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THE DUODECIMAL SOCIETY OF AMERICA

20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y.

THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve-base arithmetic. The lessons and examinations are free to those whose entrance applications are accepted. Remittance of $6, covering initiation fee ($3) and one year's dues ($3), must accompany applications.

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The Duodecimal Bulletin

A PLEA FOR THE DUODECIMAL SYSTEM

by H.G.G. Robertson, M.R.C.S., Lieut. Col., R.A.M.C.

There is no doubt that a large number of intelligent people in all parts of the world have often wondered, and are still wondering, why it is that, with such a simple and logical arrangement as the Metric System ready to their hands, and the example of many progressive nations in front of them, the British, (and, to a lesser extent, the Americans), should cling so pig-headedly to their antiquated, cumbersome, complex, and altogether obsolete system of weights and measures.

Probably it is assumed that the only reason is the well-known conservatism of the British Public which prevents their leaders from taking a step which would at once simplify calculation and bring the country into line with the rest of the civilized world.

Now I am very far from denying that the British System of weights and measures is sadly in need of overhaul. It has grown up, like so many British institutions, from primitive beginnings, and has had constant additions to it, without, however, the necessary pruning away of the dead wood of past and gone ideas.

But all this does not explain the attitude of the Americans, who are generally regarded as a progressive and go-ahead people, and there is another reason, besides the dislike of change innate in the British character, which makes these nations reluctant to abandon their time-worn methods. Before, however, we examine this reason and see whether it has anything to support it, we must go back a little, and discover how things have arrived at their present state.

THE ORIGIN OF DECIMAL NOTATION

It is obvious, though perhaps many people fail to realize its significance, that our system of notation is based on the number ten; i.e. ten is the *Radix* (Latin, a root), or *base* of our number script, and we count in multiples or fractions of ten.

When primitive man first began to count his flocks and herds, he found it easiest to number them in groups of so many. Owing to the fact that he used his fingers as a tally, and that we have five fingers on each hand, he counted them in groups of five, (both hands), as certain tribes in Paraguay do to this day. Sometimes he used his feet as well.

Children still count on their fingers, and even adults are not always immune to the habit: the very word "digit" means a finger.

The next stage of evolution was the invention of the counting-frame, or Abacus, such as is fitted to the sides of children's play-pens, for them to play with, and to learn to count on. The Chinese and Japanese still use the abacus in their everyday life, as they have done for the past five or six thousand years, and they appear to be incapable of carrying out any but the very simplest calculations without its aid. It is indeed an experience to hear the continuous clicking of abacus beads when one enters a Chinese bank or business house. Five or ten, however, still remained the basis of the group.

There are traces of this association with multiples of five to be found all over the world, notably in the English Biblical use of the "score", and in the French "quatre-vingt", examples of the use of twenty as the base.

There is no doubt, therefore, that the fortuitous circumstance of our having ten fingers is responsible for the number ten being the base of our system of notation; if man had two toes, like a a cow, or as many legs as a centipede, the history of mathematics would undoubtedly have been different.

People who are interested in puzzles based on numbers will tell you that there are all sorts of queer things about the number nine. For instance, any multiple of nine totals 9 if its digits are added up, and if 9 is added to any number the total of the digits of both the original number and the new one will be the same. Greek philosophers, and even mathematicians like Pythagoras, whose name is associated with "the square on the hypotenuse", attributed magical properties to the number ten, regarding it as "perfect".

Many people seem to be under the impression that these peculiar characteristics of numbers like nine and ten are due to something inherent in the numbers themselves, whereas, of course, they only possess these properties because ten happens to be the base of our number system; nine behaves like it does because it is one less than ten, the base: if the base were changed these peculiar attributes would then belong to some other number.

DISADVANTAGES OF DECIMAL NOTATION

As a matter of fact the number ten has serious drawbacks as a numerical base. It has only two factors - 2 and 5 and consequently cannot be divided exactly into thirds or quarters; this is a great nuisance, as we need to use three and four as divisors much more frequently than we do five. Ease of calculation is greatly facilitated if the numerical base has a large number of factors, and in this respect ten is an "also-ran". This lack of factors is responsible for the large number of recurring decimals we are always running across, and is particularly noticeable when we are using 3 as a divisor, which we are very frequently.

REASON FOR BRITISH OBSTINACY

It will now perhaps be apparent why the British retain their apparently clumsy system, and that there is, after all, some method in their madness.

The numbers which we most often use as divisors are 2, 3, 4, 6 and 8, and to a lesser extent, 10, though this last is really due to the fact that 10 is our base. To get even division with our present system is impossible, hence the British, with their genius for compromise, have evolved a method which makes the best of both worlds. They have chosen units of measurement which will provide as many factors as possible, and hence make calculation easy.

To illustrate this, let us examine the following table:

Table I

MEASURE	UNIT	SUB-UNITS	FACTORS AVAILABLE (BELOW 30)
MONEY	£	4 FARTHINGS = 1 PENNY 12 PENCE = 1 SHILLING 20 SHILLINGS = £1 240 PENCE = £1	2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24
LENGTH	YARD	12 INCHES = 1 FOOT 3 FEET = 1 YARD 220 YARDS = 1 FURLONG 8 FURLONGS = 1 MILE 5280 FEET = 1760 YDS. = 1 MILE	2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 15, 16, 18, 20, 22, 24
AREA	SQ. YARD	144 SQ. IN. = 1 SQ. FT. 9 SQ. FT. = 1 SQ. YD. 4840 SQ. YDS. = 1 ACRE 640 ACRES = 1 SQ. MILE	2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 15, 16, 18, 20, 22, 24

MEASURE	UNIT	SUB-UNITS	FACTORS AVAILABLE (BELOW 30)
VOLUME (CUBIC)	CUBIC FOOT	1728 CUBIC INS. = 1 CUBIC FT. 27 CUBIC FT. = 1 CUBIC YD.	2, 3, 4, 5, 6, 8, 9, 12, 16, 18, 24
VOLUME (LIQUID)	GALLON	5 FLUID OZ. = 1 GILL 4 GILLS = 1 PINT 2 PINTS = 1 QUART 4 QUART = 1 GALLON	2, 4, 5, 8, 10, 16, 20
DRY	BUSHEL	2 GALLONS = 1 PECK 4 PECKS = 1 BUSHEL 8 BUSHELS = 1 QUARTERN	2, 4, 8, 16
WEIGHT (AVOIRDUPOIS)	POUND (2240 LBS. 1 TON)	16 DRAMS = 1 OUNCE 16 OZ = 1 POUND 14 LBS. = 1 STONE 2 STONES = 1 QUARTER 4 QTRS. = 1 CWT 20 CWTS = 1 TON	2, 4, 5, 7, 8, 10, 14, 16, 20, 28
WEIGHT (TROY)	OUNCE	24 GRAINS = 1 PENNYWEIGHT 20 DWT = 1 OZ. 12 OZ. = 1 LB. (TROY)	2, 3, 4, 5, 6, 8, 10, 12, 15, 16, 20, 24

These examples could be multiplied, but sufficient have been given to show the truth of my statement. Of course, I do not imply that the British people are all fully conscious of the *reasons* why they prefer their own system; they only know that it works, and being, in spite of many opinions to the contrary, people of sound, practical commonsense, they are averse from altering it.

OTHER BASES

Now, you must not think that the Anglo-Saxon races are the only ones who have realized the disadvantages of ten as a number base. Far from it. One has only to look at units of measurement which are truly universal to see that twelve plays at least as big a part as ten in the business of the world.

