The Duodecimal Bulletin

Volume 2 — No. 2
September, 1946

THE DUODECIMAL SOCIETY OF AMERICA

20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y.

THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve-base arithmetic. The lessons and examinations are free to those whose entrance applications are accepted. Remittance of $6, covering initiation fee ($3) and one year's dues ($3), must accompany applications.

The Duodecimal Bulletin is the official publication of the Duodecimal Society of America, Inc., 20 Carlton Place, Staten Island 4, New York. George S. Terry, Chairman of the Board of Directors. F. Emerson Andrews, President. Ralph H. Beard, Editor. Separate subscriptions $1 a year.

CONTENTS

The Duodecimal Bulletin

REVOLVING DUODECIMALS

by F. Emerson Andrews

In the earliest days of my duodecimal experiments, before New Numbers was written or the Duodecimal Society even a dream, I came by accident upon the simplest of the duodecimal revolving numbers (also called periodic fractions, circulatory numbers.) In working out the conversion table for tenths expressed as duodecimals, I observed that all the even tenths - that is to say, the fifths - were curious repetitions:

.2 = .2497 2497 ....
.4 = .4972 4972 ....
.6 = .7249 7249 ....
.8 = .9724 9724 ....

It will be observed that each of these duodecimals is a repetition of four digits following each other in precisely the same order. They have other curious properties. Moreover, such numbers occur in apparently infinite series in number systems to any radix. My "discovery" was, in fact, seven centuries late, since such numbers had fascinated some mathematicians ever since the thirteenth century, and Dickson's History of the Theory of Numbers records at least 154 papers on the subject. But I had opened up one new tool for their investigation, the use of different number bases. This made generalization easier, and has probably resulted in several discoveries at the hands of the capable experimenters who have lately been working with them.

These numbers have a strange fascination which has brought the Duodecimal Society an extensive literature, from members and correspondents. The most recent of these is Mr. Robert Gutmann of Whangarei, New Zealand, who has supplied comprehensive tables on which he has worked for many years. But I attempt no exhaustive discussion of these strange numbers, in which may lie some of the final secrets of primitive roots and the nature of number itself; this is merely an introductory statement, to open up a new field for interesting experiment for any of our members who have not already hit upon it for themselves.

If unity is divided by a prime number (not itself a factor of the number base, as is 3 in duodecimals and 5 in the ten-system) an endlessly repeating point-form fraction will result, with revolving characteristics. Sometimes this number will have just one digit less than the prime from which it is derived, and will then be a perfect revolving number. Otherwise it will have a number of digits which is a submultiple of one-less-than the prime, and will generate the full complement by successive multiplications. For example, the repeating duodecimal for *1/7* is *186 X35*, containing *6* figures and therefore a perfect revolving number. But *1/17* does not produce a *16* place figure, but *076 £45*. This number revolves as usual if multiplied by *7, 8, £, 10,* or *16*; it changes into quite a different form *131 X8X* if multiplied by *2, 3, 5, 12, 14,* or *15*; and transmutes itself into still a third revolving number, *263 958*, if multiplied by *4, 6, 9, X,* or *13*. These three numbers together do contain the necessary *16* digits.

Mr. George S. Terry has pointed out, in his excellent discussion of such numbers in The Dozen System, that the perfect revolving numbers, running the full period, are found only in primes ending *5* or *7*, and do not include all of these. He gives the period for all primes from *5* to *5££*.

To simplify the discussion, we will eliminate the primes which result in limited revolving numbers and examine the extraordinary characteristics of the perfect revolving numbers, setting up in the form of a magic square one example, *1/7*.

1	8	6	X	3	5
3	5	1	8	6	X
5	1	8	6	X	3
6	X	3	5	1	8
8	6	X	3	5	1
X	3	5	1	8	6

1. The digits in the number are one less than the prime from which derived; for the perfect revolving number, d = p - *1* .

2. These digits circulate - repeat in the same order - when multiplied by any number less than the prime.

3. If multiplied by the prime, they come out *£££ £££* .

4. In any column, horizontal or vertical, the first half of the number bears a complementary relation to the second half, each pair of figures adding up to *£*, or in the ten-system, 9; in any case, r - *1* (radix minus *1*). This is pairing quotients.

5. If the remainders producing these quotients are paired, they will in each case add up to the original prime. In the example above, the remainders are *546 231*.

$$7\,\overline{)\,1.0^{5}0^{4}0^{6}0^{2}0^{3}0^{1}0}$$
$$1\;8\;6\;X\;3\;5$$

6. As a corollary of Point *4*, in any square from a completed revolving number, as in the sample above, the sums of all vertical or horizontal columns will be identical, and will also equal half the sum of the two diagonals. This sum will always be $\frac{1}{2}$(p - *1*)(r - *1*), where p is the prime from which the number is developed, and r is the radix, or base, of the number system employed.

7. The number patterns within such a square show many interesting correspondences; for example, patterns are identical for the pairs of numbers which add to (r - *1*), as, *1* and *X*, *3* and *8*, *5* and *6*.

