# *The* Duodecimal Bulletin

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THE DUODECIMAL SOCIETY OF AMERICA

20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y.

THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve-base arithmetic. The lessons and examinations are free to those whose entrance applications are accepted. Remittance of $6, covering initiation fee ($3) and one year's dues ($3), must accompany applications.

The Duodecimal Bulletin is the official publication of the Duodecimal Society of America, Inc., 20 Carlton Place, Staten Island 4, New York. George S. Terry, Chairman of the Board of Directors. F. Emerson Andrews, President. Ralph H. Beard, Editor.  Separate subscriptions $2.00 a year, 50¢ a copy.

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# The Duodecimal Bulletin

*All figures in italics are duodecimal.*

## AN IDEAL NUMERICAL BASE

by Nina McClelland

*Miss McClelland is a freshman, majoring in mathematics, at the University of Toledo. She submitted the following as a term paper, and it was called to our attention through the kindness of Professor Wayne Dancer, Head of the Mathematics Department.*

Ever since the beginning of civilization, ten has been used as a numerical base by many of the tribes of man. Undoubtedly the reason for this is that God gave us ten fingers on which we have learned to count. It is interesting to ponder just what we would have done if, instead of ten flexible fingers, we had been given just two inarticulate stumps. If any number system would develop at all in this case, it would most likely be one in which two was used as the base.

Now let us suppose that we are starting an entirely new civilization. One of the many problems that would eventually confront us is, "Which of all the numbers known to us would be the very best to use as a base for our number system?" Well, I'm sure that one essential characteristic we would want in our ideal number system would be simple form; that is, we would want it to have only a small number of different symbols; and, in it, we would want only relatively few figures to be required to express large quantities. The basic mathematical processes of addition, subtraction, multiplication, and division should be easily obtainable; and, of course, we would have to be able to represent exactly any conceivable quantity.

With these characteristics in mind, let us look at our present system with ten as its base. Today this system is used universally by the Eskimo, who still counts on his fingers, and by the mathematician, who makes important calculations on the slide rule. Actually, however, this ten-system is making mathematics much more complicated than it needs to be; for example, ten has only two factors besides itself and one, these two factors being two and five. Two is extremely important and is used very often, it's true, but what about three and four, which are also used a great deal? As the base of a percentage system, it's quite ridiculous; because it's impossible to divide the whole, one hundred, by three, six, seven, eight, nine, eleven, or twelve parts without involving fractions. One great mathematician argued that ten was God-sent from Mount Sinai, because we were given just *Ten* Commandments; but the reply of another equally great mathematician to this argument was that, in the *New Testament*, there were *twelve* apostles.

Of course we can't overlook eleven in our search for an ideal base number. It would serve in a very unique way when fractions were involved because it is prime; and with a prime base, all fractions would be irreducible. In this system, though, all fractions except the elevenths and their multiples would be just rotating decimals.

If, in the first place, we had found all the factors of every number from one to one hundred, that is, all the factors except one and the number itself, we would have found that the lowest number containing four factors is twelve, containing six factors is two times twelve, seven factors is three times twelve, eight, is four times twelve, ten, is five times twelve, and so on; so that we would have known immediately that our ideal base lay somewhere in the twelve-series. Upon further investigation, then, we find that, of this twelve series, twelve itself would make the best base, because it is used more often in computations than any other number that is small enough to be used as a base.

Since it is necessary to have as many separate symbols in our new number system as the number used as the base, we will need to adopt twelve symbols. We can use the same first nine symbols

## CHART I

The Twelve System

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | χ | Ɛ | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 1χ | 1Ɛ | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 2χ | 2Ɛ | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 3χ | 3Ɛ | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 4χ | 4Ɛ | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 5χ | 5Ɛ | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 6χ | 6Ɛ | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 7χ | 7Ɛ | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 8χ | 8Ɛ | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 9χ | 9Ɛ | χ0 |
| χ1 | χ2 | χ3 | χ4 | χ5 | χ6 | χ7 | χ8 | χ9 | χχ | χƐ | Ɛ0 |
| Ɛ1 | Ɛ2 | Ɛ3 | Ɛ4 | Ɛ5 | Ɛ6 | Ɛ7 | Ɛ8 | Ɛ9 | Ɛχ | ƐƐ | 100 |



in our new system as we use in our present system, but we will need to find others to take the place of our ten, eleven, and twelve. If we would use χ, dek, for our present ten, Ɛ, el, for our present eleven, and 10, do, for our present twelve, our number system would look like that which I have illustrated in Chart I, and would be pronounced one (1), two (2), three (3), four (4), five (5), six (6), seven (7), eight (8), nine (9), dek (χ), el (Ɛ), do (10), do-one (11), do-two (12), do-three (13), do-four (14), do-five (15), do-six (16), do-seven (17), do-eight (18), do-nine (19), do-dek (1χ), do-el (1Ɛ), twodo (20), and so on. Then 30 would be pronounced threedo; 90, ninedo; χ0, dekdo; Ɛ0, eldo; and 100, gro.

One very important thing we must remember when we use our new system is that the number 10 no longer means one ten and no units, but one twelve and no units; therefore 12 now means one twelve and two units, or our present 14; 58 now means five twelves and eight units, or our present 68, etc. If we remember this principle, we can easily proceed with the fundamental mathematical processes. Addition and subtraction problems are worked the same as we work them in our ordinary arithmetic. If we want to add 36, 49, and 20, our answer will be χ3. If we want to subtract 19 from 22, we will get 5, or 144 from 396 will leave us 252. I have illustrated our new multiplication table in Chart II. Looking at the chart, we can see that if we multiply 6 by 7, we will arrive at 36 for our answer; or, multiplying 7 by Ɛ, we will get 65. Division, then, will be just the opposite of multiplication. Dividing 698 by χ, we will get 82; and 5Ɛχ90 divided by 10 will give us 5Ɛχ9.

If we were to look once again at our entire present number system, we would find that some parts of it are much better than

## CHART II

Complete Multiplication Table, Base of Twelve

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | χ | Ɛ | 10 |
| 2 | 4 | 6 | 8 | χ | 10 | 12 | 14 | 16 | 18 | 1χ | 20 |
| 3 | 6 | 9 | 10 | 13 | 16 | 19 | 20 | 23 | 26 | 29 | 30 |
| 4 | 8 | 10 | 14 | 18 | 20 | 24 | 28 | 30 | 34 | 38 | 40 |
| 5 | χ | 13 | 18 | 21 | 26 | 2Ɛ | 34 | 39 | 42 | 47 | 50 |
| 6 | 10 | 16 | 20 | 26 | 30 | 36 | 40 | 46 | 50 | 56 | 60 |
| 7 | 12 | 19 | 24 | 2Ɛ | 36 | 41 | 48 | 53 | 5χ | 65 | 70 |
| 8 | 14 | 20 | 28 | 34 | 40 | 48 | 54 | 60 | 68 | 74 | 80 |
| 9 | 16 | 23 | 30 | 39 | 46 | 53 | 60 | 69 | 76 | 83 | 90 |
| χ | 18 | 26 | 34 | 42 | 50 | 5χ | 68 | 76 | 84 | 92 | χ0 |
| Ɛ | 1χ | 29 | 38 | 47 | 56 | 65 | 74 | 83 | 92 | χ1 | Ɛ0 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | χ0 | Ɛ0 | 100 |

others. As a matter of fact, our measuring system uses twelve as a base, so one-half, one-third, one-fourth, or one-sixth of a foot may be had in even inches; and our yard, which is a multiple of twelve, may be divided evenly into two, three, four, six, nine, twelve, or eighteen parts. Our time is also well divided. Our twenty-four hour day is divided into two parts of twelve hours each; so we may work in shifts of an even two, three, four, six, eight, or twelve hours each.