Take time, for example:- 60 seconds = 1 minute, 60 minutes = 1 hour, 24 hours = 1 day, 12 months = 1 year. Even the days of the week are based on six working and one rest day.

The ancient Mayas and also the Babylonians had a numerical system based on the radix 60, and coming nearer our own time we need only think of the number of things sold "by the dozen", (an example of the use of 12 as the base), and the fact that the word "dozen" has its equivalent in many languages, to realize the truth of my statement.

Of course, it is quite true that our measurement of time is not altogether the result of design. It is partly due to accident, the same sort of accident as is responsible for ten being the base of our number system.

Owing to a miscalculation, or rather, to imperfect observation due to the crude instruments at their disposal, both the Ancient Egyptians and the Babylonians reckoned the year as being 360 days. This led to the division of the circle into 360 degrees, each degree being the distance travelled by the stars per day in moving round the celestial sphere, and, by a process of repeated halving, gave rise to 15° of arc to the hour, and hence to 24 hours to the day. (24 x 15 = 360).

None the less, the advantages of the number 360 as a basis for calculation were so obvious that it has been retained.

The ancient Syriac number script was based on a radix of 2. This makes for very great simplification in calculating: only two symbols are needed - e.g. 1 and 0: thus 1 (in our notation) would be 1; 2 would be 10; 3, 11; 4, 100; 5, 101; 6, 110; 7, 111; 8, 1000; and so on. The four fundamental operations of arithmetic could be done with ease by the merest child - there would be no multiplication tables to learn, but unfortunately, enormous quantities of paper would be required owing to the large number of figures needed to express even quite small numbers.

There is, then, no magical property about the number ten, nor any special reason, save the fact that we have ten fingers and ten toes, why we should use, or should continue to use it as our number base. *Any* number can be chosen; the only necessity being the possession of a corresponding number of figure symbols.

WHAT IS THE BEST BASE?

It is obvious, of course, that no base is perfect: by the nature of things it is impossible for any one number to have all smaller numbers as factors. What then is the best base to choose? Well, there are only two important considerations - that it should be convenient in size, and that it should possess as many useful factors as possible. For example -

2 is too small; it requires far too much paper and space.
60 is too big; it needs an inconveniently large number of symbols.
10 is handy for size, but it has too few factors.

The best compromise is undoubtedly twelve; it is convenient in size, and it has as factors 2, 3, 4 and 6, all important and useful divisors, compared with the 2 and 5 of 10.

Furthermore, $12^2 = 144$, which corresponds to our 100, has no fewer than 13 factors - 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, as opposed to the seven of 100 - 2, 4, 5, 10, 20, 25, 50, and they are more useful ones at that.

THE DUODECIMAL SYSTEM

Our present method is called a *Decimal Notation*, (Latin, decem, ten), because it is based on ten as the radix: the system based on twelve is called *Duodecimal*, (Latin, duodecim, twelve). If our weights and measures, coinages, etc., were duodecimal, we should have all the advantages of the decimal system in simplicity, with the added ones of easier calculation due to the larger number of factors, fewer recurring radix fractions, and somewhat fewer digits in very large numbers or very small fractions, due to the larger base employed. Dividing by 12 would be the same as dividing by 10 is now, and fractions like 1/3rd, 1/6th and 1/9th could be converted into duodecimals without any "recurring".

What is wanted, then, is a change of our numerical base from 10 to 12: in other words, the adoption of a duodecimal as opposed to a decimal system.

WHAT WOULD IT INVOLVE?

Granted, then, the desirability of the change, what would it involve?

In the first place, two new symbols must be invented to stand for 10 and 11 respectively. Any number system requires the same number of symbols as the value of the base, so 12 would be needed: we already have ten of them, therefore, two more are necessary.

I do not propose in this paper to weigh the pros and cons of what particular symbols should be used - that is a matter for discussion when the principle of the system has been generally accepted. My purpose is merely to explain, in as simple language as possible, the effects of the change, and the general method of working. Details can be left to a later stage.

It is essential, however, to give examples, and for this purpose some symbols must be used. In a short section on the subject in his excellent little book "Mathematics for the Million", which should be on the bookshelf of every thinking person, Professor Lancelot Hogben used ♀ and ♂. I do not propose to follow his example. ♀ and ♂ are astronomical, or rather, astrological symbols,

signifying the planet Venus and Mars respectively, and they are also used in Biology to mean female and male. So I feel that they have enough work to do at present without adding to it.

The symbols I am going to use in this paper are Ɵ and γ. The first is merely the 1 and 0 of our present 10 amalgamated to form one sign, and the two horns of γ represent the two "1s" of 11. It may be argued that Ɵ is liable to be mistaken for a Greek θ, and γ for the astronomical symbol ♉, the sign of the constellation Taurus. I freely admit the possibility, though I think it unlikely, but, as I have stated above, I have no intention of proposing that these suggested symbols are ideal, or in their final form, and I leave it to others to produce better ones.

The number script of the duodecimal system would thus be:

	1 2 3 4 5 6 7 8 9 Ɵ γ 10
corresponding with present system.	1 2 3 4 5 6 7 8 9 10 11 12 of our

There are two possible methods of dealing with the extra two symbols. One is to retain the present names ten, eleven, and twelve for the old values 10, 11, and 12, and apply them to the new figures Ɵ, γ, and *10*. New names would then have to be invented for the numbers *11* and *12* in the new script, (which are the old thirteen and fourteen), because these latter names are required for the new *13* and *14*. The logical thing would be to call them oneteen and twoteen. Strictly speaking, the "'teens", which are derived from ten, should now become "'tweens," being derived from twelve, but I feel that this is a somewhat unneces-complication, which would, besides, make the pronunciation somewhat less euphonious. However, it is a minor point which could be settled later. *1Ɵ* and *1γ* would become tenteen and eleventeen (or "'tween") respectively, and *20*, (the old 24), would be twenty, or twentwy after the analogy of 'teen. The latter, however, is very distinctly less easy to say, the *w* in the middle of the word being definitely inharmonious, and it would almost certainly be dropped in time, at any rate in pronunciation.

On this basis *Ɵ0*, (the old 120), would be tenty, or tentwy, but there is a very decided danger of its being mistaken for twenty.

There are, it will be observed, several objections to this nomenclature, and for this reason I am inclined to favour the alternative method, which is to retain the names eleven and twelve for the figures *11* and *12*, (the old 13 and 14), and invent new ones for Ɵ and γ. The names I suggest for these two symbols are *Dek* and *Elf*. Dek is derived from the Greek word for ten, and elf is borrowed from the German for eleven.

The names for the numbers from *9* to *20* (old 24), by the two methods, are shown in the following table:

TABLE II

Name 1st Method	Name 2nd Method	Figures	Old Figures
Nine	Nine	*9*	9
Ten	Dek	*θ*	10
Eleven	Elf	*ɣ*	11
Twelve	Ten	*10*	12
Oneteen	Eleven	*11*	13
Twoteen	Twelve	*12*	14
Thirteen	Thirteen	*13*	15
Fourteen	Fourteen	*14*	16
Fifteen	Fifteen	*15*	17
Sixteen	Sixteen	*16*	18
Seventeen	Seventeen	*17*	19
Eighteen	Eighteen	*18*	20
Nineteen	Nineteen	*19*	21
Tenteen	Dekteen	*1θ*	22
Eleventeen	Elfteen	*1ɣ*	23
Twenty	Twenty	*20*	24

100 could either be called "a hundred", or "a gross", as it is equal to the old 144. In both methods two new names have had to be coined. I do not think it matters a great deal which method is employed, but for the purposes of this article I propose to use the second one, for the reasons given above.