8. In any perfect revolving number, or the completed series of a limited revolving number, the numerals which will appear can be predicted in advance. They are as many full sets of the digits *0* to *£* (0 to 9 in the ten-system) as there are even dozens in the prime, plus those digits which constitute the revolving number for the terminal figure of the prime. For instance, the three sets of *6* numbers generated by *1/17* have been given above. These will be found to be composed of all the numbers *0* to *£*, plus *186 X35*. The revolving number (this time perfect) for *87* is: *01493X 176X59 XX0296 783318 £79805 713466 35£374 0£2269 106-£X7 281X45 16211£ 925438 8X3042 3£64X8 755860 847£09 952X£5*. This number consists of *8* complete sets of the numerals *0* to *£* plus *186 X35*.

9. As Mr. Terry has pointed out, in duodecimals the last figure of the circulating period is the complement of the last figure of the prime; for instance, in the revolving numbers for *7* or *87*, we have seen that the last figure is *5*. This is not necessarily true in decimal revolving numbers.

The list of relationships might be extended much further, but the reader is invited to continue the explorations for himself. It will be found helpful to consider these numbers, not merely in relationship to the primes which generate them, but with reference to $r^n - 1$; that is to say, numbers of the form *££££....* or 9999.... Whenever such a number is evenly divisible by a prime, a number with circulatory characteristics results. If the even divisor is a prime of the order n + *1*, then a perfect revolving number results. For instance, *£££ £££* (or 10^6-1) is evenly divisible by *7* (or n + *1*) developing *186 X35*. I believe all the perfect revolving numbers, in any number system, are found within this formula. They are apparently infinite in number in any system, and every different base necessarily generates its own set.

With the permission of Mr. Terry and his publishers, Longmans Green and Company, we reproduce Mr. Terry's table of reciprocals of primes from Page *36*, The Dozen System.

The table lists the primes from *5* to *5££* and gives the periods of the revolving duodecimals which are their reciprocals. Where the reciprocals are limited revolving numbers, their submultiples are stated. Note the validity of Mr. Terry's corol-

Reciprocals of Primes

Periods and Submultiples

From The Dozen System by George S. Terry

P.	Per.	S.M.	P.	Per.	S.M.	P.	Per.	S.M.	P.	Per.	S.M.
11	2	6	5	4		7	6		Ɛ	1	X
31	9	4	15	14		17	6	3	1Ɛ	Ɛ	2
51	13	4	25	4	7	27	26		3Ɛ	1Ɛ	2
61	30	2	35	34		37	36		4Ɛ	25	2
81	14	6	45	44		57	56		5Ɛ	2Ɛ	2
91	46	2	75	8	Ɛ	67	22	3	6Ɛ	35	2
			85	84		87	86		8Ɛ	45	2
			95	94		X7	X6		XƐ	55	2
			Ɛ5	Ɛ4		Ɛ7	Ɛ6				
111	3	44	105	104		107	106		11Ɛ	6Ɛ	2
131	76	2	125	124		117	116		12Ɛ	75	2
141	20	8	145	144		147	56	3	13Ɛ	7Ɛ	2
171	96	2	175	8	25	157	12	13	16Ɛ	95	2
181	X0	2	195	194		167	166		17Ɛ	9Ɛ	2
1Ɛ1	Ɛ6	2	1X5	1X4		1X7	46	5	18Ɛ	X5	2
			1Ɛ5	1Ɛ4		1Ɛ7	1Ɛ6		19Ɛ	XƐ	2
221	66	4	205	204		217	86	3	21Ɛ	10Ɛ	2
241	120	2	225	224		237	92	3	24Ɛ	125	2
251	73	4	255	254		267	266		25Ɛ	12Ɛ	2
271	27	10	285	284		277	276		27Ɛ	13Ɛ	2
291	83	4	295	294					2XƐ	155	2
2X1	150	2							2ƐƐ	37	X
2Ɛ1	26	12									
301	90	4	315	314		307	102	3	30Ɛ	165	2
321	170	2	325	324		327	10X	3	32Ɛ	175	2
391	1X6	2	365	364		347	46	9	33Ɛ	17Ɛ	2
			375	34	11	357	11X	3	34Ɛ	2Ɛ	12
			3X5	3X4		377	376		35Ɛ	18Ɛ	2
			3Ɛ5	3Ɛ4		397	396		3XƐ	1Ɛ5	2
						3Ɛ7	3Ɛ6				
401	100	4	415	414		427	426		40Ɛ	205	2
421	63	8	435	434		437	436		41Ɛ	20Ɛ	2
431	109	4	455	454		447	446		45Ɛ	22Ɛ	2
471	13	38	465	464		457	15X	3	46Ɛ	7	7X
481	24	20	485	44	11	497	496		48Ɛ	245	2
4Ɛ1	9X	6	4X5	4X4					4ƐƐ	25Ɛ	2
511	266	2	535	534		507	182	3	51Ɛ	45	12
531	276	2	545	544		517	516		58Ɛ	2X5	2
541	54	10	565	564		527	526		59Ɛ	2XƐ	2
591	3X	16	575	574		557	556		5ƐƐ	2ƐƐ	2
5Ɛ1	159	4	585	584		577	116	5			
			5Ɛ5	5Ɛ4		587	1XX	3			
						5Ɛ7	66	Ɛ			

lary: "Only primes ending *5* or *7* produce the maximum period of recurrence in their reciprocals." These primes are of the form of *10n* ± *5*, and serve to illustrate another of the advantages of the use of the duodecimal base.