Perhaps the most useful invention in mathematics since the zero is the decimal. The ten system is quite poor for the use of decimals, because many numbers have remainders that are infinitely long; so they can never be expressed with complete accuracy as decimals. Duodecimals are usually more accurate than decimals, even when they are both carried out to the same number of places; they are usually simpler too. They are particularly good for expressing the smaller fractions, which are most frequently used: *(1/2 = .6, 1/3 = .4, 1/4 = .3, 1/6 = .2, 1/8 = .16, 1/9 = .14,* etc.*)*. A percentage system based on one hundred forty-four as the whole would be able to express more accurately many more of our much-used fractions than our present system with one hundred as its whole can express; for example, *1/3* would equal an even *40%*, *2/3*, an even *80%*, *1/4*, *30%*, *3/4*, *90%*, and so on.

If a base of twelve were adopted, much of the drudgery of mathematics would disappear, for all calculations could be made with much simpler figures. Therefore, I believe it would be a good idea to change - gradually, of course - our numerical system with the base of ten, to this easier system with the base of twelve. Just what method should be used to make this change is debatable. Possibly, if duodecimals were introduced to school children in the form of mental exercises, enough could be learned about them so that when the masses of people finally did learn of their value and decided to change to this more sensible numerical system, some of those who learned them in school as children could immediately begin using them to advantage in business accounts, and others could help to educate the masses in their newly acquired mathematical system. It's true that, if the change were made today, the present generation would have quite a difficult time adjusting itself to the use of this new system, but wouldn't these present day sacrifices be worth-while if we were to make mathematics so much easier for the future generations? I think they would be; but, sadly enough, we must realize that man bases his life on his experiences and habits rather than on his own power of reasoning, so he will probably continue to count on his fingers for the rest of his days on earth.



## THE ANNUAL MEETING

This year, the Annual Meeting was held at the Gramercy Park Hotel in New York City, and was convened at 8:30 P.M., January 22nd.

It was the best meeting we have had. Even distant members came to New York for the event, the longest trip being Harry Robert's journey from Atlanta.

The reports of officers and committees was the first business considered. The Treasurer reported expenditures for the year of *1278.46*, which exceeded receipts by *$388.96*. He expressed the Society's grateful acknowledgment of donations of *$625.75*. A balance of *$625.87* is carried forward into 1948.

The Secretary announced that duodecimals were attracting wider attention, and that, especially among undergraduates, our publicity was securing a more active response. Mary Lloyd reported that six Aspirants had been advanced to full membership during the year.

The Nominating Committee proposed the re-election of the Directors whose terms were expiring, and named Harry C. Robert for the vacancy in the Board occasioned by the death of F. Morton Smith. These nominees were elected, and a new Nominating Committee for 1948 was elected, consisting of Albert De Valve, Chairman, Thomas W. Delaney, and Edward W. Pharo, Jr.

Chairman Terry reviewed the record of the Society's Annual Awards. They have always marked our recognition of some major accomplishment in duodecimal development. It is to be expected that there will be some years in which no development will occur which is deemed deserving of this special recognition. This year there could be no such question. The Committee was happy to confer the Annual Award for 1948 upon Harry C. Robert, Jr., for his many excellent papers on the mathematics of duodecimals, with especial citation of his discoveries in that part of the Theory of Numbers dealing with Square Sums of Consecutive Squares.

Mr. Robert's explorations have resulted in many articles in the Duodecimal Bulletin, which have considerably enriched our technological literature. Following the presentation of the Award, Mr. Robert addressed the meeting on "An Engineer Looks at Mathematics and Duodecimals." His remarks on the research possibilities offered by the use of duodecimals were lightened by an occasional anecdote which the meeting found most enjoyable.

Thereafter the meeting was opened to general discussion and refreshments were served. The lively chatter which followed was interrupted to announce the receipt of a telegram from John Selfridge at the University of Washington, Seattle, which read:

"Heartiest congratulations to the Society. We know the Annual Meeting will be successful. Hope to send delegate next year. Will be formally organized here soon.

Washington Duodecimal Society."

This stimulating announcement was enthusiastically applauded. It set a high key for the interest during the rest of the meeting. Dr. Nathan Lazar suggested that the Duodecimal Bibliography be reprinted in pamphlet form, and be distributed to the chairmen of mathematics sections in high school faculties, and to the teachers of the history of mathematics in the colleges and universities.

As is the custom, a meeting of the Board of Directors was bracketed with the Annual Meeting. They re-elected the present officers to serve for another year, but devoted considerable attention to the possibilities of effecting a major change in our official personnel. It was thought that this was desirable for the good of the Society, and to facilitate such change, it was decided to advise the new Nominating Committee to provide for the selection of two additional Directors next year. The difficulty to be met, is that the effective transaction of the Society's business seems to require a centralization in the New York area. As much decentralization as can be achieved within these requirements is to be accomplished.

Our progress in membership was seriously reviewed. The quality of our member personnel is highly satisfactory. Numerically, we are not making as rapid progress as we should. A program of promotion is to be undertaken, which will effect a wider spread of our stimulation.

As a step in effecting a wider appeal, the Board decided to accord tentative acceptance to the system of weights and measures which has been proposed as "The Do-Metric System." This action is to be subject to any further modification the Board may deem desirable. Our introductory folder is to be revised to include an outline of this metric system, and a further exposition of the system is to be carried in an early Bulletin.

President Andrews announced the committee appointments for 1948 as in the appended list. These official actions put into effect a plan of progressive action which make 1948 a year of hard work and fullest promise. In the period following these official meetings, we have received eight new applications for membership.



## OFFICIAL ROSTER for 1948

| | | |
|---|---|---|
| *Chairman of the Board* | George S. Terry | Class of 1950 |
| *President* | F. Emerson Andrews | 1951 |
| *Vice-President* | Paul E. Friedemann | 1949 |
| *Secretary-Treasurer* | Ralph H. Beard | 1950 |
| *Members of the Board* | William S. Crosby | 1951 |
| | Harry C. Robert, Jr. | 1949 |

*Committee on Awards*

To decide whether a 1949 Award is to be made, and if so, to recommend for that Award a person of outstanding achievement in mathematical research, with relation to duodecimals.

George S. Terry, Chairman, 507 Main St., Hingham, Mass.
F. Emerson Andrews, Ralph H. Beard.

*Committee on Bibliography*

To assemble, from all sources, an annotated bibliography on the duodecimal system.

Lewis Carl Seelbach, Chairman, 163 Davidson Ave.,
Buffalo 15, N.Y.

*Committee on Finance*

To review the Society's budget, act upon major expenditure proposals, and audit the report of the Treasurer.

George S. Terry, Chairman, 507 Main St., Hingham, Mass.
Ralph H. Beard, H. K. Humphrey.

*Committee on Mathematical Recreations*

To prepare material in this field for the Bulletin, and to be a center of member correspondence on the subject.

Mary B. Lloyd, Chairman, 2304 Forest Park Ave.,
Baltimore 7, Md.
George S. Terry, Ralph H. Beard.

*Committee on Mathematical Research*

To conduct original research, to check Bulletin submissions for accuracy and validity, and to be an advisory center on the application of duodecimals in any field of mathematical research.

Harry C. Robert, Jr., Chairman, 1683 Johnson Rd., N. E.,
Atlanta 6, Ga.
Donald Meeker Brown, William S. Crosby.

*Committee on Membership*

To promote new memberships, to handle membership examinations and encourage qualification for advanced standing, and to endeavor to fit members into useful work and acquaintanceships.

Mary B. Lloyd, Chairman, 2304 Forest Park Ave.,
Baltimore 7, Md.

W. B. Campbell, Paul E. Friedemann

*Committee on Public Information*

To spread information concerning the duodecimal system through correspondence, meetings and discussion groups, publication, preparation of news material, encouragement of special articles, work through schools and colleges, and other appropriate means.

Paul Van Buskirk, Chairman, 1524 Clairmount Ave.,
Detroit 6, Mich.

George Carreras, John Selfridge

*Committee on Weights and Measures*

To promote discussion and use of the Do-Metric System, to investigate further proposals in weights and measures, including problems of navigation and avigation, and to be an advisory body in this field.