Now it is obvious that the addition of two new symbols to our present ten is going to alter our tables of addition, multiplication, etc., and will also mean revising such things as logarithm tables, trigonometrical ratios, and so on. This will not matter very much because the new tables are, if anything, easier to learn than the old, and modern logarithms are not calculated direct, but are first worked out to the base "e", (natural logarithms), and are subsequently converted to the common base of 10. It is just as easy to convert them to the base 12, and will give the mathematicians something to do.

A complete list of Addition, Subtraction, and Multiplication Tables, Methods of Conversion from one scale to the other, Factorial Rules, etc. and a Conversion Table for whole numbers up to *10,000*, and for Radix Fractions down to *.0001* correct to four places of duodecimals, can be obtained if required.

HOW WOULD IT WORK?

Once the tables have been mastered calculating in the new scale is precisely the same as in the old. All mathematical processes and rules remain unaltered, but as already pointed out, much of the working would be simpler. I therefore do not intend to go further into this aspect of the matter.

In order, however, to obtain the full value from the duodecimal system it is essential that the British System of Weights and Measures be revised on the lines of the Metric System, but adjusted to the new base.

There are three ways in which this might be done.

(1) To drop the present system entirely, and replace it by the metric system, but alter the latter to a base of 12.

(2) To establish a completely new system, with new units and new names, on the lines of the establishment of the Metric System in the French Revolution, abolishing all the old ones.

(3) To retain the names and some of the standards of the old British System, adjusting the values to the new base so that the units go up in twelves, thus retaining for the duodecimal system the advantages at present enjoyed by the Metric.

The following table shows a suggested new System on these lines for some of the present British Weights and Measures, the figures being given in duodecimal notation, with the decimal equivalents in brackets:

TABLE III

MEASURE	UNIT	SUB-UNITS
Money	£	*100* (144) pence *£1* *10* (12) pence = *1* shilling *10* (12) S = *£1.*
		REMARKS: If desired a duodecimal subdivision of the penny could be also used.

MEASURE	UNIT	SUB-UNITS
Length	Yard	*10* (12) inches = *1* foot, *10* (12) ft. = *1* yd. *10* (12) yds. = *1* chain; *10* (12) ch. = *1* furlong *10* (12) fur. = *1* mile = *100* (144) chains = *1000* (1728) yds. = *10,000* (20,736) feet = *100,000* (248,832) inches.

REMARKS: If the yard remained the standard the new mile would be *28* (32) yds. shorter than the present one; the new foot would equal 3 of our present inches, and the inch just over ½ cm (¼")

MEASURE	UNIT	SUB-UNITS
Area	Square Yard	*100* (144) sq. ins. = *1* sq. ft. *100* (144) sq. ft. = *1* sq. yd. *100* (144) sq. yds. = *1* Rood. *100* (144) Roods = *1* Acre *100* (144) Acres = *1* Sq. Mile = *10,000* (20,736) Roods = *1,000,000* (2,985,984) sq. yds.

REMARKS: At present 1210 sq. yds. = 1 Rood, 4 Roods = 1 acre, 640 acres = 1 Sq. Mile. This gives 3,097,600 sq. yds. to the square mile.

MEASURE	UNIT	SUB-UNITS
Volume (Cubic)	Cubic Foot	*1,000* (1728) cub. ins. = *1* cubic foot *1,000* cubic feet = *1* cubic yard = *1,000,000* (2,985,984) cubic ins.
Volume (Liquid)	Gallon	*10* (12) ounces = *1* pint *10* pts. = *1* gallon (1 pt. = 1 c.ft. of water at max. density)

REMARKS: 1 c. ft. = *23* (27) of our present c. inches, as a gallon would be (324) old inches, (at present about (277¼) and would weigh *10* (12) lbs.

MEASURE	UNIT	SUB-UNITS
Weight	Pound (wt. of 1 c.ft. of water)	*10* (12) drams = 1 oz. *10* (12) oz. = 1 lb. *10* (12) lbs. = 1 stone *10* (12) stones = 1 cwt. *10* (12) cwt. = 1 ton = *100* (144) stones = *1000* lbs. = *10,000* oz. = *100,000* drs. (1728) (20,736) (248,832)

REMARKS: The new lb. would occupy *23* (27) old c. in. of water as against (27.725). The new cwt. would occupy (3811) o.c.in. as against (3105.2). The new ton would occupy (45,732) o.c.in. as against (62,104). Thus the new lb. would be almost identical with the old, while the new cwt. would be about (23%) greater and the new ton about (26%) less than the old.

MEASURE	UNIT	SUB-UNITS
Temperature	Degree	Melting Ice = 0^{o} Boiling Water = *100*^{o} (144).

Thermometers are calibrated at Normal Atmospheric Pressure, i.e. on our present scale 760 m.m. of mercury or 29.9218 inches. This is equivalent to *25.χ09* old inches in D.D. notation. To simplify calculation this would probably be adjusted to 26, giving *θ0* inches (new scale) of mercury as Normal Pressure. Water reaches its maximum density at 4°C = *5.915*^{o} D.D. Owing, however, to the alteration of Normal Pressure from 29.92 old inches to *θ0* new inches, the correspondence of the scales given in (2) above will not be absolutely exact, and for all practical purposes the maximum density of water can be taken as at 6^{o} D.D.

It is often assumed that Great Britain is the outstanding obstacle to the world wide adoption of the Metric System, but this is not strictly true, though undoubtedly her example has considerable influence.

Apart from the Dominions, the U.S.A. uses British (or nearly British) weights and measures, although her monetary system is decimal; China, with a population of over 400 millions, has native units of weights and measures; India, whose population is nearly as large, is in the same position, the units varying from province to province, with, in addition, a monetary system of 12 pies to the anna, and 16 annas to the rupee; Japan uses native units side by side with Metric, though it must be admitted that most of these are decimal; Russia has only legalized the Metric System since the Revolution; in fact, by far the greater part of the world's population use methods other than Metric, so there is no valid reason, if ten is no longer the base, for preferring this to other forms.

My own preference is for the third method I have outlined. As the table shows, the adjustment necessary is small and I consider it an advantage to use the old names, but this again is a matter for future discussion. What matters at the moment is for the general public to become "duodecimal conscious", and to demand the adoption of this advantageous system. Once that has been done the rest will follow.

THE CHANGE OVER

Among the things of everyday life that will be profoundly affected by the change from decimal to duodecimal notation are dates. When it is realized that the present year of grace (1946) is only 1162 on the D.D. scale, it will be appreciated how much the dates will be altered.

The reform of the Calendar is long overdue, and the introduction of the Duodecimal System would be a splendid opportunity to bring it about, but the details of the alterations required are beyond the scope of this paper, and I propose, therefore, to deal with them in a subsequent one.

Meanwhile, let us see how the change-over from a number system based on 10 to one based on 12 can be brought about, and what its effect will be on the ordinary life of the individual.

There is no doubt that such a change, revolutionary as it is, would be very difficult to accomplish in the ordinary way, and would probably require many months, if not years, of argument and discussion before the various National Governments could be prevailed upon to agree.

Fortunately, however, a heaven-sent opportunity is available in the shape of the enormous amount of reconstruction necessary after the War, in the midst of which the change of notation may well be regarded as merely one of a large number of other changes. Furthermore, there is a splendid chance of achieving international unity over the question by incorporating a clause in the Peace

Treaties providing for the setting-up of a Commission or Sub-Committee to discuss the matter and decide on the details I have mentioned, and the final form the system is to take.