DUODECIMAL TOOLS FOR THE MACHINIST

by M. Irving Chriswell, Ed. D.

In the process of applying the units called dometric to fields other than the purely scientific, a new principle must be recognized. As yet most writers have considered purely logical principles. A psychological principle now enters the picture. The best illustration of this is perhaps the design and numerical system of inspection tools of the machine trades. Toolmakers and machinists will be very particular about a new tool proposed for their use. The psychological principle referred to is not merely the whim of a machinist. Every feature of these fundamental tools of industry evolved through practical use and facility of measurement.

The Micrometer

The micrometer caliper has its primary divisions marked longitudinally upon the sleeve, a stationary steel cylinder; the subdivisions are marked upon the thimble, the part which revolves at the touch of the finger. The following psychological principles seem inherent:-

1. Graduations marked upon either the sleeve or the thimble must not approach the limit of visual acuity. 1/64" is too fine a graduation.

2. The mechanical design of the micrometer is almost a fixed quantity, and the diameter of the thimble cannot be increased appreciably to allow for more subdivisions.

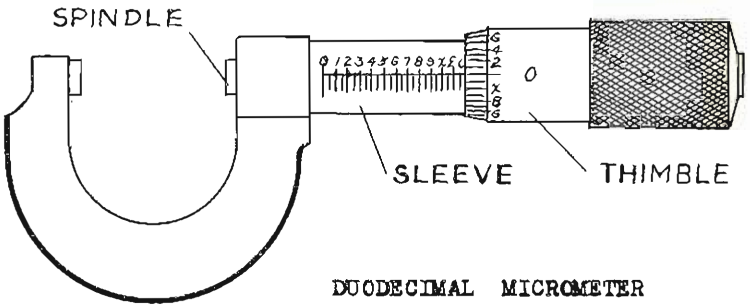
3. The smallest subdivision must be:- one unit, in the third fractional place of whatever number system is used, or *.001* .

A standard 1" English micrometer has a screw pitch of .025 inch. One revolution of the spindle advances it a distance of .025, or one fortieth of an inch. The sleeve is graduated in 40ths of an inch from 0 - 40. The thimble is graduated in 25 divisions, every fifth line being numbered, from 0 - 25. Rotating the thimble from one graduation to the next therefore advances the spindle one twenty-fifth of .025", or .001".

A standard Metric micrometer has a screw pitch of .5 millimeters. One revolution of the spindle advances it a distance of .5 mm. The sleeve is graduated in millimeters from 0 - 25. Therefore it takes two revolutions of the thimble to advance the spindle 1 mm. The thimble is graduated in 50 divisions, every fifth line being numbered, from 0 - 50. Rotating the thimble from one graduation to the next therefore advances the spindle one fiftieth of .50 mm., or .01 mm.

It appears from the above comparison that there is considerable leeway in graduating a micrometer, but to divide a quarter-inch, or quen, into *.001* for a duodecimal tool is impractical. Even a half-inch as the basic unit renders the subdivisions smaller than the metric. A vernier scale on the sleeve, such as on micrometers for reading ten-thousandths, might be suggested, but for all around work a machinist would not welcome this refinement.

The following specifications for the future standard *1*" duodecimal micrometer caliper are suggested:-



DUODECIMAL MICROMETER

Let the pitch of the screw be *.03*". One revolution of the spindle would advance it a distance of *.03*". Let the sleeve be graduated in half-edo inches, i.e., *20*ths of an inch, from *0 - 20*. Therefore it would take two revolutions to advance the spindle one-*20*th, or *.06* inch. The thimble would be graduated in *30* divisions, every *6*th line being numbered, from *0 - 30*. Rotating the thimble from one graduation to the next would advance the spindle one-*30*th of *.030*", or *.001* inch.

Advantages of the Suggested Micrometer

1. Greater visual ease in reading graduations on the sleeve than the English micrometer, the same ease as the metric.

2. Greater visual ease in reading divisions on the thimble than the metric micrometer.

3. Relative accuracy compared in decimal inches:-

English	.001"
Metric	.0004"
Duodecimal	.0006"

The duodecimal presents a happy medium between the two in use. Specifications on blueprints could easily be altered to indicate the tolerances required under the new system.