Ralph H. Beard, Chairman, 20 Carlton Place
Staten Island 4, N.Y.

M. Irving Chriswell, William S. Crosby, Thomas W. Delaney, Eugene M. Scifres.

## FOUR FOURS

The problem of the Four Fours is: To express the consecutive numbers as far upwards as possible in terms of four 4's, using the ordinary practices and notation of arithmetic and algebra. It is a problem of absorbing interest. An article in the Bulletin of April, 1946, (Vol. 2, No. 1,) explained four different sets of concepts as to what the ordinary practices and notations of arithmetic and algebra might be considered to be. For each, of course, there is a different range of attainment possible. Different possibilities also will exist for the different number bases. We are interested in stimulating the exploration of the possibilities for the duodecimal base.

The simplest set of conditions, and the most limited, is to confine the expression to the use of +, −, x, and ÷ . In duodecimals, the list has been carried as far as *26* for this class, which is termed Condition A.

Condition B adds the use of powers and of square roots and similar radicals. Condition C includes factorials. Condition D enlarges the limits to include integral indices, and subfactorials.

We list below the statements under Condition A as far as *26*, for duodecimals. Can you extend this list further?

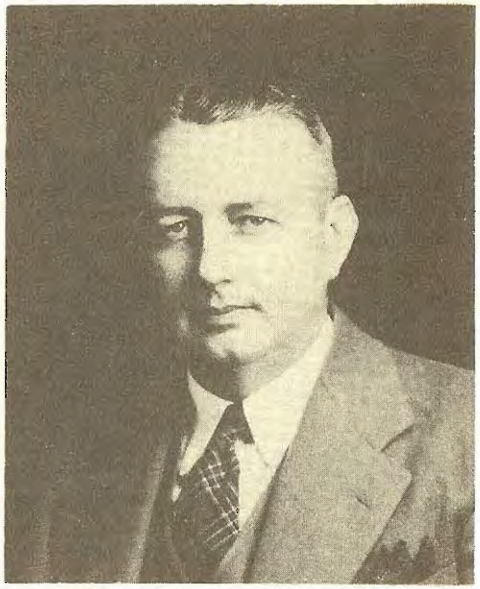


| | | | | | |
|---|---|---|---|---|---|
| *1* = | $4 - 4 + \frac{4}{4}$ | *11* = | $\frac{4}{.4} + \frac{4}{4}$ | *21* = | $\frac{4.4 + 4}{.4}$ |
| *2* | $\frac{4}{4} + \frac{4}{4}$ | *12* | $\frac{44 + 4}{4}$ | *22* | $\frac{4 + 4}{.4} + 4$ |
| *3* | $\frac{4 + 4 + 4}{4}$ | *13* | $4(4) - \frac{4}{4}$ | *23* | $4(4) + \frac{4}{.4}$ |
| *4* | $4 - 4(4-4)$ | *14* | $4 + 4 + 4 + 4$ | *24* | $4(4) + \frac{4}{.4}$ |
| *5* | $4(4) - \frac{4}{.4}$ | *15* | $\frac{44}{4} + 4$ | *25* | $\frac{4}{.4(.\dot{4})} - 4$ |
| *6* | $4 + \frac{4 + 4}{4}$ | *16* | $\frac{4 + 4}{.\dot{4}} - 4$ | *26* | $\frac{4 - .\dot{4}}{.4(.\dot{4})}$ |
| *7* | $4 + 4 - \frac{4}{4}$ | *17* | $\frac{4}{.4} + 4 + 4$ | *27* | |
| *8* | $4 + 4 + 4 - 4$ | *18* | $\frac{4}{.4} + 4 + 4$ | *28* | |
| *9* | $\frac{44}{4} - 4$ | *19* | $\frac{\frac{4}{.\dot{4}} - 4}{.4}$ | *29* | |
| *χ* | $\frac{4}{.\dot{4}} - \frac{4}{4}$ | *1χ* | $\frac{4}{.\dot{4}} + \frac{4}{.\dot{4}}$ | *2χ* | |
| *Ɛ* | $\frac{4}{.\dot{4}} - 4 + 4$ | *1Ɛ* | $\frac{4}{.\dot{4}} + \frac{4}{.4}$ | *2Ɛ* | |
| *10* | $\frac{4\ .\dot{4}}{.4\dot{4}}$ | *20* | $4(4) + 4 + 4$ | *30* | |

## THE ANNUAL AWARD

Each year, the Society confers an Award in recognition of outstanding accomplishment in research, in development of duodecimal technology, or other exceptional contribution to the advancement of duodecimals, or of the Society.

The Award for 1944 was conferred upon F. Emerson Andrews, as the author of "New Numbers," and of many articles on duodecimals, including "An Excursion in Numbers," which appeared in the Atlantic Monthly for October, 1934.



The 1945 Award went to George S. Terry for his fundamental work on the mathematical tables of duodecimals, published as the "Duodecimal Arithmetic," and, in more popular form, as "The Dozen System."

In 1946, the work of the late F. Howard Seely on the still unpublished elementary arithmetic, and in preparation of the tests for our Aspirants, was similarly recognized.

Ralph H. Beard was honored with the Award for 1947, for his work on the Do-Metric System.

This year, the Society has conferred the Annual Award for 1948 upon Harry C. Robert, Jr., of Atlanta, Ga., for his many excellent papers on the mathematics of duodecimals, and for his discoveries in the Theory of Numbers, relating specifically to the Square Sums of Consecutive Squares.

Mr. Robert is chairman of the Society's Committee on Mathematical Research. His paper on "The Fibonacci Series," in the Duodecimal Bulletin for February, 1947, (Vol. 3, No. 1,) attracted considerable public comment, and has awakened new interest in that series and its applications.

Duodecimals are an exceptional tool for research in numbers because of their peculiar aptitude in developing latent numerical patterns. Mr. Robert is putting them to good use, and the Society honors itself as well as Mr. Robert in making this Award.



*Editorial Note: The following is an abstract of a booklet published in 1913 by P. S. King and Son, London. Our efforts to reach the author and the publisher have been unsuccessful.*

## THE NEW ENGLISH SYSTEM OF MONEY, WEIGHTS AND MEASURES AND OF ARITHMETIC

by Engineer Rear Admiral G. Elbrow, R.N.,
with an introduction by George Moores, F.S.S.

### Introduction

Attempts have been made from time to time during the last hundred years or more to introduce compulsorily into the United Kingdom the decimal system of reckoning. The only success that has attended these efforts has been to introduce a new coin—the tenth part of a sovereign, or the florin, as it is named. Over 50 years have elapsed since the two-shilling piece was first minted, and the prophecies that its introduction would cause a "rush for decimal coinage" have all been falsified. The double florin, or one-fifth of a pound, has proved so useless, that it has been decided not to coin any more, and the older and much more useful half-crown, or one eighth of a pound, is to-day so much better liked than the florin that that coin, the only decimal coin we possess, might well be dropped, for its presence is only confusing. Here we have a case of the survival of the fittest, and the freedom to use whichever coin is most serviceable has overwhelmingly demonstrated which is the fittest—the decimal or the binary part of the pound. So it is with weights and measures. Since 1897 the decimal system based on the metre has been lawful in the United Kingdom for all purposes, and thousands of pounds have been spent in having metric standards made and distributed in various parts of the Kingdom in order to secure its adoption. It is all money wasted. No practical man wants the metre or any decimal derivative of it. It is "put up with" in some cases, but never adopted, from choice.

To make a long story short, experience—Continental experience as well as experience in the United Kingdom—proves that decimal divisions are bad for the practical pursuits of life, but, inasmuch as decimals agree with our radix of arithmetical notation, they are often very serviceable for computative purposes.