It is therefore imperative that the subject should be in front of the public eye NOW, before it is too late and the opportunity is gone, perhaps for a generation or longer.

If the Duodecimal System should be taken up by the United Nations it would simplify the task of reaching agreement over the matter of notation, and, in turn, the adoption of the Duodecimal System should react favourably on the establishment of a truly international currency, an urgent need which would go far in removing many of the economic tendencies which have made for wars in the past.

Now as to the effect on the individual.

It cannot be denied that a certain amount of inconvenience is bound to be caused, particularly to the older generation, by the necessity for learning new tables of addition and multiplication, and by the need for renewing such things as slide rules, books of reference, instruments, etc., indeed, it is quite possible that to some of the more conservative the change will never quite be wholly acceptable, nor will they ever quite get used to it.

To the main bulk of the population, however, the inconvenience will be transitory, and should not cause more worry than is occasioned by the learning of a new language, or the difficulties in adjustment experienced by our ancestors by the alteration in the calendar or the introduction of the Metric System or of steam traction.

So much has been destroyed, too, in the War, and so much will have to be renewed and replaced, that the scrapping of a few instruments and books is not likely to cause much trouble. So much that is new and revolutionary will be going on that it is conceivable that the change may pass almost unnoticed.

For a time, no doubt, until we have got used to it, it will be necessary to use the two systems side by side, but the greater flexibility of the new method, and its obvious advantages, will, I am convinced, quickly prove its superiority, and smooth over what might, at other times, have proved a difficult period.

To the younger generation, of course, there will be no difficulty at all. They will learn the new system at school, and will know no other except as a historical relic, though they will probably have to do sums in the scale of ten (among others), as we had to in the scale of twelve.

In a few years the old system will be forgotten as other things have been. How many people to-day have ever handled a sovereign or ridden in a hansom-cab? Yet in their hey-day they were considered immutable. Tout passe.

CONCLUSION

The world to-day is a world of science, and is becoming more in need of and more dependent on science as every year passes. And of all sciences mathematics is the one most bound up with human welfare.

Just as the program of Mathematics has mirrored the progress of civilization, so to-day the sciences we are most in need of are becoming more and more dependent on mathematics. Physics, Statistics, Economics, Biology, Genetics, to mention but a few, are more and more only understandable in terms of mathematics.

The crying need to-day is more and better education, for *everybody*. Anything, therefore, that can help in our education, that can simplify our understanding and our use of mathematics is going to contribute to the welfare of the human species.

I do not doubt, and I hope that this paper may have helped to dispel some of *your* doubts, that the introduction of the Duodecimal System will do this very thing. It will simplify calculation, and so assist our understanding and our use of Mathematics. It will be easier to learn than the old method, and so will help our education. And for this reason I am convinced that it will materially add to the benefits that Science can confer upon mankind.

ANNUAL MEETING

The fourth annual meeting of the Duodecimal Society will be held at 8:30 P.M., Thursday, January 22nd, 1948, in the Gramercy Park Hotel, Lexington Avenue and 21st Street, New York City. All members and friends of the Society are invited to attend.

The meeting will receive the reports of the officers and committees covering the past year, and will act on the recommendations of the Nominating Committee for directors, to re-elect, replace, or supplement those whose terms expire this year, namely: F. Emerson Andrews, and William S. Crosby, - and for the new Nominating Committee for 1948.

After the formal business of the meeting has been transacted, there will be a talk by Harry C. Robert, Jr., on, "An Engineer Looks at Mathematics and Duodecimals," and other informal addresses on duodecimal subjects. There will be a period of open general discussion when comments and questions from anyone present will be welcomed. Refreshments will be served at the close of the meeting.

MATHEMATICAL RECREATIONS

by Mary Lloyd, Editor

One popular type of mathematical recreation deals with the formation of geometrical designs, or mosaics, covering a plane area by the use of tiles of given geometrical forms.

There are many forms of these tesselation problems. If only one type of regular polygon is to be used, there are three possible patterns, - triangles, squares, and hexagons. But if several types of polygons may be used, a variety of patterns may be found within the limits imposed by these conditions. There is a considerable literature devoted to the analysis of these patterns, setting forth the mathematical principles involved.

We have received several brief papers from George S. Terry on this subject, and an occasional comment from Harry C. Robert, Jr. It is an amusing recreation to draw these tessellations, and we believe you will find this material interesting. So we will attempt to assemble these essays into an informal presentation.

TESSELLATION

George S. Terry

We dozeners are accustomed to considering angles as some fraction of the circle; that is, as that portion of the whole plane that is enclosed by the angle. This is a simpler concept of the angle than that commonly used. For this reason, the principles governing tessellation can be more easily understood.

The sum of the angles of a polygon is two half-circles less than the number of sides. The sum of the angles of a triangle is one half-circle; of a square, it is two half-circles; of the pentagon, three half-circles, of the hexagon, four, and so on.

The internal angle of any polygon equals this sum divided by the number of angles, or sides. Thus the internal angle of an equilateral triangle is $.2^c$; of a square $.3^c$; of a pentagon $.37249^c$; of a hexagon $.4^c$; of an octagon $.46^c$; of the dodecagon, or *10*-gon, $.5^c$; and of the *20*-gon $.56^c$.

Then, naturally, when we wish to form a tessellation of polygons of only one kind, we are limited to those polygons whose internal angles are some simple fraction of the circle; that is, to the triangle, the square, and the hexagon, whose respective angles are $.2^c$, $.3^c$, and $.4^c$. It is easy to see that we can surround a point with six triangles, or four squares, or three hexagons. These, and only these, are possible.

When we use two kinds of polygon to form our tessellation, there are five ways of adding up internal angles to complete the circle about a point, namely:-

$3[.2^c] + 2[.3^c]$	or three triangles and two squares;
$2[.2^c] + 2[.4^c]$	or two triangles and two hexagons;
$4[.2^c] + [.4^c]$	or four triangles and a hexagon;
$[.3^c] + 2[.46^c]$	or a square and two octagons;
$[.2^c] + 2[.5^c]$	or a triangle and two *10*-gons.

When we use three kinds of polygon, there are three ways of making the angles add-up to the complete circle.

$[.2^c] + 2[.3^c] + [.4^c]$	or a triangle, two squares, and a hexagon;
$[.3^c] + [.4^c] + [.5^c]$	or a square, a hexagon, and a *10*-gon;
$[.3^c] + 2[.2^c] + [.5^c]$	or a square, two triangles, and a *10*-gon.

The last of these will not make a tessellation covering the plane. Perhaps someone will be able to tell me why.

Harry C. Robert, Jr.

I cannot explain why the point-closure expression mentioned, $[.3^c] + 2[.2^c] + [.5^c]$, does not lead to a regular tessellation. This is not as unique as it might seem, since the tessellations $3[.2^c] + 2[.3^c]$, and $2[.2^c] + 2[.4^c]$ each provide only one "regular" tessellation, although the elements can be arranged in two different cyclic orders. Only $[.2^c] + 2[.3^c] + [.4^c]$ provides patterns for two different cyclic orders of the elements.