4. The outstanding advantage of the duodecimal micrometer, which would be welcomed by all shop men, is that the equivalents for the binary fractions come out even in the third place. By contrast, all 64ths must be carried to 6 places, and 32nds to 5 places in the decimal equivalent scale. For example:-

Fraction	Decimal	Duodecimal
1/64	.015625	*.023*
1/32	.03125	*.046*
3/64	.046875	*.069*
1/16	.0625	*.09*

5. Another advantage of the duodecimal micrometer may be illustrated by a sample setting. To set 47/64", or *.899*", we turn the thimble until the graduation *8* appears on the sleeve; we turn the thimble *3* revolutions further (one revolution for every *.03*",) until the *0* appears on the thimble; we turn the thimble *9* graduations beyond the *0*, and mentally add:

```
 .8
 .09      (3 x .03)
 .009
 ----
 .899
```

By contrast, to set 47/64" on the English decimal scale, .734375" is approximated to .734". We turn the thimble until the graduation 7 appears on the sleeve; we turn the thimble one revolution further (one revolution for every .025",) until the 0 appears on the thimble; we turn the thimble 9 graduations beyond the 0 and add:

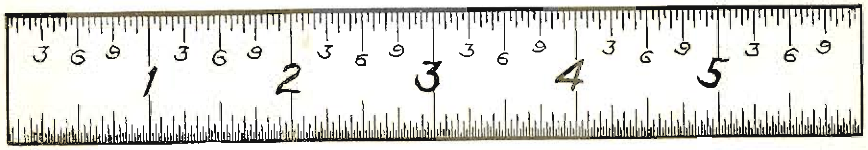
```
 .7
 .025
 .009
 ----
 .734
```

The operation is much simpler on the duodecimal micrometer because the fractional expressions and the settings are clearer, and the mental addition is easier.

The Scale

The psychological principle applied to the micrometer applies with equal force to the steel six-inch scale. Graduations to the 64th of an inch are too fine for visual acuity. They are still marked upon the scales because they offer a more accurate setting for dividers than 32nds, but complaints are often made against the degree of fineness of the division. Scales measuring in 50ths of an inch on the basis of decimal division of the inch are distributed in the United States in limited quantities, and the division into 50 parts is apparently more accurate than 32nds and more readable than 64ths.

With the coming of duodecimals, *40* divisions to the inch are admirably suited to visual acuity. Emphasis upon the *3*, *6*, and *9* subdivisions of the inch seems desirable. Down to the quarter inch, our traditional scale might agree with the duodecimal. The novel subdivision of the quarter inch into thirds will immediately differentiate the duodecimal scale from the traditional scale. These twelfths should then be halved on the one side of the scale, and divided into four parts on the other set of finer graduations.



DUODECIMAL SCALE

To illustrate the use of the new scale, *3 3/4"* will be read *3.9"*, and *5 1/2"* will be read *5.6* . The fine scale will give us the following graduations up to the quarter inch:-

.03 .06 .09 .1 .13 .16 .19 .2 .23 .26 .29 .3

The sixteenths may be read directly from the scale (as *.09* for one sixteenth, *.16* for one eighth, *.23* for three sixteenths,) and there are three finer subdivisions between them. Thus the 32nds may be interpolated, and the 64ths approximated. For example, 5/32" is *.1X6"*, which may be interpolated between *.19* and *.2*, and 9/64" is *.183"*, which is just less than *.19*.

Although, as shown, the duodecimal scale readily accomodates the use of the customary binary subdivisions of the inch, its advantages will become most apparent when used with the duodecimal subdivisions, (lines, and points,) which can be read directly from the scales. The numbered subdivisions are lines, and between each line there are subdivisions of *6* points, and *3* points respectively, as there are *10* lines to the inch, and *10* points to the line.

EXTRACT

from "Base du Systeme Metrique Decimal"

by Mechain & Delambre

Reply to Editorial of The Edinburgh Review, reprinted in The Duodecimal Bulletin, Vol. 1, No. 3

"This report, made to a large assemblage, excluded all detail. Thus the critics who had no other foundation were at least premature, and we are excused from answering them. The only criticism that seems worthy of attention is found in an English journal, The Edinburgh Review. The author who edited it had before him the first volume of our work. His objections, which denote an excellent spirit and much kindness, are proposed only as doubts, and we hope he will be satisfied with our answers.

"His first reproach to us is that we have been too timid, not to have completely profited from the mania for innovation which affected all French minds, and not to have substituted the duodecimal scale for the one which is the foundation of our arithmetic. This idea, indeed, came first when it was thought of founding a new metric system, and if it was not adopted at all, it was not without some very good reasons. Four equal dividers would have resulted, in place of the two for the decimal system. But with how many inconveniences would have been bought this slight advantage. It would have been necessary to reform the spoken arithmetic as well as the written arithmetic. And if the substitution of decimes (tenth part) and centimes (hundredth part) instead of sols and deniers has been effected only in the transactions which are more directly under control, - if the people, and even the more educated class, refuse this so simple and convenient reform, what resistance would have developed to the reform suggested to us?

