitudes must hold.

The smallest solution for this case appears to be:

$a = 16 \quad b = 261 \quad c = 201 \quad d = 21$ and $s = \frac{(5736)^2}{2}$

$A_1 = 11\ Ɛ693XƐ$	$A_2 = 4\ 2022XƐ$
$B_1 = 3\ 1Ɛ7790$	$B_2 = 11\ 3X1790$
$C_1 = 12\ 399071$	$C_2 = 11\ Ɛ5X171$
$p_1 = 27\ 542030$ =	$p_2 = 27\ 542030 = (5736)^2 = 2s$

In a similar manner we may extend the foregoing to the case of perimeters equal to cubes and other higher powers.

If it is desired to admit solutions for which one or two sides of one or more of the triangles is negative, the same general forms of the solutions hold and only the inequalities which limit the relative magnitudes of the parameters are ignored.

It should be noted that in order to obtain all non-primitive solutions, it does not suffice to use the method developed for primitive solutions and simply eliminate the requirements that a, b, c and d be relatively prime. This will give some non-primitive solutions but not all. To obtain all non-primitive solutions the first method must be used since non-square odd multiples of primitive and non-primitive triangles are produced only by irrational generators.

LIST OF PRIMES ≯ 12855 AND POWERS OF PRIMES ≯ 5954

WITH PERIODS NOT GREATER THAN 100

BASES 2, 3, 5, 6, 7, χ, Ɛ, AND 10. BASE XII NOTATION

Powers of Primes, Overscored

BASE	2	3	5	6	7	χ	Ɛ	10
PERIOD 1	-	2	2 $\overline{4}$	5	2	3 $\overline{9}$	2 5	Ɛ
2	3	$\overline{4}$ $\overline{8}$	3 $\overline{8}$	7	3 $\overline{4}$ $\overline{8}$ $\overline{14}$	Ɛ	3 $\overline{4}$ $\overline{8}$	11
3	7	11	27	37	17 $\overline{9}$	31 $\overline{23}$	7 17	111
4	5	5 $\overline{14}$	11 $\overline{14}$	31	5 $\overline{21}$ $\overline{28}$	85	51 $\overline{14}$	5 25
5	27	Ɛ $\overline{\chi 1}$	Ɛ 5Ɛ	21Ɛ $\overline{21}$	1755	35 1χ7	1χ45 $\overline{21}$	11111
6	$\overline{9}$	7	7 $\overline{9}$	27	37	7 11	31 $\overline{9}$	7 17
7	χ7	771	Ɛ377	-	25 28χ5	17Ɛ 2835	37	46Ɛ 2χ3Ɛ
8	15	35 $\overline{28}$	221 $\overline{28}$	901	841 $\overline{54}$	61 Ɛ5 $\overline{69}$	42χ1 $\overline{28}$	75 175
9	61	531	17 591	17 1517	31 747 $\overline{23}$		-	31
χ	Ɛ	51	375	Ɛ 85	Ɛ 13Ɛ	5317	7925	Ɛ0Ɛ1
Ɛ	1Ɛ 75	1Ɛ 228Ɛ	-	1Ɛ	797	10641	9185	1Ɛ $\overline{\chi 1}$
10	11	61	421	11 81	11 131	-	11 791	ƐƐ01
11	48χ7	-	-	1Ɛχ1	-	45 67	771	-
12	37	397	25 315	25 145 $\overline{41}$	95 63Ɛ	-	-	157 7687
13	107	2781	131 1011	817 841	27	27	-	51 471 57Ɛ1
14	195	15 $\overline{54}$	15 6795 $\overline{54}$	15	15 $\overline{\chi 8}$	15	15 $\overline{54}$	15 81