What is it, then, that metrological reformers want? They are seeking for a universal system of weights, measures, and coinage which shall be adaptable for all peoples and serviceable for all purposes. Such a system can only be brought about in one way, and that is the way shown by the gallant author of this little book. The authorities who preside over the decimal-metric headquarters at Sevres know quite well that their system is a retrograde system, and they are equally alive to the fact that sound, lasting progress to the goal which their compatriots of the eighteenth century set out to attain, can only be reached by adding two more figures to our notation and thus open the way for the introduction of a system which shall be suitable alike to the scientists, the manufacturers, the builders, and the traders ofevery country.

GEORGE MOORES.

For many years past there has existed a feeling of profound dissatisfaction with the present system of Weights and Measures, and of the Compound Arithmetic they render necessary. Now and again an attempt has been made to stir the nation to throw off the yoke of this bondage, and adopt some other system that will save their time and spare their brains. It must be admitted that the introduction of a new system of metrology would cause great temporary inconvenience to the whole community, but this fact does not sufficiently explain the hesitation previously shown to making any change. The real objection appears to be that no other system has been hitherto devised that appears sufficiently good, from every point of view, to warrant such an upheaval as its adoption would produce. As regards simplicity, the French Metric system is generally regarded as the best in extensive use, but from an English point of view that is the only recommendation, and this explains the failure of those who during many years have vainly endeavoured to convince this nation that they should adopt the Metric system in the place of their own. In the early 'eighties an important discussion took place in the House of Commons on this question, when the following fatal objections were expressed to our adoption of the Metric system. Mr. W. E. Gladstone said that no change should be made which would affect the value of the English penny. Sir W. Harcourt stated that, while the English pound sterling was too high a unit for National finance and large business transactions, the franc was much too low in value. Another distinguished statesman, happily still living, objected that the base ten of this system is not suited for its purpose, saying that the half of ten is five, and the half of five is two-and-a-half, and that a base capable of better subdivision is essential. In these three statements lie the chief obstacles to our adoption of the Metric system, viz.: — that a more suitable base is required, and that the names and values of the units fail altogether to satisfy English requirements, and, we may add, English sentiment. In the pages of this book is set forth a new system, devised by the writer, with the view of its being free from any of the above objections, which is called the Duodenal system because it is based on twelve, in the same way as the Decimal system is based on ten. The practical difficulty with the decimal system is that the base ten cannot be divided by three and four without fractions, hence that system is inapplicable to the division of the circle and time, and consequently cannot be used in Navigation, Geography and Astronomy, whereas the Duodenal system is equally applicable in all branches of metrology.



The number twelve is greatly used in the present English system; thus we have twelve pence in a shilling, twelve inches in a foot, twelve ounces in a Troy pound, twelve hours on the face of a clock, twelve months in a year, twelve signs in the zodiac. Twelve is known as a dozen, twelve dozen is a gross, and twelve gross is a great gross. Every school child learns the Tables up to twelve times twelve.

In the Tables of Money, Weights and Measures set forth in this book, every principal unit after the lowest is equal in value to that of the principal unit preceding it multiplied by twelve, or twelve times twelve, or some higher multiple of twelve, and after the remarks in the preceding paragraph it is thought that this will appeal to English people as the most simple and natural mode of progression.

The number twelve appears to be the best suited for an arithmetic base for the reason that it can be divided by *2*, *3*, *4* and *6* without leaving fractions. But, in order that Duodenal arithmetic may be equally simple with Decimal arithmetic, it is necessary that the base twelve in the former shall be represented by the figures *10*, in the same manner as the base ten is in the latter. And further, that the numbers ten and eleven shall each be represented by a new single figure; the figures adopted are *Χ* for ten and *Ɛ* for eleven. (Note: Where the author has used script capitals for X and E, we have used *Χ* and *Ɛ*.) Further, their denominations have been changed to teen and lin respectively. There are obvious objections to this new method of notation, but after well weighing them this decision has been arrived at. Several names have been considered for the new base number, but twelve has been finally adopted because this will certainly tend to make the transition from the present system to the new one so much easier. There are many reasons for changing ten to teen, a main one being that tenty and twenty sound too much alike. Eleven is changed to lin chiefly because eleventy eleven is more cumbrous than necessary.

It is proposed that the number expressed by the word twenty and by the figures *20* shall equal twice twelve; thirty and *30*, three times twelve; and so on to teenty and *X0*, teen times twelve; linty and *Ɛ0*, lin times twelve; one hundred and *100*, twelve times twelve.

Two hundred and *200* will represent twice this new hundred, and so on to teen hundred and *X00*, lin hundred and *Ɛ00*, one thousand and *1,000*, the last representing twelve hundred.

Similarly for thousands, twelves of thousands, etc.

Between *10* and *20* there will be lin numbers named tyone expressed by *11*, tytwo by *12*, tythree by *13*, and so on to tyteen by *1X*, and tylin by *1Ɛ*. Other additional numbers will be twenty teen expressed by *2X*, twentylin by *2Ɛ*, and so on to linty teen by *ƐX*, and linty lin by *ƐƐ*, the next number after the last being *100*.

From the foregoing it will be seen that the same figures represent different numbers in the present and new systems, and as it is necessary in this book to employ figures for both, it must be clearly indicated to which system they belong; all figures, therefore, which refer to the Decimal system of notation are placed in brackets, while those not in brackets refer to the Duodenal system. When figures of the two systems are placed together it implies that they are equal, that is, they represent the same number of units; thus *10* (12), *100* (144), *1,000* (1,728), *84* (100), *6Ɛ4* (1,000), *XƐ* (131), *9ƐX* (1,438) and *1,135* (1,913) are examples of equal numbers under the two systems. Children learning the Tables should take no notice of the figures in brackets, as they will not be required after the new system is established, except for reference.

We now proceed to make out a Numeration Table, and also Tables of Addition and Multiplication, on the Duodenal system, and by means of these all questions in Simple Arithmetic are solved in an exactly similar manner to that of the Decimal system. But the great and all-important recommendation of the Duodenal system is the fact that Compound Arithmetic is also worked by it on the simple method of the Metric system. This follows necessarily from the fact of the conditions being the same in these two systems, namely, that the figures of the numeration table and those of the tables of money, weights and measures, range themselves together respectively in the same relative proportion of value to each other, and so they can be dealt with alike by the same simple arithmetic.

A few words of explanation with regard to the proposed new Tables are now necessary. The principles adopted are the following:—



1. To compile them on the Duodenal system.
2. To establish a direct and simple relationship between the units of length, capacity, and weight.
3. To utilise to the utmost extent possible the existing Tables, making as few changes as are absolutely necessary, and those of least inconvenience. While practically all the former terms have been retained, only *6* new denominations have been introduced, viz., Point, Quin, Burden, Duor, Mil, and Royal. The Quin is a quarter-inch, and the Duor = *2* hours. The term Burden will appear, it is thought, very appropriate for the amount of the weight, viz., *100* (144) pounds; the terms Hundredweight, New hundredweight, and Cental may well be left behind. The Royal is recommended to replace the present pound or sovereign. As regards the Mil, though it is not suggested that an actual coin should be created, still merely as an abstract unit, it is necessary for Arithmetic, and should be valuable in providing a finer graduation for rates of sale and purchase. Unfortunately, it has been necessary to alter considerably the actual and relative values of some of the units, but this has been done with the utmost care, so that the values which matter most in daily life and business are changed the least. Thus the Yard, Foot, Palm, Inch (with their Squares and Cubes), Hour, Shilling and Penny are not altered at all; while the Pound, Pennyweight, Carat, Mile, and Minim are only slightly changed in value. The new Grain and the new Second of time are each about one-third their present values; it is thought this will be an advantage.

The new pound is the weight of $3^3$ or (27) cubic inches of water, as compared with (27.7274) cubic inches for the present pound; it is thus about two-fifths of the present ounce less than the present pound, a fact to be regretted, but which will scarcely be apparent in ordinary practice, and only represents the value of half a farthing on a four-pound loaf.