"How much time would have been needed to accustom calculators to see in the figure 10 the expression of the number twelve, and in 20, 30, and 40, those of numbers twenty-four, thirty-six, and forty-eight, - to see in 50 the number sixty, in 60 the number seventy-two, and in 80 and 90 the numbers ninety-six, and one hundred eight? Numbers to express one hundred and twenty, and one hundred and thirty-two would have to be entirely new, and this is a disadvantage of a different type. They tell us that the introduction of two more characters are enough for this change, - and this is very true in theory.

"We have read a German work in two volumes, which is a complete treatise of duodecimal arithmetic. All the precepts are clear and enlightening, -nothing seems easier or better reasoned out. But after having approved the author's system, let the most ordinary figuring be tried, - a problem using the simple rule of three, - and you will think as we do, that the best way to have failed completely would have been to have given to our plan this more specious than really useful extension.

"The duodecimal division of the circle is especially regretted. They object that the 60 degree angle, or hexagon, is not found in our division. This disadvantage, if it is one, concerns only at the most the makers of instruments. And we do not see that our Borda circles are less well divided, though they have not a single arc whose chord is exactly equal to the radius. As to the calculators, they will never have to use the sixty degree angle when they have given up the sexagesimal division. Of all the sines which made up the old table, only one, that of 30 degrees, was an aliquot part of the radius, and all the logarithms are only approximate numbers. Thus, in this respect, the new division is just as convenient as the old one, and no calculator will notice the difference.

"If astronomers seem little disposed, in general, to accept our centesimal division, it is less on account of the real inconveniences it might have, than by a sort of impossibility of changing, at the same time, their instruments and the subsidiary tables which they now have, - by the difficulty of replacing so soon numbers which are familiar to them and which they need continually, - and, finally, it is a little by prejudice, and because they have made no trial of the new method."

(We gratefully acknowledge our debt to Prof. Jaime Malakis, for)
(the translation of the above article from the original French.)

ADDITIONS TO DUODECIMAL BIBLIOGRAPHY

Bakst, Aaron
Mathematics, Its Magic and Mystery
Van Nostrand, N.Y., 1943, p. 23-29.

Beard, Ralph H.
The Dozen System of Mathematics
Office Appliences, Chicago, June 1946.

Bell, Eric Temple
Men of Mathematics, (Lagrange)
Simon and Schuster, N.Y., 1937.

Buffon, Georges Louis Leclerc, Comte de
Essai d'Arithmetique Morale, Paris 1777.
(Suppl. 4, Histoire Naturelle, Generale et Particuliere)

Caramuel, Joannis
Mathesis Biceps, Vetus et Nova
Officini Episcopali, Campaniae 1670.

Christofferson, H. C.
A New Number System
School Science and Mathematics, December 1924, p. 913-916.

Clark, Jacob M.
Joktanic Arithmetic, (Duodecimal)
Trans. A.S.C.E., Vol. 11, 1882, p. 408-417.

Colles, George Wetmore
Duodecimal System
Letter to the Editor, Mechanical Engineering, October 1945.

De Morgan, Augustus
Arithmetical Books from the Invention of Printing to the Present Time. Taylor and Walton, London 1849.
Elements of Arithmetic
Taylor, Walton and Maberly, London 1846
On the Study and Difficulties of Mathematics
Society for the Diffusion of Useful Knowledge, London 1831

Duodecimal Bibliography
Duodecimal Bulletin, Vol. *1*, No. *3*, p. *16*
Additions, Vol. *2*, No. *1*, p. *6*

Encyclopedie des Sciences Mathematiques
Gauthier-Villars, Paris 1908.

Gelin, E.
Du Meilleur Systeme de Numeration
Mathesis (2) 6 1896 p. 161

Glazier, Harriet E.
Arithmetic for Teachers
McGraw Hill, N. Y., 1923,
(Number Bases, p. 23)

Halliday, G.
Suggestions for a Duodecimal System of Notation, Weights and Measures, Electrical Engineer, London, April, 1896.

Hammer, H. K.
Table of Reciprocals of Primes 2 to 97 (Bases 2 to 12)
Mathematical Tables and Other Aids to Computation, April '46
National Research Council, Washington, D.C.

Jones, Burton W.
Elementary Concepts of Mathematics
Macmillan, N.Y., 1941.

Krause, Karl Christian Friederich
Grundlage der Arithmetike, Jena 1803.

Lagrange, Joseph Louis
Elementary Mathematics, Paris 1795, p. 32.

Lehmer, Derrick H.
Review of Tables of V. Thebault
Table of Squares of Integers 1 to 1000 (Bases 2 to 12)
Mathematical Tables and Other Aids to Computation, April '46
National Research Council, Washington, D.C.

Mechanical Engineering, (Monthly, A.S.M.E.)
Duodecimals
Editorial by George A. Stetson, October 1945, p. 675.

Pascal, Blaise
De Numeriis Multiplicibus, Paris 1665.