BASE	2	3	5	6	7	χ	Ɛ	10
PER. 15	-	10ƐƐ	2χ1	17Ɛ 2χ1 797	-	-	-	-
16	17 $\overline{23}$	17 31	2Ɛχ7 $\overline{23}$	-	-	17	$\overline{23}$	Ɛ61 1061
17	-	Ɛ11	13Ɛ 3767	13Ɛ	2χƐ	-	-	-
18	35 $\overline{21}$	825 $\overline{21}$	35 5375	181 3Ɛ11	1Ɛ5 43Ɛ1 $\overline{\chi 5}$	2071	-	$\overline{21}$
19	241 $\overline{41}$	-	277	-	-	37 1151	ƐƐ7 4Ɛ27 $\overline{41}$	-
1χ	48Ɛ	57 471	1Ɛ 57 3081	-	1Ɛ	1Ɛ 2451 50Ɛ7 $\overline{\chi 1}$	1Ɛ 75 147	-
1Ɛ	3Ɛ	3Ɛ	5237	3Ɛ Ɛ7 1χ45	3Ɛ 194Ɛ	-	-	3Ɛ
20	181	3901	-	-	61 141 2χ1	-	6201 Ɛ781	141
21	421 1061	-	85 18Ɛ 295	χ5	1587	10475	18χ1 11Ɛ87 $\overline{\chi 5}$	-
22	16Ɛ7	-	3037	45 661	45	5Ɛ7	45 63Ɛ	67 $\overline{121}$
23	-	91 301 4901	91 1χ7 24χ7	117	91 577 1461 $\overline{69}$	531 $\overline{183}$	-	-
24	25 95	25 9665	-	2Ɛ1	-	25 1Ɛ5	25 1151	481
25	175 77Ɛ 1261	4Ɛ	4Ɛ	-	4Ɛ	1χ1Ɛ 984Ɛ	377	4Ɛ 123ƐƐ
26	237	27 1χ7	51 44Ɛ1	-	-	157 181 1301	27	27 2Ɛ1
27	-	48Ɛ	10Ɛ1	3105	21Ɛ 1029Ɛ	1747	-	271
28	-	$\overline{\chi 8}$	1601 $\overline{\chi 8}$	255 Ɛ95	255 $\overline{194}$	255 315 455 995	$\overline{\chi 8}$	
29	-	-	147	57	2127	57	471 χ11	-

BASE	2	3	5	6	7	Χ	Ɛ	10
PER 2Χ	-	87 217 711	1931	-	-	87	-	1587
2Ɛ	5Ɛ	5Ɛ	157 447 2521	5Ɛ		5Ɛ	157	5Ɛ 34Ɛ
30	31 91	-	31	61 391	-	-	-	61
31	167	-	105	105 46ƐƐ 7097	167 1807		15ƐƐ	
32	-	1797	535 Ɛ421	104Ɛ	-	-	13Ɛ 171	1ΧƐ7 4901
33	67	221 3961 42Ɛ1 121	67	-	-	-	67 111 397	-
34	-	-	181	35	35	-	35	35 375
35	789Ɛ	6Ɛ	-	-	6Ɛ	6Ɛ 867	6Ɛ 867	6Ɛ
36	3177	37 1391 41	37 Χ7 4497 41	-	-	Χ7 1681 41	-	37 9Χ17 41
37	2ƐƐ 575Ɛ	2ƐƐ	-	125 2ƐƐ	-	125	-	2ƐƐ
38	291 1281	3225	75	-	471 995	75	-	1461 2181 3801
39	447 115Χ7	131 Ɛ31	817	1301	-	-	-	-
3Χ	-	-	3Ɛ	-	-	3Ɛ Ɛ7 156Ɛ	3Ɛ 705	7231
3Ɛ	143Ɛ 2741	85Ɛ 10891	-	-	-	-	1245	-
40	81 481	81 401 541	-	3541	-	-	81 181 1041	4541
41	-	34Ɛ 23ΧƐ 4921	-	-	2061	-	-	-
42	18Ɛ 2417	107	1125	-	-	18Ɛ	-	1167
43	87 12Χ7	-	-	217	87	431	6247	-

BASE	2	3	5	6	7	Χ	Ɛ	10
PER 44	45 111 Ɛ25	45	45 121	221 1431	111	375		45 485
45	3821	8Ɛ 11ƐΧ1	-	7Ɛ85	4951	8Ɛ	8Ɛ	8Ɛ 51Ɛ
46	69	Ɛ301 Ɛ647	117 347 69	-	-	-	277 69	91 1Χ7 347
47	615 1Χ1Ɛ	921 92Ɛ	Χ1	-	-	921	12785	3081
48	-	-	-	1Ɛ5 241 435	241 1295	4655	95 315 1561	-
49	-	171	-	-	Ɛ421 261	10407	261	-
4Χ	4Ɛ	377 3637	-	4Ɛ	-	4Ɛ	4Ɛ	-
4Ɛ	-	-	-	-	-	-	825	1092Ɛ
50	51 921	-	13Χ1	51 131 2071	51	51	-	-
51	-	-	4Χ57	-	267 2907	511 2825	-	-
52	-	3Ɛ97	907	-	271	-	21Ɛ 907 62Ɛ1	-
53	-	-	-	277	-	6331 115Χ7	Χ7 4991	-
54	455	194	455 194	1715	368	Ɛ595	141 195 194	541
55	-	ΧƐ	ΧƐ	6Χ1Ɛ	ΧƐ	-	ΧƐ	ΧƐ
56	57 100Χ1	-	-	327	57	-	57 5567	57 147 Χ11 3207
57	-	-	1Χ5 Ɛ21	-	-	-	-	-
58	Ɛ5 675	-	1185 Ɛ795	-	Ɛ5	-	Ɛ5	-
59	-	1Ɛ1	Ɛ7 3717	6957	Ɛ7	1Ɛ1	Ɛ7	-
5Χ	1Ɛ5	-	-	447 4Χ5 1407 5421	5Ɛ	-	5Ɛ 4461 9Χ17 2Ɛ01	-
5Ɛ	-	-	3Ɛ5	-	-	-	109Ɛ	2Ɛ61 Ɛ411