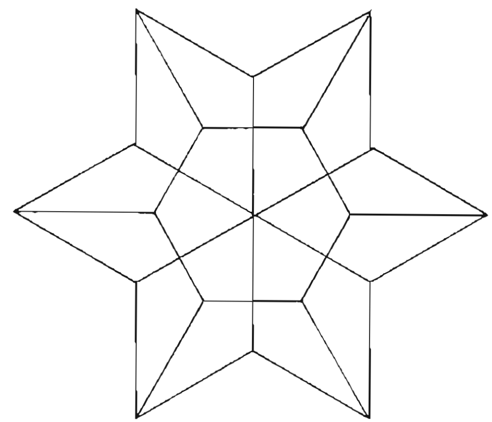
That is, point closure alone is not sufficient to produce regular tessellations. The irregular tessellations can be symmetrical and as interesting as the regular patterns. To illustrate this, a sketch is attached combining a regular tessellation, using the point-closure $[.2^c] + 2[.5^c]$, with an irregular tessellation, using $2[.2^c] + [.3^c] + [.5^c]$.

SOLID TESSELLATION

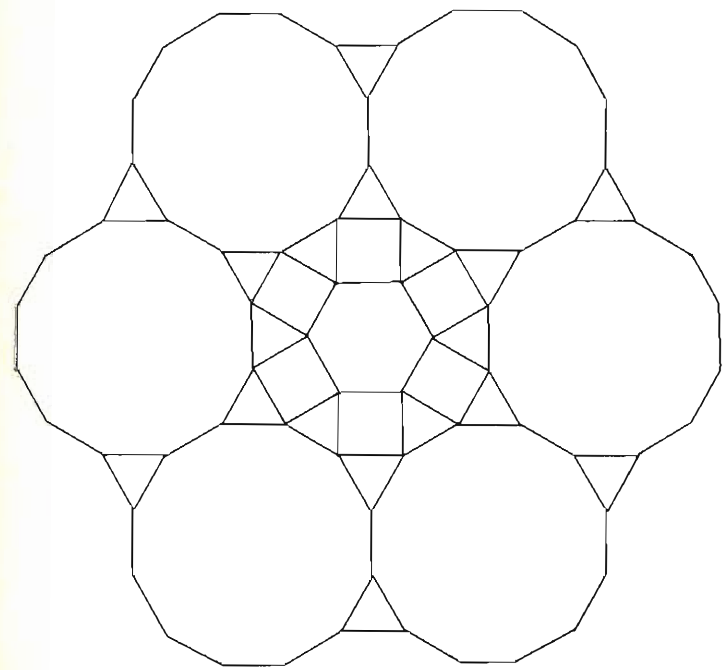
George S. Terry

We may not yet be accustomed to think of solid angles as duodecimal fractions of a sphere. A solid angle is the proportion of space enclosed by the angle; or, with the angle at the center of the sphere, it is the proportion of the spherical surface cut off to the total surface of the sphere.

It is evident that eight cubes meeting at a point fill all space about that point. So the solid angle of the cube is one-eighth



Partial Inverse of Irregular Tessellation on Left. (Figure formed by lines connecting centers of adjacent areas - crossing sides at Right Angles)



Tessellation [.2] + 2[.5] with center dodecagon subdivided with Tessellation [.2] + 2[.3] + [.4]
Border points = 2[.2] + [.3] + [.5]

sphere, or $.16^s$. The solid angles of the other four regular polyhedra will tell us whether they can be space-filling or not.

Consider a regular tetrahedron, with one angle at the center of a sphere of radius R. The angle cuts off a spherical triangle on the surface of the sphere, and its area is found from $(A + B + C - .6^c)2\pi R$ where A, B, and C, are the plane angles between adjacent faces of our solid angle. In this case, $A = B = C$, and $\cos A = 1/3$, as may be readily seen by drawing a section of the tetrahedron. So, $A = B = C = .2426\ldots^c$, and the area of our spherical triangle is $(.7076\ldots - .6)2\pi R^2$, while the area of the whole sphere is $4\pi R^2$. Thus, our solid angle is $.0639\ldots^s$, and no whole number of these will fill space.

For the cube, $A = B = C = .3^c$. And the solid angle will be $.6(A + B + C - .6) = .16^s$.

For the octahedron, $A = B = C = D = \cos^{-1}(-1/3) = .3796\ldots^c$. The solid angle is twice $.6(A + .6B + .6C - .6)$, and this equals $(.7370\ldots - .6) = .1370\ldots^s$; or, looking at the solid angle as made up of four pieces, it equals four times $.6(.6A + .6B + .3 - .6)$, which equals $.1370\ldots^s$, and is clearly not capable of filling space.

For the dodecahedron,

$$A = B = C = \cos^{-1}\left(-\frac{1}{\sqrt{5}}\right) = .3X76\ldots^c$$

$$\text{Solid angle} = .6(.Ɛ7X6\ldots - .6) = .29Ɛ3\ldots^s$$

For the icosahedron, (five triangles meet at the vertex,)

$$A = B = C = D = E = \cos^{-1}\left(-\frac{\sqrt{5}}{3}\right) = .4734\ldots^c$$

$$\text{Solid angle} = 5 \text{ times } .6(.6A + .6B + .\dot{2}49\dot{7} - .6)$$

$$= 5(.0605X) = .\dot{2}623\ldots^s$$

Hence, it is evident that only cubes can fill space.

But what about semi-regular polyhedra, notably the rhombic dodecahedron? And what about compressing spheres to fill space?

Try piling marbles. There are two ways, when you get to the third layer. But more on this later.

Harry C. Robert, Jr.

If only one kind of regular polyhedron is used, only cubes can fill space; but if two kinds of polyhedra are used, we can fill space with alternating tetrahedra, and octahedra. Each vertex will be common to six octahedra and eight tetrahedra. Our solid angle summation will be:

$$\begin{aligned}
&6 \text{ octahedra} &&= 6(.1370\ldots^s) &&= .7960\ldots^s\\
&8 \text{ tetrahedra} &&= 8(.0639\ldots^s) &&= .4260\ldots^s\\
&\text{Total angles} && &&= 1.0000\ldots^s
\end{aligned}$$

These are the only two space filling arrangements with regular Platonic polyhedra. Several other arrangements are possible with truncated and other semi-regular polyhedra.

See W. W. Rouse Ball's, "Mathematical Recreations and Essays," Revised American Edition, 1947, Macmillan, New York.

PRODUCTS REPEATING INTEGERS OF FACTORS

Lewis Carl Seelbach brings to our attention one of Bob Ripley's Believe It Or Not columns, which lists a series of numbers whose integers repeat the integers of their factors. These were numbers in the decimal notation. We list them here with the hope that some one will work up a similar duodecimal list, and, perhaps work out a general statement for their occurence and formation.

15 x 93 = 1395	201 x 627 = 126027	327 x 591 = 193257
21 x 87 = 1827	201 x 897 = 180297	465 x 831 = 386415
152 x 824 = 125248	231 x 543 = 125433	524 x 623 = 326452
161 x 725 = 116725	251 x 608 = 152608	776 x 992 = 769792
167 x 701 = 117067	317 x 461 = 146137	894 x 906 = 809964

F. MORTON SMITH

Morton Smith passed away peacefully at his home in Hingham on September 9th, 1947, at the age of sixty-one.

Lawyer, trustee, wharf-executive, - the wharf of the Boston Tea Party being among those in his care, - he spent much of his time in unselfish service for his neighbors.

His passing is a great loss to many people, among them the members of the Duodecimal Society. He played an important part in the formation of the Society as a Charter Member, and as a member of the Board of Directors. While disclaiming any ability in mathematics, he was an able financier and a most kindly adviser. Without his influence, and his friendship for the late John Benbow, it is doubtful if we would have been so fortunate as to have our duodecimal tables published by Longmans, Green.

He will be long remembered in this part of New England as a gentleman of the old school.

G.S.T., Hingham, Mass.

LECTURE CHART

Paul Van Buskirk has delivered a number of talks on duodecimals to interested special groups in his community.