Ullrich, E
Rechnen mit Duodecimalzahlen, Heidelberg 1891.

Uspenski, J. V., and Heaslet, M. A.
Elementary Number Theory
McGraw Hill, N. Y., 1939,
(Scales of Notation, p. 13)

Van Buskirk, Paul
Discrepancies in Metric System
Civil Engineering, May 1946, p. 216.

UNITS OF WEIGHTS AND MEASURES

Dr. Walter Renton Ingalls, President of the American Institute of Weights and Measures, has written another excellent monograph on standards of measurement, which has just been published under the above title by the Institute.

This review of the English and American standard units calls to our attention the naturalness and flexibility of the measures in common use among the English-speaking peoples of the world. While there is an absence of system among them, this is directly attributable to the freedom of every trade and industry to adopt that standard and scale of measurement best suited to its needs in practical use. The natural survival of these measures is a forceful testimony to the importance of that freedom. These practical units noticeably disregard the decimal base in preserving their flexibility of structure, yet decimalization is freely used whenever there is any advantage in conforming to the scale of notation.

The irregularity of design among the different basic measures affects the problems of conversion in small degree. The definition of these units is adequately precise and well maintained, and the conversion factors are familiar constants to all who use them, says Dr. Ingalls.

Any coercion in requiring adherence to a fundamental plan of systemization could have had only unattractive results. And, it must be realized that the naturalness of our measures has aided the progress of England and America in world trade.

Dr. Ingalls presents these factors with a simplicity of statement that has its own force. There would be impressive economic benefit in a world standard of weights and measures, and the French metric system, notwithstanding the awkward inflexibility of its decimal base, has proven the desirebility of conforming to the number scale. But, in achieving these results, there may be no sacrifice of facility and flexibility, nor may compulsive authority transgress the freedom to shape our standards to the forms best adapted to our needs.

Any standard must earn its adoption, and its survival, by its own merits. We believe that, for exactly these reasons, the only prospect of achieving a world standard of weights and measures is offered in the duodecimal metric system, based upon our customary natural units.

MEMBERSHIP STATUS

We have neglected to announce the advancement of quite a number of our Aspirants to full status as Members. To correct this omission, the present list of Members and Aspirants follows.

Members

Richard L. Akers
F. Emerson Andrews
Lesbia S. Beard
Ralph H. Beard
Kingsland Camp
W. B. Campbell
M. Irving Chriswell
William S. Crosby
Dudley A. George
P. E. Friedemann
Jamison Handy, Jr.
Robert H. Hoskins
H. K. Humphrey
Dallas H. Lien
Mary Lloyd
Robert C. Lloyd
Alfred Norland
H. C. Robert, Jr.
Eugene M. Scifres
Lewis Carl Seelbach
F. Morton Smith
George S. Terry
Paul Van Buskirk
R. C. Williamson

Aspirants

Gertrude Carpenter
P. A. Chamberlain, Jr.
J. C. Codrington
Albert De Valve
H. Harold Hartzler
Raymond Kassler
Martha E. Martin
Thomas H. Nash
Edward W. Pharo, Jr.
Harry G. Riblet
Daniel Saint
Thomas J. Seaton
Charles P. Stewart
Milton Charles Telles

Arguments are Temins (.01^c) and Minettes (.001^c), paralleled with Degrees and Minutes of Arc.