BASE	2	3	5	6	7	Ӿ	Ɛ	10
PER. 60	301	-	61	401 1Ɛ01	-	1Ӿ01	61	-
61	307	Ɛ7Ɛ1	-	-	307	-	-	-
62	1041	Ӿ7Ɛ7	5327	232Ɛ	105	4245	-	-
63	-	421 5681	107 1ӾƐ1	421	-	107 2521	107	421
64	171 321	-	-	-	-	-	-	-
65	-	-	-	-	-	3045	-	2811
66	-	67 111 1807	1981	67 52Ӿ1	67	111 3851 121	221	211 5Ɛ7
67	167Ɛ	-	-	-	-	225 3697 5Ɛ3Ɛ	225	-
68	-	8401	-	Ӿ341	615	-	3081	-
69	1601	2311	2991	-	Ɛ31 21Ӿ7 183	117 5531 509	-	-
6Ӿ	6Ɛ	-	6Ɛ	6Ɛ	-	-	-	7151
6Ɛ	11Ɛ	11Ɛ 701Ɛ	-	11Ɛ 357 Ӿ225	11Ɛ	-	11Ɛ 701Ɛ	11Ɛ 6Ɛ1 11Ӿ1 1093Ɛ
70	9Ɛ1 8441	17Ӿ1	1561	-	-	-	Ɛ051	-
71	-	-	Ӿ77	-	Ӿ77 2Ӿ2Ɛ	-	-	711 10Ӿ7Ɛ
72	-	-	Ӿ91 5637	-	66Ɛ Ɛ5Ɛ7 10117	-	3105	-
73	2501	-	3281 4041	-	-	2397	-	251 1011
74	255	75	-	75	75 5061	435	255	-
75	-	-	-	-	-	-	-	12Ɛ 12Ӿ1
76	-	-	-	-	391	-	131 447	131
77	63Ɛ	-	-	-	-	397 8271 Ӿ3Ӿ5	-	-
78	1Ɛ1 705 Ɛ61	-	-	3621	-	8Ɛ5	-	-
79	-	791	681	-	-	-	1747 19Ɛ7	5201
7Ӿ	1Ɛ7	9961	1245	-	-	378Ɛ	-	Ӿ541

BASE	2	3	5	6	7	Ӿ	Ɛ	10
PER. 7Ɛ	13Ɛ	13Ɛ	1797	3Ɛ7 1125	-	13Ɛ	-	13Ɛ
80	141	-	81	141 5001	81	81	401	2401
81	-	-	285	285 1017	285 68Ɛ	-	285	-
82	-	-	-	247	145 617 2267	145	-	-
83	147	-	-	9761	147	147 291	291 1461	291 1Ӿ01
84	Ӿ5	85 Ӿ5	-	-	85 7635 12785 441	-	85	85 841
85	-	-	-	-	427 575 375Ɛ	-	-	-
86	217 17Ӿ1	431	87 4Ӿ97	87 647	-	-	87	87 217
87	-	-	2191	1521	Ӿ457	71Ɛ	11Ɛ47	2191 4361
88	-	-	-	-	221	-	-	-
89	-	2Ɛ1 3791	-	157	-	-	2Ɛ1	-
8Ӿ	8Ɛ	-	8Ɛ	8Ɛ	8Ɛ	-	-	1Ӿ11
8Ɛ	-	-	3871	457	-	457	-	-
90	-	-	-	91	-	91	91	301
91	-	76Ɛ	76Ɛ	-	-	-	-	-
92	1877 Ӿ1	-	-	Ӿ1	237 1407 Ӿ1	237	-	237 3767 Ӿ5
93	-	-	1971	-	3721	961	-	-
94	2Ɛ95	95 Ɛ341	95 241	95 2955	315 481	95	241	95 1295
95	1Ɛ67 1157Ɛ	ӾƐƐ	184Ɛ 37Ɛ5	-	16Ɛ	16Ɛ	16Ɛ	16Ɛ
96	3Ɛ7	1947	171	321	321	-	35Ɛ7	171 261
97	879Ɛ	-	325 497	325 7ƐƐ	7ƐƐ	-	-	156Ɛ
98	-	7225	-	251	175	251	251	8Ӿ5 5Ӿ1
99	661 3961	-	-	-	1101	-	5347	3961
9Ӿ	1781	1Ӿ17	58Ɛ	-	-	-	1092Ɛ	4Ɛ1 747