Probably the alterations that will come most under notice are those made to the units of Dry and Liquid Measures, the Pint, Quart, Gallon, Barrel, Bushel, Quarter, and Wey; the only apology must be that this appeared the best method of bringing them into the Duodenal system. The liberty has been taken of reversing the relative values of the Barrel and Kilderkin; this is to bring the Barrel into the list of principal units, as the term is so much better known and understood than Kilderkin.

The principal units in each Table are those which follow one another in multiples of twelve. The intermediate units have been made to fit in as suitable multiples of the principal units. But it may be asked, why have any intermediate units? The answer is (i.) because it is the English way; (ii), because in certain businesses they are most convenient if not absolutely necessary;

and (iii.), because they do not prevent the Duodenal system of arithmetic being fully and easily carried out. Just as sixpences and florins are necessary coins, but a day's takings or spendings are all reduced to royals, shillings, and pence, so may a similar reduction be easily made of other intermediate units.

The Palm = *3* inches must be universally recognised as an all-important unit of measurement in the Duodenal system. For this reason it is essential that the Palm and its smaller relation, the quin, should take the place of the foot and inch in drawings and descriptions of buildings, bridges, ships, engines, guns, etc. They should be used, too, for small measurements in the drapery and allied businesses, instead of fractions of the yard; also in the hardware and other businesses instead of the foot and inch. For this purpose the ordinary yard measure should be divided and marked in palms and quins; also the present two-foot and foot rules should be replaced by six-palm and three-palm rules divided and marked in palms, quins, and lines; this would make all calculations of prices perfectly simple.

A bitter cry often goes up for something easier, some simpler system than our present English one. Here then is a system which is strongly recommended for adoption by all the English speaking peoples. In the preparation of this book the writer is necessarily indebted to the authors of published works for many of the ideas and methods herein adopted, and to all of them he begs to express his obligations and thanks. May he appeal to them, also to teachers in schools, the scientific and commercial societies of the country, members of Parliament, and the general public to treat this venture kindly, and give it a thorough consideration and trial.

What are the alternative systems as far as we know?

(i.) Present English system.
(ii.) Metric system.
(iii.) Duodenal system, as proposed in this book.
(iv.) The present English system decimalised.

The writer is not aware whether the latter has yet been attempted or not, but the objection to it is as before stated to the Metric system, viz., that the base ten cannot be sufficiently subdivided to be useful for commercial purposes, and it cannot be applied to time, the circle, geography, navigation and astronomy.

But it would possess in common with the Duodenal System this unspeakable advantage over the Metric System, as far as Englishmen are concerned, viz., that instead of using the monotonous, unpoetic terms of the latter, we should retain the old familiar names that have come down to us through the centuries, and are woven into our literature and proverbs.



Certain results of the adoption of the new system may be mentioned.

A new Table of Logarithms and a revised Nautical Almanac become necessary. The appearance of the face of a clock will be very little changed; the figures will require some alteration, and each space between successive figures will represent *6* minutes instead of *5*, there being *60* (72) minutes in the hour. The even numbers will represent, as before, *10*, *20*, *30*, *40*, and *50* minutes past the hour; but the odd numbers will represent *6*, *16*, *26*, *36*, *46*, and *56* minutes past. The seconds hand must travel round its new period.

The length of the new Statute Mile is to that of the old mile as *46* (54) is to *47* (55); so that in a length of railway, for instance, *1* mile must be added for every *46* (54) of present measurement to get the new length. For new distance in Nautical Miles, subtract *1* for every *21* (25) miles in present measurement. Similarly for the new area of a plot of land subtract *1* for every *13* (15) Acres in the present measurement. The new Quarter of corn,,etc. (containing *4* new Bushels) is seven-eighths the capacity of the present quarter. The new Ton is three-quarters the weight of the present ton.

As the new number *100*, read one hundred, equals (144) of the present system, it follows that an Interest of nearly *1½%* will have to be paid for the loan of money, to correspond in value with each *1%* paid under the present system. The Watt and Horse Power are brought under the new system, and the new value of each is approximately three-quarters its present value, their relative value remaining about the same.

## TABLES OF WEIGHTS AND MEASURES

In the following tables it should be remembered that—

| | | | |
|---|---|---|---|
| *10* | reads twelve | = | 1 dozen. |
| *100* | reads hundred | = | twelve times twelve, or 1 gross. |
| *1,000* | reads thousand | = | twelve hundred, or 1 great gross. |
| *10,000* | reads twelve thousand, and so on. | | |

All figures which refer to the Decimal system of notation placed in brackets, while all those not in brackets belong to the Duodenal system. When numbers of the two systems are placed together, e.g., *50* (60), *100* (144), it implies that they are equal, that is they represent the same number of units.

### *Measures of Length*

| | |
|---|---|
| *10*-twelve Points (pt) make | 1 Line (ln) |
| *10* Lines | 1 Quin (qn) |
| *4* Quins | 1 Inch |
| *10* Quins, or *3* Inches | 1 Palm (pm) |
| *4* Palms | 1 Foot |
| *10* Palms, or *3* Feet | 1 Yard (yd) |
| *10* Yards | 1 Pole (pol) |
| *10* Poles | 1 Furlong (fur) |
| *10* Furlongs | 1 Mile (ml) |
| *4* Miles | 1 League |

The Land Chain (proposed) used in surveying is *3* poles, or *30* Yards Long, divided into *100* (144) links joined by rings. Each Link with ring attached measures *3* Palms. A square plot of land of which the side is a chain in length measures *1* Rood.

The Mile measures *1,000* (1,728) Yards.

The Nautical Mile (proposed) measures *1,280* (2,112) Yards. As this = *1* Minute of Longitude at the Equator, or

$\frac{1}{10{,}000}$ part of the circumference of the earth, the latter therefore = *12,800* (25,344) new Statute miles, and the mean diameter of the earth approximately = *4,800* (8064) miles.

A Degree of the Meridian measures *100* (144) Nautical miles or *128* (176) Statute Miles.

The new Nautical mile is to the new Statute mile as *£* is to *9*.

The Fathom = *2* Yards is used in measuring the depths of the sea.

The Hand of *4* Inches is used in measuring horses.

### *Measures of Surface*

| | |
|---|---|
| *100* Square Quins (sq. qn.) make | 1 Square Palm (sq. pm.) |
| *14* (16) Sq. Palms, or *100* Sq. Inches | 1 Sq. Foot |
| *100* Sq. Palms | 1 Sq. Yard |
| *100* Sq. Yards | 1 Sq. Pole |
| *9* Sq. Poles, or *1* Sq. Chain | 1 Rood |
| *4* Roods, or *3,000* (5,184) Sq. Yards | 1 Acre |
| *100* Sq. Poles | 1 Sq. Furlong |
| *100* Sq. Furlongs, or *400* Acres | 1 Sq. Mile (sq. ml.) |

The Chain (proposed) used in surveying is *3* Poles or *30* (36) Yards long.



### *Measures of Solidity*

| | |
|---|---|
| *1,000* cubic Quins (cub. qn.) make | 1 cubic Palm (cub. pm.) |
| *54* (64) cubic Palms, or *1,000* Cub. Ins. | 1 Cub. Foot |
| *1,000* cub. Palms, or *23* (27) cub. Feet. | 1 Cub. Yard |

| | |
|---|---|
| A cubic Yard of distilled water weighs | 1 Ton |
| The Standard Bushel measures | 100 cubic Palms |
| The Imperial Gallon | 10 cubic Palms |
| The Standard Pint or Pound of Water | 1 cubic Palm |

### *Weight*

| | |
|---|---|
| *10* Grains (gr.) make | 1 Carat (ct.) or Scruple (scr.) |
| *6* Carats | 1 Pennyweight |
| *10* Carats, or Scruples | 1 Dram (dr.) |
| *10* Drams | 1 Ounce (oz.) |
| *10* Ounces | 1 Pound (lb.) |
| *10* Pounds | 1 Stone (st.) |
| *10* Stones | 1 Burden (bur.) |
| *10* Burdens | 1 Ton |

*1* Pound is the weight of *1* cubic Palm of distilled water.
*1* Ton " " *1* cubic Yard " "
The Specific Gravity of any substance is the weight in Pounds of a cubic Palm of the substance.