Degrees & Minutes	Temins	0	12.5	25	37.5	50	62.5	75	87.5	100	112.5	125	137.5	150		
Minettes		0	1	2	3	4	5	6	7	8	9	X	£	'0		
00	00	0000	0063	0107	016X	0212	0275	0318	0380	0423	0486	052X	0591	0634	2£	87 30
02 30	01	0634	0698	073£	07X2	0846	08X9	0950	09£3	0X57	0X£X	0£61	1004	1067	2X	85
05	02	1067	110X	1171	1214	1278	131X	1381	1424	1487	152X	1591	1634	1697	29	82 30
07 30	03	1697	1739	17X0	1843	18X5	1948	19XX	1X51	1X£3	1£56	1££8	205X	2101	28	80
10	04	2101	2163	2205	2267	2309	236£	2411	2473	2515	2577	2619	267X	2720	27	77 30
12 30	05	2720	2782	2823	2885	2926	2987	2X29	2X8X	2£2£	2£90	3031	3092	3133	26	75
15	06	3133	3194	3234	3295	3336	3396	3437	3497	3537	3597	3637	3697	3737	25	72 30
17 30	07	3737	3797	3837	3897	3936	3996	3X35	3X95	3£34	3£93	4032	4091	4130	24	70
20	08	4130	418£	422X	4288	4327	4385	4424	4482	4520	457X	4618	4676	4713	23	67 30
22 30	09	4713	4771	480X	4868	4905	4962	49££	4X58	4X£5	4£52	4£X£	5047	50X3	22	65
25	0X	50X3	5140	5198	5234	5290	5328	5383	541£	5476	5512	5569	5604	565£	21	62 30
27 30	0£	565£	56£6	5750	57X7	5841	5898	5932	5988	5X22	5X77	5£11	5£67	6000	20	60
30	10	6000	6055	60XX	6143	6198	6230	6285	631X	6372	6406	645X	64£2	6545	1£	57 30
32 30	11	6545	6599	6630	6684	6717	676X	6800	6853	68X5	6938	698X	6X20	6X72	1X	55
35	12	6X72	6£03	6£55	6£X6	7037	7089	7119	716X	71££	724£	729£	732£	737£	19	52 30
37 30	13	737£	740£	745X	74XX	7539	7588	7617	7666	76£4	7743	7791	781£	7869	18	50
40	14	7869	78£7	7944	7991	7X1X	7X67	7X£4	7£41	7£89	8015	8061	80X9	8135	17	47 30
42 30	15	8135	8181	8208	8253	829X	8325	836£	83£6	8440	8486	8510	8555	859£	16	45
45	16	859£	8624	8669	86£2	8736	877£	8803	8847	888£	8912	8956	8999	8X20	15	42 30
47 30	17	8X20	8X63	8XX6	8£28	8£6X	8££0	9032	9074	90£5	9136	9177	91£8	9239	14	40
50	18	9239	9279	92£9	9339	9379	93£8	9438	9477	94£6	9534	9573	95£1	962£	13	37 30
52 30	19	962£	9669	96X6	9724	9761	979X	9816	9853	988£	9907	9943	997£	99£6	12	35
55	1X	99£6	9X31	9X68	9XX3	9£18	9£53	9£89	X003	X039	X072	X0X7	X120	X155	11	32 30
57 30	1£	X155	X189	X201	X235	X269	X2X0	X313	X346	X379	X3£0	X422	X454	X486	10	30
60	20	X486	X4£7	X529	X55X	X58£	X5££	X630	X660	X690	X700	X72£	X75X	X789	0£	27 30
62 30	21	X789	X7£8	X826	X854	X882	X8£0	X91X	X947	X974	X9X0	XX09	XX35	XX61	0X	25
65	22	XX61	XX89	XX£4	X£20	X£47	X£71	X£98	£002	£028	£052	£077	£0X1	£106	09	22 30
67 30	23	£106	£12X	£153	£177	£19£	£203	£226	£249	£270	£293	£2£5	£318	£339	08	20
70	24	£339	£35£	£381	£3X2	£403	£423	£444	£464	£483	£4X3	£502	£521	£540	07	17 30
72 30	25	£540	£55X	£579	£597	£5£5	£612	£630	£649	£665	£682	£69X	£6£6	£711	06	15
75	26	£711	£729	£744	£75£	£775	£790	£7X6	£800	£815	£82X	£843	£858	£870	05	12 30
77 30	27	£870	£885	£899	£8£0	£904	£917	£92X	£940	£952	£964	£976	£988	£999	04	10
80	28	£999	£9X£	£9££	£X0£	£X1£	£X2£	£X3£	£X4X	£X59	£X68	£X76	£X85	£X93	03	07 30
82 30	29	£X93	£XX0	£XXX	£X£7	££04	££10	££19	££25	££31	££38	££43	££4X	££55	02	05
85	2X	££55	££60	££66	££70	££75	££7£	££84	££88	££91	££95	££99	££X1	££X4	01	02 30
87 30	2£	££X4	££X7	££XX	£££0	£££3	£££5	£££7	£££9	£££X	££££	££££	1.0000	1.0000	00	00
Minettes		'0	£	X	9	8	7	6	5	4	3	2	1	0	Temins	Degrees & Minutes
Minutes		150	137.5	125	112.5	100	87.5	75	67.5	50	37.5	25	12.5	0		

DUODECIMAL NATURAL COSINES

Arguments are Temins (.01) and Minettes (.001), paralleled with Degrees and Minutes of Arc.