As an effective visual aid, he has developed a chart on the "Analysis of Number Systems," that facilitates ready comprehension of the use of the duodecimal base and its advantages. The chart covers many details of numbers on the bases of six, ten, twelve, and twenty-four.

He has presented to the Society a blue print of this chart approximately *3.3'* high by *3.0'* wide which affords excellent legibility to members of an audience. Since further reproductions would present few difficulties, you might be able to secure reproductions from him at the cost of reproduction and mailing. Address: Paul Van Buskirk, 1524 Clairmont, Detroit 6, Mich. The small scale reproduction shown will give you a good idea of the set-up.

ANALYSIS OF NUMBER SYSTEMS

BASE	SIX		TEN	TWELVE		TWENTY-FOUR	
b	10	6	10	10 en	12	a0	24
b^2		36	100	ɛ = el	144	x	576
b^3	5	216	1,000	ᘔ = dek	1,728	v	13,284
b^4		1,296	10,000	9	20,736	t	331,776
b^5	4	7,776	100,000	8	248,832	r	7,962,624
b^6		46,656	1,000,000	7	2,985,984		191,102,976
FRACTIONS	SEXIMALS		DECIMALS	UNCIALS		SEMUNCIALS	
1/2	.3		.5	.6		.m	
1/3	.2		.333··	.4		.h	
1/4	.13		.25	.3		.f	
1/6	.1		.166··	.2		.d	
1/8	.043		.125	.16		.c	
1/16	.0213		.0625	.09		.am	
1/32	.01043		.03125	.046		.ot	
1/64	.003213		.015625	.023		.oj	
$\sqrt{2}$			1.4142136	1.4ɛ79			
π			3.1415926	3.1848094			
$\frac{\pi}{4}$			.7853981	.9512024			

$\frac{57°}{60}$ $\frac{18'}{60 \times 60}$ = .9 5 5 — 10 10^2 10^3

	FACTORS	TOTAL
60	2 3 4 5 6 10 12 15 20 30	10
100	2 4 5 10 20 25 50	7
144	2 3 4 6 8 9 12 16 18 24 36 48 72	13

PAUL VAN BUSKIRK
Detroit, Mich.

Harry C. Robert, Jr.

LEAST FACTOR TABLE

Numbers 6 n ± 1 Base XII From 3001 to 5ƐƐƐ

Terminations 01 to 5Ɛ

	01	05	07	0Ɛ	11	15	17	1Ɛ	21	25	27	2Ɛ	31	35	37	3Ɛ	41	45	47	4Ɛ	51	55	57	5Ɛ	
30	5		25	5		7	Ɛ	35		11	5	15	1Ɛ	5					11	7	5	25	4Ɛ	5	30
31	61		5	17	7	5			45	Ɛ	1Ɛ	27	5	7	35	5	17		7		15		5		31
32	11				5	Ɛ	15	5	1Ɛ				7	31	5			5			Ɛ	7	25	1Ɛ	32
33	35	7		15	11	37	5			5						7	5		45	5	7	11		Ɛ	33
34	7	5	61	25	1Ɛ	45			5	7		5	Ɛ		7		31		5	Ɛ		5		7	34
35	5	17	1Ɛ	5	51	27			7	15	5		11	5	17	Ɛ		7	4Ɛ	57	5	3Ɛ	7	5	35
36	1Ɛ		5	61	Ɛ	5		11		4Ɛ		7	5			5	7		15	27	35		5	25	36
37	Ɛ				5	7		5			7	11		1Ɛ	5	15	67	5		7	11		Ɛ		37
38		15		Ɛ	7		5			5		1Ɛ		7		11	5		7	5		31	17	37	38
39		5	11		37	61	57	7	5	1Ɛ	15	5	7		Ɛ	51		3Ɛ	5	11	27	5			39
3X	5	7	17	5		25	7	15	51		5			5	4Ɛ	7		Ɛ		35	5			5	3X
3Ɛ	7	11	5			5	Ɛ			7	11		5	Ɛ	7	5	15	17					5	7	3Ɛ
40	27		Ɛ	7	5	11	25	5	7	Ɛ	45			15	5			5			17		7		40
41		1Ɛ	7	31		Ɛ	5		61	5	17	7	35	3Ɛ	27		5		11	5	Ɛ		15		41
42	17	5				7		27	5		7	5		11			Ɛ		5	7	45	5	11	Ɛ	42
43	5			5	7	15	31	45		61	5	3Ɛ	Ɛ	5	6Ɛ	17		11	7	Ɛ	5	27		5	43
44		4Ɛ	5		11	5		7	Ɛ		61		5		15	5			17			7	5		44
45	15	7			5		7	5	11	3Ɛ	67	Ɛ			5	7		5			7	37			45
46	7	27	37	11			5	Ɛ	25	5	31	61	11		7		5		35	5	15		Ɛ	7	46
47	75	5		7			15	11	5			5	61	17		27	11	7	5	67	1Ɛ	5	7	51	47
48	5		7	5	35		4Ɛ				5	7		5	Ɛ		7				5	Ɛ	3Ɛ	5	48
49		37	5			5	17				7		5	61	31	5	1Ɛ	Ɛ		7			5		49
4X		51	11		5		Ɛ	5		15	6Ɛ			7	5	31	27	5	7	11	3Ɛ	17			4X
4Ɛ	25		Ɛ	3Ɛ	57		5	7		5		17	7				5	6Ɛ	15	5	37	7		11	4Ɛ
50		5		35	15	Ɛ	7		5		11	5			17	7			5		7	5		27	50
51	5	Ɛ	4Ɛ	5	17	11			1Ɛ	7	5			5	7		Ɛ			31	5		45	5	51
52			5	7		5	1Ɛ		7	11	15		5			5	3Ɛ	7	11	Ɛ	75	15	5		52
53	37	25	7	27	5	51		5	Ɛ	17		7		11	5	Ɛ	7	5		1Ɛ			11	35	53
54	11		1Ɛ		Ɛ	7	5			5	7	Ɛ	17		3Ɛ	4Ɛ	5	11	61	5				31	54
55	Ɛ	5	15		7		6Ɛ	Ɛ	5	35		5		7		1Ɛ	81		5			5	Ɛ		55
56	5	31		5	27		75	7	11		5		7	5			35	17	Ɛ	61	5	7	15	5	56
57		7	5	11		5	7	17	15			1Ɛ	5		Ɛ	5		75	27	15	7	Ɛ	5		57
58	7	81	35		5	15		5		7	Ɛ	27			5		11	5	37		4Ɛ			7	58
59	17		51	7		31	5	1Ɛ	7	5		11		Ɛ	15	57	5	7	81	5	11	61	7		59
5X	15	5	7			1Ɛ			5	Ɛ		5	57	25	45	11	7		5			5	61		5X
5Ɛ	5	45	11	5	25	7					5		27	5				37	17	7	5		35	5	5Ɛ

Harry C. Robert, Jr.