## STANDARD MEASURE OF CAPACITY

The standard measure of capacity, both for liquids and dry goods, is the Gallon, which measures twelve cubic palms, and contains twelve pints or twelve pounds of distilled water, weighed in the air, at *20* degress of the Thermometer,* the barometer being at *X0* quins (30 ins.).

The standard Bushel equals *10* (12) gallons, contains *100* (144) cubic palms, and measures *75* (89) quins in diameter, and *34* (40) quins deep.

The standard Peck contains *30* (36) cubic palms, and measures *50* (60) quins in diameter, and *1X* (22) quins deep.

The standard Pint measures *1* cubic palm.

* The proposed Thermometer is graduated from *0*° at freezing point to *100*° (144°) at boiling point.

### *Dry Measure*

| | |
|---|---|
| *3* Pints make | 1 Quart |
| *10* Pints (pt.) or *4* Quarts | 1 Gallon (gall.) |
| *3* Gallons | 1 Peck |
| *10* Gallons, or *4* Pecks | 1 Bushel (bush.) |
| *2* Bushels | 1 Coomb |
| *4* Bushels | 1 Quarter |
| *10* Bushels | 1 Wey (wey) or Chaldron (chdn.) |
| *4* Weys | 1 Last |

| | |
|---|---|
| *1* Pint measures | 1 cubic Palm |
| *1* Quarter | 9 cubic Feet |
| *1* Wey, or *3* Quarters | 1 cubic Yard |

Corn, Grain, Pulse, Fruit, Vegetables, Coke, etc., are sold by this Measure.

### *Liquid Measure*

| | |
|---|---|
| *10* Fluid Grains (fl. gr.) or *4* Minims make | 1 Fluid Scruple (fl. scr.) |
| *10* Fl. Scruples | 1 Fl. Dram (fl. dr.) |
| *10* Fl. Drams | 1 Fl. Ounce (fl. oz.) |
| *3* Fl. Ounces | 1 Gill |
| *10* Fl. Ounces, or *4* Gills | 1 Pint (pt.) |
| *3* Pints | 1 Quart |
| *10* Pints, or *4* Quarts | 1 Gallon (gall.) |
| *6* Gallons | 1 Firkin |
| *10* Gallons | 1 Barrel (brl.) |
| *2* Barrels | 1 Kilderkin |
| *4* Barrels | 1 Hogshead |
| *6* Barrels | 1 Puncheon |
| *8* Barrels | 1 Pipe or Butt |
| *10* Barrels | 1 Tun |

A Fluid Scruple of distilled water weights *1* Scruple and measures *1* cubic quin; a Minim (or a *drop*) is one-fourth of this quantity.

| | | | |
|---|---|---|---|
| *1* Pint | of distilled | water weights | *1* Pound |
| *1* Gallons | " | " | *1* Stone |
| *1* Barrel | " | " | *1* Burden |
| *1* Tun | " | " | *1* Ton |



### *Time*

| | |
|---|---|
| *100* Seconds (sec.) make | 1 Minute (min.) |
| *60* (72) Minutes | 1 Hour |
| *100* Minutes | 1 Duor (dr.) |
| *10* Duors, or *20* (24) hours | 1 Day |
| *7* Days | 1 Week |
| *265* (365) Days | 1 Year |
| *266* (366) Days | 1 Leap Year |
| *10* Months | 1 Year |

### *Angular Divisions of the Circle*

| | |
|---|---|
| *100* Seconds (″) make | 1 Minute (′) |
| *100* Minutes | 1 Degree (°) |
| *100* Degrees | 1 Circle |

| | |
|---|---|
| *10* Degrees make | 1 Sign |
| *10* Signs | 1 Circle of the Zodiac |
| *30* (36) Degrees | 1 Right Angle |

*4½* Degrees between two adjacent points of the Compass.

The English Nautical Mile (proposed), which equals *1* minute of Longitude at the Equator, measures *1,280* (2,112) Yards.

There are *100°* of Longitude, viz.: *60°* E. and *60°* W. of Greenwich.

*1* Duor difference of time at two places corresponds to *10°* difference of Longitude.

*1* Hour corresponds to *6°*.

### *Money*

| | |
|---|---|
| *10* Mils (m) make | 1 Penny (p.) |
| *10* Pence | 1 Shilling (s.) |
| *10* Shillings | 1 Royal (R.) |

## MISCELLANEOUS

The cubic Palm, the Pint, and a Pound of distilled water are all of the same capacity. So are the Bushel, the Barrel, and a Burden of water, viz., *100* cubic Palms, So also are the Wey, the Tun, and a Ton of water, viz., a cubic Yard.

The Specific Gravity of a solid of liquid is the weight in Tons of a cubic Yard of the substance.

One quin of rainfall = *10,000* tons per sq. mile.

One inch of rainfall = *100* tons per acre.

One Atmosphere = *13.14* or $13\frac{1}{9}$ lbs. per sq. in.

A system of absolute units in which the Quin, Grain, and Second are the fundamental units is known as the Q.G.S. system.

Acceleration of gravity = *136.4* quins per second in *1* second.

The weight of *1* grain gives to the mass of a grain a velocity of *136.4* qns. per sec. in *1* sec.

$$\text{Unit of force, or } 1 \text{ Vig} = \frac{1}{136.4} \text{ grain.}$$

Unit of work, or *1* Comp = *1* Vig — quin.

Unit of rate of working
or *1* Watt ...... = $10^7$ Comps per second.
∴ *1* Watt = *93.35* palm-pounds per minute.
*1* Horse Power = *40,000* pm. lbs. per min.
= *10,000* ft. lbs. per min.
= *4* yard-tons per min.
= *521* (745) Watts.

The British Thermal Unit, or the Heat necessary to raise *1* pound of water, at maximum density, *1* degree of temperature = *2,300* palm-pounds.

The Thermometer is graduated from *0°* at freezing point to *100°* at boiling point.

Sound travels at the rate of *Ɛ0* yards per second.

Light travels at the rate of *32,000* miles per second.

*The Values of the Following Units*
*have not been Altered*

| | | |
|---|---|---|
| Yard | Palm | Hour |
| Foot | Inch | Shilling |
| | | Penny |



The following list shows in decimal figures the relative values of units of the same denomination in the present Tables and the proposed new Tables respectively:

| *Denomination* | *Ratio of New Value to Present Value* | *Ratio of Present Value to New Value* |
|---|---|---|
| League | .76 | 1.31 |
| Mile | .98 | 1.02 |
| Furlong | .65 | 1.53 |
| Pole | 2.18 | .46 |
| Line | .25 | 4 |
| Square mile | .96 | 1.04 |
| Acre and Rood | 1.07 | .93 |
| Square Pole | 4.76 | .21 |
| Ton | .75 | 1.33 |
| Stone | .83 | 1.2 |
| Pound (Avoir) | .974 | 1.027 |
| Ounce (Avoir) | 1.3 | .77 |
| Pound and Ounce (Troy and Apoth.) | 1.183 | .845 |
| Dram | .79 | 1.267 |
| Scruple | .197 | 5.07 |
| Pennyweight and Carat | .986 | 1.014 |
| Grain | .329 | 3.042 |
| Last | 1.05 | .95 |
| Wey | .52 | 1.9 |
| Chaldron | .58 | 1.71 |
| Quarter and Coomb | .876 | 1.14 |
| Bushel and Peck | 1.75 | .57 |
| Tun, Firkin, Pint and Gill | .78 | 1.28 |
| Puncheon, Gallon and Quart | 1.17 | .85 |
| Butt and Hogshead | 1.04 | .96 |
| Kilderkin | 1.56 | .64 |
| Barrel | .39 | 2.56 |
| Fluid Ounce | 1.298 | .77 |
| Fluid Dram | .865 | 1.155 |
| Minim | 1.082 | .924 |
| Minute of Time | .833 | 1.2 |
| Second of Time | .347 | 2.88 |
| Degree | 2.5 | .4 |
| Minute of Angle and Nautical Mile | 1.042 | .96 |
| Second of Angle | .434 | 2.304 |
| Horse Power | .733 | 1.363 |
| Watt | .736 | 1.358 |
| British Thermal Unit | 1.217 | .821 |

## AN AID TO CALCULATING 1/*N*

by H. C. R.