BASE	2	3	5	6	7	X	Ɛ	10
PER. 9Ɛ	17Ɛ Ɛ85Ɛ	17Ɛ	17Ɛ 9Ɛ1 2X87	9785	71Ɛ5	-	17Ɛ	17Ɛ 9Ɛ1
X0	-	181	-	7941	-	-	-	181
X1	507	-	7005	-	X11	92Ɛ1	-	408Ɛ 92Ɛ
X2	-	267	267	5367	-	-	-	-
X3	-	-	517	-	-	-	-	-
X4	3291 5035	-	-	-	-	-	-	-
X5	-	18Ɛ	-	441	18Ɛ	527	441	18Ɛ 7Ɛ5Ɛ
X6	-	X7 617 1561	617	X7	X7 5831	-	-	X7
X7	-	346Ɛ	-	-	-	XX65	-	-
X8	-	368	541 368	-	714	-	368	-
X9	-	2481	-	10X1	6331	-	-	-
XX	XƐ	-	X825	XƐ	-	XƐ	-	-
XƐ	19Ɛ	19Ɛ	1625	19Ɛ 2337	-	-	19Ɛ	19Ɛ
Ɛ0	-	-	X11	1911	-	-	-	-
Ɛ1	-	-	547Ɛ	6Ɛ17	-	Ɛ11	-	-
Ɛ2	-	-	-	1X5	1002Ɛ	-	1X5	-
Ɛ3	1X7	391	391	-	1X7	-	1X7	-
Ɛ4	201	Ɛ5	Ɛ5	Ɛ5 3475	-	-	2X1 6Ɛ15	Ɛ5
Ɛ5	-	2X31	ƐX87	-	1705	1517	4911	-
Ɛ6	Ɛ7	Ɛ7 2Ɛ57 5331	-	-	1Ɛ1	-	497	Ɛ7 1Ɛ1
Ɛ7	-	-	-	3X5 1047Ɛ	-	-	-	-
Ɛ8	-	-	1Ɛ5	-	-	-	-	-
Ɛ9	-	-	-	1Ɛ7 7X01	1Ɛ7	1Ɛ7	1Ɛ7	-
ƐX	-	5Ɛ1 1297	2571	-	-	-	-	-
ƐƐ	-	-	-	5Ɛ7	-	-	1ƐX1	-
100	401	-	901	-	-	5201	-	401

Compiled from Cunningham and Creak,
"Haupt-Exponents, Residue-Indices, Etc."
Hodgson. London. 1922.

THE MAIL BAG

Velizar Godjevatz, author of the interesting proposal for a duodecimal musical notation, in this issue, was private secretary to former King Peter of Yugoslavia. He holds degrees from the New College, Oxford, from Berlin University, was Cand. Phil., at the Masters School of Piano, Conservatory of Hanover, and studied conducting under Scherchen.

. . .

Dudley George, of Ukiah, California, writes:- "For fun, I have made and used an abacus with four beads strung on each rod below the dividing bar, and three beads above. This gadget requires a lot more mental application to operate than an adding machine, but it can be used for a lot of calculating." In line with this idea, Dr. Nathan Lazar, of Teachers College, Columbia University, says that the regular abacus, with five beads below the bar, and two above, can be used for duodecimal operations. For the sixth bead, the bottom string is emptied, and one bead is counted above the bar; for the twelfth count, of course, both top and bottom strings are emptied, and one bead at the bottom of the next row is counted.

. . .

We sadly report the death of W. B. Campbell, of Cincinnati, Ohio. He has been a member of the Society for many years, and we will miss him.

. . .