LEAST FACTOR TABLE

Numbers 6 n ± 1 Base XII From 3001 to 5ƐƐƐ

Terminations 61 to ƐƐ

	61	65	67	6Ɛ	71	75	77	7Ɛ	81	85	87	8Ɛ	91	95	97	9Ɛ	X1	X5	X7	XƐ	Ɛ1	Ɛ5	Ɛ7	ƐƐ	
30	7		17	1Ɛ	Ɛ		5			5	15	Ɛ	57		7		5		3Ɛ	5	11	15		7	30
31	Ɛ	5		7				Ɛ	5	51		5				11		7	5	45	37	5	7		31
32	5	27	7	5		57		17			5	7		5	31		7	25	Ɛ	11	5	5Ɛ	27	5	32
33			5	35		5	11		25		7	4Ɛ	5	15	Ɛ	5				7		Ɛ	5	11	33
34	17	11			5			5			Ɛ			7	5			5	7	37	5Ɛ		15		34
35	37		27		45	11	5	7	15	5			7	Ɛ	11	17	5		31	5		7			35
36		5	Ɛ			15	7		5	Ɛ		5	3Ɛ	51		7	27		5	31	7	5	1Ɛ	35	36
37	5			5		Ɛ	51		17	7	5			5	7		4Ɛ		5Ɛ		5		11	5	37
38	11	Ɛ	5	7		5		4Ɛ	7	35	3Ɛ	15	5			5	Ɛ	7	1Ɛ	25			5	Ɛ	38
39		67	7		5			5			25	7	Ɛ	17	5		7	5		Ɛ	15	11		31	39
3X	31			17		7	5		Ɛ	5	7	45			1Ɛ	Ɛ	5	15	37	5	25			57	3X
3Ɛ		5	35	11	7		17		5			5	11	7		5Ɛ	6Ɛ	51	5		57	5			3Ɛ
40	5	25		5			3Ɛ	7	37		5		7	5		67	11	27			5	7	Ɛ	5	40
41		7	5	Ɛ	31	5	7		1Ɛ	15		11	5	57	5Ɛ	5		37	Ɛ		7		5	1Ɛ	41
42	7	17	25		5	31	1Ɛ	5		7	57			5Ɛ	5	11		5	15			Ɛ	35	7	42
43		35	11	7	15		5	37	7	5	Ɛ		25			15	5	7	27	5			7		43
44		5	7	57			Ɛ		5			5	5Ɛ	Ɛ			7	1Ɛ	5	17		5	25	11	44
45	5	11	Ɛ	5		7			4Ɛ	Ɛ	5	5Ɛ		5	51	1Ɛ				7	5	15	17	5	45
46	3Ɛ		5	25	7	5		15					5	7	11	5	45		7		Ɛ	35	5		46
47		Ɛ	17	45	5			5		11	5Ɛ	1Ɛ	7	25	5		Ɛ	5	11	6Ɛ		7		Ɛ	47
48	67	7	15		25	27	5	35		5			Ɛ	11		7	5	17		5	7	4Ɛ	11	25	48
49	7	5					37	17	5	7		5		45	7	Ɛ		11	5	27	17	5	15	7	49
4X	5			5	Ɛ	1Ɛ			7	67	5	Ɛ		5		37	31	7	51	15	5	11	7	5	4X
4Ɛ	Ɛ		5	1Ɛ		5	27	Ɛ	11			7	5		67	5	7	31				75	5	45	4Ɛ
50		1Ɛ		Ɛ	5	7		5			7		11		5	17		5	Ɛ	7	27	57			50
51	15				7	17	5	11	6Ɛ	5		15		7	Ɛ	25	5	4Ɛ	7	5	31	Ɛ		67	51
52		5				5Ɛ	25	7	5		Ɛ	5	7			6Ɛ		Ɛ	5		11	5		3Ɛ	52
53	5	7		5			7	75	45		5	57		5		7	25	15			5		51	5	53
54	7		5	15	5Ɛ	5	35		57	7			5	17	7	5				11		3Ɛ	5	7	54
55				7	5	Ɛ	11	5	7				15		5		17	5	45		Ɛ		7	11	55
56	51	Ɛ	7		37	45	5	25		5	11	7		4Ɛ			5			5	1Ɛ	27		Ɛ	56
57		5	5Ɛ	31		7			5		7	5	Ɛ	37	11			25	5	7		5			57
58	5	5Ɛ		5	7	35			Ɛ	11	5	17		5		Ɛ	1Ɛ	3Ɛ	7		5			5	58
59		15	5	37	Ɛ	5	31	7	67			Ɛ	5	11	17	5	75		25			7	5		59
5X	Ɛ	7			5		7	5			15	51	1Ɛ		5	7	85	5	4Ɛ		7	15	Ɛ		5X
5Ɛ	7			Ɛ	11		5	15		5	1Ɛ				7		5	67	Ɛ	5		11	37	7	5Ɛ

"ELEMENTARY CONCEPTS OF MATHEMATICS"

Book Review by F.E.A.

The Macmillan Company has just issued *Elementary Concepts of Mathematics* by Burton W. Jones, Professor of Mathematics, Cornell University. It is intended as a text for college students who wish a firmer grounding in the mathematics they "may find interesting and/or useful in later life," but who do not intend to make mathematics a specialty. Its specific interest to the Duodecimal Society lies in the considerable emphasis it gives to counting by dozens, with reference to the papers and books members of the Society have produced.

Professor Jones sometimes writes with a light touch, quoting on his first page L. E. Dickson's dictum that "the most essential tool of a mathematician is a large wastebasket." By the second chapter he is discussing number systems to various bases, beginning with the system to the base ten, "natural to us who can wiggle our fingers."

A considerable discussion of base twelve follows, for which he proposes, with tongue perhaps slightly in cheek, the name *dozal*, "pronounced duzzal," and suggests that "the horses in *Gulliver's Travels*, having three toes on a foot, would perhaps have counted by twelves." For the two additional numerals he would use *t* and *e*, and he would count ... eight, nine, ten, eleven, dozen, doza-one, doza-two . . . doza- eleven, two dozen, two doza-one ... gross ... great gross.

In succeeding pages he presents the rudiments of the fundamental operations to the dozen base. His conversion formulas, however, seem unnecessarily complicated. He concludes this section with a number of exercises in duodecimal arithmetic, one of which is the following:

> In the decimal system, every number which is a perfect square and whose last digit is 0 has 00 as its last two digits. Find a number in the dozal system which is a perfect square, whose last digit is 0 but yet whose next to the last digit is not 0.

Other possible number bases are briefly treated, including the values of the binary base for the game Nim. This is the well-known game in which counters are arranged in three piles, the players drawing successively as many counters as each desires from any one pile; each must draw at least one. The winner is the one who draws the last of the counters. The combinations are far more complicated than at first appears, but Professor Jones points out that winning combinations can be identified by expressing the counters in each pile as binary numbers, and placing these numbers in columns for ordinary addition. Losing combinations are those in which the sum of every column is even; otherwise it is a winning combination. Duodecimalians, with their facility in converting numbers to other bases, may find the theory of Nim an interesting exercise.

Although Professor Jones presents duodecimals "without seeking to change the habit of counting by tens," he points to the "dozens of times when we find counting by twelves useful," and devotes enough attention to the subject so that the student will be able to try the system out for himself. Since the chief obstacle to duodecimal progress in the past has been the fact that very few people have tried it out, this new college text should prove enlightening to many and a distinct aid to our Society's aims.

INTEGER SUMS OF NUMBERS AND THEIR POWERS.

by George S. Terry

In Scripta Mathematica Vol. XIII, No. 1-2, p.117, are given those numbers for which the sum of the integers of a power is equal to the number, i.e., sum of integers in $A^n = A$ omitting the obvious results $1^n = 1$ we have for Base X:-

$9^2 = 81$	$8^3 = 512$	$7^4 = 2401$
	$17^3 = 4913$	$22^4 = 234256$
	$18^3 = 5832$	$25^4 = 390625$
	$26^3 = 17576$	$28^4 = 614656$
	$27^3 = 19683$	$36^4 = 1679616$

The corresponding list for Base XII: is:-

Ɛ^2 = X1	19^3 = 5439	1Ɛ^4 = 115Ɛ41
	1X^3 = 61Ɛ4	
	1Ɛ^3 = 705Ɛ	
	28^3 = 16Ɛ68	29^4 = 429369
	29^3 = 18969	
	2X^3 = 1X8Ɛ4	

The squares given above are $(\text{Base}-1)^2$ and this holds for any num number base. For base Four $3^2 = 21$, for base Seven $6^2 = 51$ and so on.