1. The American (1947) edition of Ball's "Mathematical Recreations and Essays", pages 53-54, presents an interesting device for calculating $1/N$ decimally (Base X) which is credited to E. Schiffner's articles in "Sphinx", page 115 - 1937 and page - 206 - 1938.

2. In adapting the device to duodecimal (Base XII) calculations we find that its general usefulness is doubled, being in proportion to the number of factors of the number base. It appears that the principle of the device is independent of the number base and therefore applicable to calculations using any *composite* base, although the writer has verified this only for Base XII and Base VIII. The explanation which follows for the dozenal base is adapted from Ball's presentation for the decimal base.

3. The first few digits of $1/N$ are calculated in the usual manner; but after a certain stage we pass from division by $N$ to division by a power of *2*, of *3*, or of *6*. (When the character of $N$ results in this new divisor being *2*, *3*, *4*, *6*, *8*, or even *14* or *23* our work will probably be materially reduced.) Ordinary division by $N$ is continued until the remainder happens to be equal to $2^m$, $3^m$, $4^m$ or $6^m$ or is congruent to some power of a factor of *10*, Modulo $N$. (Such a remainder will always arise for cases of the maximum period of $(N-1)$ digits and generally will arise for other cases.) Assume that we find such a remainder after determining $k$ digits of our fraction, $1/N$, it being necessary $k \geq m+1$. We write the first $(m+1)$ digits in a row from left to right and the rest (if any) in a column below the $(m+1)$th. If the remainder is in the form $a^m$ (Mod. $N$), the new divisor for the balance of the process will be $(10/a)^m$. Using the new divisor, we divide the first $(m+1)$ digits, (top row of our arrangement) and place the units-digit (only) of its quotient in the column below the $k$th digit, and write the full remainder to the left of the $(m+2)$th digit. We then divide the row containing the $(m+2)$th digit (the second row of our arrangement) and write our quotient's digit in the column below the $(k+1)$th and the remainder to the left of the $(m+3)$th digit. The successive quotients and remainders can now be written down rapidly (particularly if our divisor <*10*) until the complete cycle of $1/N$ is obtained. The process can be continued for additional cycles by any of those who lack faith in the recurrence of such fractions. Our final fraction will now be found (written in proper order) by starting at the top left corner of our arrangement, reading across the top row to the right hand column and reading down the right hand column. Several explanatory examples follow.



4. *Examples:* (A) $N = 15$

We calculate $1/N = .08579\ldots$, passing by the first remainder, $8 = 2^3$, because $(m+1) = 4 > k = 2$, and also pass-up the remainder $11 \equiv 4^3$ (Mod. *15*) because it would require use of $(10/4)^3 = 3^3 = 23$ as our new divisor (which would be no improvement on *15*). After finding the 5th digit, *9*, our remainder is *3*, so that we can complete our calculation using $(10/3)^1 = 4$ as our divisor. Since $(m+1) = 2$ we write down the first part of our fraction and then proceed as shown by the arrangement on the right. $08 \div 4 = 2$ which we place under the *9* and the remainder $= 0$ we write to left of *5* in the second row. $05 \div 4 = 1$ which we place under *2* and the remainder, *1*, we write to the left of *7*. $17 \div 4 = 4$ plus $R = 3$, etc.,

| | |
|---|---|
| 08 | 24 |
| 05 | 02 |
| 17 | 29 |
| 39 | 1X |
| 12 | 27 |
| 21 | 30 – Cycle |
| 14 | 08 |
| 0Ɛ | 05 |
| 33 | 7 |
| 36 | 9 |
| | 2 |

so $1/15 = .08579214Ɛ36429X7085\ldots$

(B) Three other simple cases: $N = 1Ɛ$ $1/N = .06$ we obtain a remainder of *6*. New divisor $= D = 10/6 = 2$.

| | | | |
|---|---|---|---|
| $N = 2ƐƐ$ | $1/N = .004$ | $R = 4$ | $D = 3$ |
| $N = 5Ɛ$ | $1/N = .02$ | $R = 2$ | $D = 6$ |

Tabulations follow:

$1/N = 1/1Ɛ$

| |
|---|
| .06 |
| .03 |
| 11 |
| 16 |
| 09 |
| 14 |
| 08 |
| 04 |
| 02 |
| 01 |
| 10 – Cycle |
| 06 |

$1/N = 1/5Ɛ$

| | | |
|---|---|---|
| .02 | 39 | 46 |
| 20 | 37 | 09 |
| 04 | 17 | 31 |
| 40 | 13 | 16 |
| 08 | 32 | 03 |
| 21 | 26 | 30 |
| 14 | 05 | 06 |
| 42 | 50 | 01 |
| 28 | 0X | 10 – Cycle |
| 25 | 41 | 02 |
| 54 | 18 | 20 |
| 4X | 23 | 04 |
| 49 | 34 | |

$1/N - 1/2ƐƐ$

| | | | |
|---|---|---|---|
| .004 | 04 | 28 | 00 |
| 10 | 16 | 20 | 01 |
| 01 | 01 | 0X | 10 – Cycle |
| 14 | 16 | 18 | 00 |
| 10 | 00 | 23 | 04 |
| 05 | 06 | 06 | 10 |
| 24 | 00 | 09 | 01 |
| 11 | 02 | 02 | 4 |
| 19 | 20 | 23 | |
| 04 | 00 | 00 | |
| 17 | 08 | 09 | |
| 11 | 20 | 00 | |
| 16 | 02 | 03 | |

Our results:

$1/1Ɛ = .\dot{0}63\ 169\ 484\ 2\dot{1}06\ldots$

$1/5Ɛ = .\dot{0}20\ 408\ 142\ 854\ Χ99\ 773\ 26$
$50Χ\ 183\ 469\ 163\ 06\dot{1}\ 0204\ldots$

$1/2ƐƐ = .\dot{0}04\ 014\ 054\ 194\ 716\ 461\ 606\ 0200$
$802\ 80Χ\ 836\ 923\ 090\ 30\dot{1}\ 004\ 014\ldots$

(C) We now tackle a slightly more complicated example to complete our explanation, $N = 85$.

$1/N = 1/85 = .0151382462$ $R = 12 \equiv 6^3 \equiv 160 \ (Mod.\ 85)$

$D = (10/6)^3 = 2^3 = 8$ $m + 1 = 4$

| | | | | | | |
|---|---|---|---|---|---|---|
| | .0151 | | 01 | 47 | 52 | 77 |
| $R_1 = 5$ | 53 | | 1Χ | 71 | 68 | 32 |
| $R_2 = 7$ | 78 | | 69 | 56 | 09 | 5Ɛ |
| | 42 | | 18 | 26 | 16 | 37 |
| | 24 | | 4Χ | 64 | 20 | 38 |
| | 46 | | 2Ɛ | 4Ɛ | 08 | 48 |
| | 62 | Start | 32 | 3Χ | 06 | 00 |
| | 21 | $Q_1 = 1$ | 60 | 66 | 67 | 0Ɛ |
| | 17 | $Q_2 = 7$ | 02 | 6Χ | 7Χ | 34 |
| | 3Ɛ | | 2Χ | 28 | 61 | 0Χ |
| | 76 | | 22 | 03 | 12 | 25 |
| | 23 | | 27 | 39 | 63 | 55 |
| | 36 | | 74 | 57 | 31 | 17 |
| | 29 | | 04 | 35 | 50 | Cycle 30 |
| | 13 | | 49 | 19 | 49 | 41 |
| | 72 | | 10 | 5Χ | 1Ɛ | 15 |
| | 65 | | 44 | 64 | 79 | 1 |
| | 5Ɛ | | 43 | 40 | 51 | 3 |
| | 73 | | 33 | 05 | 59 | 8 |
| | 75 | | 7Ɛ | 58 | 54 | 2 |
| | 14 | | 70 | 45 | 07 | 4 |
| | | | | | | 6 |

$1/85 = .\dot{0}15\ 138\ 246\ 217\ ƐƐ3\ 693\ 25Ɛ\ 354$
$1Χ9\ 8ΧƐ\ 202\ Χ27\ 449\ 043\ 3Ɛ0\ 716$
$64Ɛ\ Χ6Χ\ 839\ 759\ Χ40\ 585\ 289\ 608\ 67Χ1$
$231\ 09Ɛ\ 919\ 477\ 2Ɛ7\ 880\ Ɛ4Χ\ 55\dot{7}\ 0151\ldots$

5. Although in all of the examples $N$ has been prime, this is not essential as may be readily found by trying $N = Χ$ or $N = 2Ɛ$. In trying $N = Χ$ however you must skip the first remainder, 2, because the requirement $k \geqq (m + 1)$ cannot be ignored without complicated modifications of the device.