Professor Donald M. Brown, of Central Michigan College, Mount Pleasant, Michigan, has called our attention to errors in the paper on the Four Fours, in the last Bulletin. In the values shown for 5, 17, 22, and 23, the symbol .4 was incorrectly shown as .4 .

. . .

Paul and Camilla Adams gleefully announce the arrival of a new dodekaphile, in the person of Thomas Henderson Adams. May we suggest to our younger members that this is an excellent way to provide for the future of our Society. Our congratulations to Paul and Cam, and to Cap. Delaney, as Tom's godfather.

. . .

One of the summer's most delightful high lights, was the visit of H. K. Humphrey, of Winnetka, Illinois. Mr. Humphrey is a member of our Finance Committee, but this has been our first opportunity to meet personally. His trip was a wide eastern swing for the pleasure of seeing many relatives and friends again, and included a talk with George Terry at Hingham, Mass.

Mr. Humphrey made a pertinent comment about the telephone service. In Winnetka, they are changing the exchange designation from WIN to W16, which would probably have been unnecessary if telephone numbers were duodecimal instead of decimal, since *10^7* is nearly three and a half times as large as 10^7. Under a duodecimal arrangement, there would naturally be twelve holes in the dial in place of ten, and there would be only two letters per hole instead of three, thus providing much greater flexibility in the selection of exchange names. The size of a central office, in capacity of exchange lines, would be enlarged from 10,000, (or 100^2,) to 20,736.

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Paul Friedemann, in sending us notice of the change of his address to 904 Mifflin Avenue, Pittsburgh 21, Pa., remarked that he had made a rediscovery of some of the qualities of Pythagorean triangles. He found a triangle with the sides:

Hypotenuse	c =	*6X1*
Leg	a =	*4X1*
Leg	b =	*4X0*

He says:- "One glance at my table told me there was something remarkable about the similarity in appearance representing this triangle. I discovered how to generate all triangles whose legs differ by only one digit."

We hope that Paul's enjoyment of his explorations will lead him into careful review of the recent work of our Mathematical Research Committee and Messers. Robert and Terry. Their work is admirable, extending our mathematical frontiers in some of its phases.

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Arthur Coldewe, of San Anselmo, California, has sent us a sizable check, expressly designated as the start of a fund for the production of the duodecimal slide rule. There has been much discussion of the type of rule we should have made. It is true that the more complex types of rule will be more expensive to manufacture, but the wisest course seems to be the provision of a rule that will meet all possible requirements. The inclusion of additional scales does not interfere in any way with the use of the rule for the simpler operations.

In the interest of developing the specifications for this rule, Kingsland Camp, Chairman of the Slide Rule Committee, has prepared the comprehensive paper in this issue. He is most anxious to receive your reactions to his proposals. Also, we would like to know which of our members will wish to acquire one of these duodecimal slide rules, in whatever form may be finally set.

Ye Ed.

COUNTING IN DOZENS

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*X*	*Ɛ*	*10*
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	*136*	Five ft. nine in.	*5.9'*
31	*694*	Three ft. two in.	*3.2'*
96	*3Ɛ2*	Two ft. eight in.	*2.8'*
19Ɛ	*1000*	Eleven ft. seven in.	*Ɛ.7'*

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2Ɛ*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

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12 ) 365
12 )  30 + 5
12 )   2 + 6
       0 + 2    Answer: 265
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Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *X*, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *X*.

Numerical Progression

1	One		
10	Do	*.1*	Edo
100	Gro	*.01*	Egro
1,000	Mo	*.001*	Emo
10,000	Do-mo	*.000,1*	Edo-mo
100,000	Gro-mo	*.000,01*	Egro-mo
1,000,000	Bi-mo	*.000,001*	Ebi-mo
1,000,000,000	Tri-mo	and so on.	

Multiplication Table

1	2	3	4	5	6	7	8	9	X	Ɛ
2	4	6	8	X	10	12	14	16	18	1X
3	6	9	10	13	16	19	20	23	26	29
4	8	10	14	18	20	24	28	30	34	38
5	X	13	18	21	26	2Ɛ	34	39	42	47
6	10	16	20	26	30	36	40	46	50	56
7	12	19	24	2Ɛ	36	41	48	53	5X	65
8	14	20	28	34	40	48	54	60	68	74
9	16	23	30	39	46	53	60	69	76	83
X	18	26	34	42	50	5X	68	76	84	92
Ɛ	1X	29	38	47	56	65	74	83	92	X1