On what bases (up to base Twelve) are there other results beside these obvious ones? Some answers appear to be:-

Base III	Base V	Base VI	Base VII	Base XI
				$5^2 = 23$
$11^2 = 121$				$6^2 = 33$
$12^2 = 221$	$13^2 = 224$	$14^2 = 244$	$15^2 = 264$	$14^2 = 195$

And for cubes:-

Base III		Base IV		Base V		Base VI	
		2^3	20	3^3	102		
11^3	2101	12^3	3120	13^3	4022	13^3	3213
		13^3	11113	14^3	10404	15^3	10055
22^3	200222	21^3	23121	22^3	23403	23^3	23343
				23^3	32242	24^3	30544

Base VII	Base VIII	Base IX	Base XI
$2^3 = 11$	$6^3 = 330$	$3^3 = 30$	$5^3 = 104$
$4^3 = 121$		$7^3 = 421$	$9^3 = 603$
$11^3 = 1331$			
$12^3 = 2061$			$13^3 = 2075$
$14^3 = 3611$	$15^3 = 4225$	$16^3 = 4560$	$15^3 = 3094$
$15^3 = 5016$	$16^3 = 5270$	$17^3 = 5551$	$18^3 = 5176$
$21^3 = 12561$			$22^3 = X428$
$22^3 = 14641$			

DONATIONS

The work of the Society constantly involves the spending of considerably more money than is received from dues and initiation fees. The expenses of the Secretary's office absorb almost all of our ordinary revenues, and the costs of the Bulletin and of our periodic mail campaigns must be defrayed from other funds. This year, the cost of the Bulletin has been greatly increased in several ways. The costs of paper and printing are naturally much greater, and we have published four issues this year for the first time. In addition, the work of typing and mounting each issue is now being done commercially. The appearance of the Bulletin is much improved, it is more readable, and the grade of the paper is better. However, our expenses have been correspondingly increased.

Our other funds consist of the revenue from the Endowment Fund established through the generosity of George S. Terry, and of donations received from our members who find, in the expansion of the Society's activities, the justification for the personal sacrifices involved. Our receipts in other funds this year have been $112.50 from the Endowment Fund, and donations of $600 from George S. Terry, $50 from Paul E. Friedemann, and $75 from Ralph H. Beard.

The Society wishes to express its gratitude for these generous contributions. It is grateful, too, for the many contributions in time, thoughtful work, and personal effort that our faithful members have made to the progress of duodecimals, and to the welfare of the Society. Its establishment and growth, its increasing influence, bear testimony to the effectiveness of their support.

THE MAIL BAG

It is a pleasure to be able to publish Col. Robertson's "Plea for the Duodecimal System." Our pleasure lies not only in the article itself, but also in learning of further interest in duodecimals in England. The history of duodecimals attests to a considerable English interest in the past, and the possibilities of a reawakened activity there are stimulating.

There are many gaps in our knowledge of the early works on our subject. Many of the early arithmetics contain expositions of the duodecimal base, and of methods of converting numbers from one base to the other. But the source of their very similar presentations is still obscure. Clearly, the idea of different number bases did not originate in England. It may have arisen in Italy, or possibly in Arabia or India. The earliest works on the Hindu-Arabic notation are Italian and Spanish (through the Moorish influence.) It is possible that the concept of a number base may have arisen then.

We know that there were early German works on the duodecimal base, but we have been unable to identify them, or their authors. The Germans are traditionally excellent in bibliography. And the cultural ties between Germany and England in those early days were strong. This is one of the reasons why we look hopefully toward the formation of a duodecimal society in Europe. The work of collating this early material would be greatly facilitated, aside from the great benefit to be derived from another center for the furthering of the education of the public in the knowledge and use of duodecimals.

We know you will enjoy Col. Robertson's article.

. . . We learn that Harry Robert has been active in the formulation and establishment of a building safety code for his native Georgia. An article of his on the safety code and fire prevention appeared in the October issue of the American Engineer. It is an old saying, that if you want to get anything done, turn it over to a busy man.

. . . Paul Van Buskirk brought us up to date on his activities in a letter calling our attention to the article in Science Illustrated on the Japanese abacus. "Why isn't there a duodecimal version of the abacus available" . . . He addressed the April meeting of the Michigan section of the A.S.C.E., on the "Origin of our Weights and Measures," and was highly complimented for an interesting talk. . . . The chart which he developed as an aid to his talks on duodecimals, is undergoing revision, and we will report on the final outcome, later.

. . . Mary Lloyd expresses a certain embarrassment in serving as editor of Mathematical Recreations. She states that most of the papers for that department are way over her head, and that she feels that the author of any one of these articles could more properly be made editor.

We understand her attitude perfectly, but we are surprised, and somewhat complimented, that she should feel that her position is unique in this regard. Our own math is a little shaky when we get beyond trig, but we find great excitement in struggling with the novel ideas presented in some of these papers, and in the amazing properties of the duodecimal base that always emerge in these researches.

We would say to Mary Lloyd that she is not the professor of Mathematical Recreations, but a sort of chairman who has the job of keeping the meeting in order, and getting the transactions properly entered in the minutes.

. . . We have not had papers from some of our old hands for some time. This is most regrettable. We understand the pressures of condensed training that some of these friends are going through, and suppose that similar conditions of adjustment and readjustment are facing many of us. Our own job is being more engrossing than ever.

But, a breather, now and then, is the healthiest thing possible. There is nothing more freshening than to let a mathematical problem absorb our thoughts, to search out the functional relations and factors involved, and to seek to reduce to general terms both the statement of the problem and its solution.

So, TIME OUT! And let's have a letter from you, if only an envelope full of scraps of your old papers with the seeds of some problem scribbled out.

Ye Ed.

COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	X	Ɛ	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	136	Five ft. nine in.	5.9'
31	694	Three ft. two in.	3.2'
96	3Ɛ2	Two ft. eight in.	2.8'
19Ɛ	1000	Eleven ft. seven in.	Ɛ.7'

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2Ɛ*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
 12 ) 30  + 5
  12 ) 2  + 6
       0  + 2     Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by X, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or X.

Numerical Progression

1	One		
10	Do	.1	Edo
100	Gro	.01	Egro
1,000	Mo	.001	Emo
10,000	Do-mo	.000,1	Edo-mo
100,000	Gro-mo	.000,01	Egro-mo
1,000,000	Bi-mo	.000,001	Ebi-mo
1,000,000,000	Tri-mo	and so on.	

Multiplication Table

1	2	3	4	5	6	7	8	9	X	Ɛ
2	4	6	8	X	10	12	14	16	18	1X
3	6	9	10	13	16	19	20	23	26	29
4	8	10	14	18	20	24	28	30	34	38
5	X	13	18	21	26	2Ɛ	34	39	42	47
6	10	16	20	26	30	36	40	46	50	56
7	12	19	24	2Ɛ	36	41	48	53	5X	65
8	14	20	28	34	40	48	54	60	68	74
9	16	23	30	39	46	53	60	69	76	83
X	18	26	34	42	50	5X	68	76	84	92
Ɛ	1X	29	38	47	56	65	74	83	92	X1