**MAGIC SQUARE** Numbers 1–100 T. More, Jr.

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 1 | 6 | Ɛ2 | Χ7 | Ɛ0 | Χ5 | 9Χ | Χ3 | 2Ɛ | 24 | 29 |
| 3 | 5 | 7 | Χ9 | ΧƐ | Ɛ1 | Χ0 | Χ2 | Χ4 | 26 | 28 | 2Χ |
| 4 | 9 | 2 | ΧΧ | Ɛ3 | Χ8 | Χ1 | Χ6 | 9Ɛ | 27 | 30 | 25 |
| 8Ɛ | 84 | 89 | 45 | 3Χ | 43 | 52 | 47 | 50 | 68 | 61 | 66 |
| 86 | 88 | 8Χ | 40 | 42 | 44 | 49 | 4Ɛ | 51 | 63 | 65 | 67 |
| 87 | 90 | 85 | 41 | 46 | 3Ɛ | 4Χ | 53 | 48 | 64 | 69 | 62 |
| 5Ɛ | 54 | 59 | 75 | 6Χ | 73 | 82 | 77 | 80 | 38 | 31 | 36 |
| 56 | 58 | 5Χ | 70 | 72 | 74 | 79 | 7Ɛ | 81 | 33 | 35 | 37 |
| 57 | 60 | 55 | 71 | 76 | 6Ɛ | 7Χ | 83 | 78 | 34 | 39 | 32 |
| 98 | 91 | 96 | 22 | 17 | 20 | 15 | Χ | 13 | ƐƐ | Ɛ4 | Ɛ9 |
| 93 | 95 | 97 | 19 | 1Ɛ | 21 | 10 | 12 | 14 | Ɛ6 | Ɛ8 | ƐΧ |
| 94 | 99 | 92 | 1Χ | 23 | 18 | 11 | 16 | Ɛ | Ɛ7 | 100 | Ɛ5 |

The sum of any row or column is 606. The sum of any number and its geometric opposite is 101. Below are the squares of which it is composed.

13 =

| | | |
|---|---|---|
| 8 | 1 | 6 |
| 3 | 5 | 7 |
| 4 | 9 | 2 |

2Χ =

| | | | |
|---|---|---|---|
| 1 | 13 | 12 | 4 |
| 10 | 6 | 7 | 9 |
| 8 | Χ | Ɛ | 5 |
| 11 | 3 | 2 | 14 |

## THE MAIL BAG

In a recent letter, William Shaw Crosby informed us that he had met Eric Temple Bell after a mathematical lecture at Berkeley, and had loaned him copies of the Bulletins which contained comments on excerpts from Dr. Bell's books. In returning the Bulletins, Dr. Bell accompanied them with a most gracious letter, stating that he had read them with interest, but that, duodecimally, he was a heretic.

There can be little doubt that Dr. Bell does not think much of the duodecimal idea. In his "Men of Mathematics," he says that we duodecimal propagandists escape the circle-squaring fraternity by a hairsbreadth; and that "a base of 12 superimposed on the 10 of our number system would be a hexagonal peg in a pentagonal hole." It might well be doubted that Dr. Bell ever gave the subject much thought.

There are many ways in which mankind has been in a sort of slavery to the ten-base. This can be illustrated by our adoption of terms like "decimals," and "decimal-point," for constructions and practices that are operable under any number base. A clear concept of number would have led to the ostracism of these ineptitudes.

The dawn of man's realization of this decimal slavery is associated with the origin of the dozen, the gross, and the scale twelve in weights and measures. Their use was quite prevalent in Roman times, though their beginnings were probably much earlier. If the Romans had had our concept of positional numerical notation, their dozenalized weights and measures would have been a true metric system. The word "dozen" is probably a corruption of the Latin "duodecem."

The adoption of twelve as the base of the French metric system was blocked, according to Dr. Bell, by the *irony* and common sense of Lagrange. It is ironical that this *sensible* decision should have become the specific factor that limits the general adoption of the French system.

The first step in the correction of this demotic compromise is the tentative adoption of the Do-Metric System, as official, by the Duodecimal Society. The Society's folder has now been revised to include a brief and elementary description of the system, and is available for general distribution. Copies have been mailed to members for circulation among their friends, and there will shortly be a general mailing to all those interested.

The task of establishing these standards in the governmental bureaus, in the encyclopedias and dictionaries, and in the laboratories may now be initiated. You can assist in this work by spreading the news as far as possible.

Ye Ed.



# COUNTING IN DOZENS

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | X | Ɛ | 10 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| one | two | three | four | five | six | seven | eight | nine | dek | el | do |

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

| | | | |
|---|---|---|---|
| 94 | 136 | Five ft. nine in. | 5.9' |
| 31 | 694 | Three ft. two in. | 3.2' |
| 96 | 3Ɛ2 | Two ft. eight in. | 2.8' |
| 19Ɛ | 1000 | Eleven ft. seven in. | Ɛ.7' |

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2Ɛ*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

12 ) 365
12 ) 30 + 5
12 ) 2 + 6
0 + 2 Answer: 265

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus $12^2$ (or 144) times the third figure, plus $12^3$ (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by X, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or X.

Numerical Progression

| | | | |
|---|---|---|---|
| 1 | One | | |
| 10 | Do | .1 | Edo |
| 100 | Gro | .01 | Egro |
| 1,000 | Mo | .001 | Emo |
| 10,000 | Do-mo | .000,1 | Edo-mo |
| 100,000 | Gro-mo | .000,01 | Egro-mo |
| 1,000,000 | Bi-mo | .000,001 | Ebi-mo |
| 1,000,000,000 | Tri-mo | and so on. | |

Multiplication Table

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | X | Ɛ |
| 2 | 4 | 6 | 8 | X | 10 | 12 | 14 | 16 | 18 | 1X |
| 3 | 6 | 9 | 10 | 13 | 16 | 19 | 20 | 23 | 26 | 29 |
| 4 | 8 | 10 | 14 | 18 | 20 | 24 | 28 | 30 | 34 | 38 |
| 5 | X | 13 | 18 | 21 | 26 | 2Ɛ | 34 | 39 | 42 | 47 |
| 6 | 10 | 16 | 20 | 26 | 30 | 36 | 40 | 46 | 50 | 56 |
| 7 | 12 | 19 | 24 | 2Ɛ | 36 | 41 | 48 | 53 | 5X | 65 |
| 8 | 14 | 20 | 28 | 34 | 40 | 48 | 54 | 60 | 68 | 74 |
| 9 | 16 | 23 | 30 | 39 | 46 | 53 | 60 | 69 | 76 | 83 |
| X | 18 | 26 | 34 | 42 | 50 | 5X | 68 | 76 | 84 | 92 |
| Ɛ | 1X | 29 | 38 | 47 | 56 | 65 | 74 | 83 | 92 | X1 |