Degrees & Minutes	Temins	0	12.5	25	37.5	50	62.5	75	87.5	100	112.5	125	137.5	150		
Minettes		0	1	2	3	4	5	6	7	8	9	X	£	'0		
00	00	0000	0063	0107	016X	0212	0275	0318	0380	0423	0487	052X	0592	0635	2£	87 30
02 30	01	0635	0698	0740	07X4	0848	08X£	0953	09£7	0X5£	0£02	0£66	100X	1072	2X	85
05	02	1072	1116	117X	1222	1286	132£	1393	1437	14X0	1544	15X9	1651	16£6	29	82 30
07 30	03	16£6	175£	1804	1868	1911	1976	1X20	1X85	1£2X	1£94	2039	20X3	2148	28	80
10	04	2148	21£2	2258	2302	2368	2412	2479	2523	259X	2634	269£	2746	27£1	27	77 30
12 30	05	27£1	2858	2904	296£	2X17	2X82	2£2X	2£96	3042	30XX	3156	3203	3270	26	75
15	06	3270	3319	3386	3433	34X1	354X	35£8	3666	3714	3782	3831	389X	394X	25	72 30
17 30	07	394X	39£9	3X68	3£17	3£87	4037	40X7	4157	4208	4278	4329	439X	444£	24	70
20	08	444£	4501	4573	4624	4697	4749	4800	4873	4926	4999	4X51	4£05	4£79	23	67 30
22 30	09	4£79	5032	50X6	515£	5215	528X	5344	53£X	5475	552£	55X6	5661	5719	22	65
25	0X	5719	5795	5852	590X	5987	5X44	5£02	5£80	603X	60£9	6178	6237	62£6	21	62 30
27 30	0£	62£6	6376	6437	64£8	6579	663X	6700	6782	6845	6908	6990	6X54	6£18	20	60
30	10	6£18	6£X1	7066	712£	71£5	7280	7347	7412	749X	7566	7633	7701	778X	1£	57 30
32 30	11	778X	7859	7927	79£7	7X86	7£57	8027	80£9	818£	8261	8334	8407	84X0	1X	55
35	12	84X0	8574	8649	8723	87£X	8895	8970	8X49	8£25	9003	90X1	9180	925£	19	52 30
37 30	13	925£	933£	9420	9502	95X4	9687	976X	9853	9938	9X22	9£08	9££3	X0X0	18	50
40	14	X0X0	X188	X276	X365	X454	X544	X635	X727	X819	X911	XX05	XX££	X££5	17	47 30
42 30	15	X££5	£0£0	£1X8	£2X5	£3X3	£4X2	£5X2	£6X3	£7X5	£8X8	£9£1	£X£6	1.0000	16	45
45	16	1.0000	1.0107	1.0214	1.0321	1.0430	1.0540	1.0651	1.0763	1.0877	1.0990	1.0XX6	1.1001	1.1119	15	42 30
47 30	17	1.1119	1.1237	1.1356	1.1477	1.1599	1.1700	1.1825	1.194£	1.1X77	1.1£X4	1.2112	1.2243	1.2374	14	40
50	18	1.2374	1.24X7	1.2620	1.2757	1.2893	1.2X11	1.2£51	1.3092	1.3215	1.335X	1.34X5	1.3631	1.3780	13	37 30
52 30	19	1.3780	1.3910	1.3X62	1.3££7	1.4151	1.42XX	1.4448	1.45X8	1.4750	1.48£5	1.4X60	1.500X	1.517X	12	35
55	1X	1.517X	1.5331	1.54X5	1.5661	1.581X	1.599£	1.5£62	1.6127	1.62£4	1.6484	1.6655	1.6829	1.6X05	11	32 30
57 30	1£	1.6X05	1.6£X4	1.7185	1.736X	1.7556	1.7745	1.7938	1.7£32	1.812£	1.8330	1.8534	1.8740	1.8950	10	30
60	20	1.8950	1.8£63	1.917£	1.939X	1.9600	1.982X	1.9X59	1.X091	1.X309	1.X54X	1.X794	1.XX22	1.£076	0£	27 30
62 30	21	1.£076	1.£312	1.£573	1.£819	1.£X89	2.0142	2.0400	2.0685	2.0953	2.1027	2.1305	2.15XX	2.1899	0X	25
65	22	2.1899	2.1£92	2.2292	2.2599	2.28£0	2.3009	2.3332	2.3663	2.399£	2.4124	2.4476	2.4813	2.4£79	09	22 30
67 30	23	2.4£79	2.5335	2.56£1	2.5X7£	2.6258	2.6642	2.6X37	2.723X	2.7650	2.7X72	2.82X4	2.8726	2.8£79	08	20
70	24	2.8£79	2.941£	2.9893	2.X159	2.X635	2.X£24	2.£426	2.£940	3.026X	3.07£0	3.1148	3.16£X	3.2086	07	17 30
72 30	25	3.2086	3.266X	3.3069	3.3686	3.40££	3.4753	3.5206	3.5899	3.6391	3.6XX7	3.7624	3.8185	3.8950	06	15
75	26	3.8950	3.9541	3.X15X	3.X9X4	3.£659	4.0342	4.1059	4.19X8	4.2770	4.3570	4.43X9	4.5267	4.6166	05	12 30
77 30	27	4.6166	4.70X£	4.807X	4.9099	4.X149	4.£252	5.03£3	5.15£5	5.2860	5.3£73	5.5339	5.677X	5.8080	04	10
80	28	5.8080	5.9649	5.£0X8	6.0824	6.2446	6.4159	6.5£70	6.7X98	6.9£10	7.007X	7.2366	7.47X7	7.7209	03	07 30
82 30	29	7.7209	7.9849	8.06X3	8.363£	8.6800	8.X013	9.1730	9.5468	9.9568	X.1X09	X.667£	X.£792	£.51£3	02	05
85	2X	£.51£3	£.£1X3	10.5839	11.0X25	11.8870	12.550£	13.3107	14.1X76	15.205£	16.3978	17.7642	19.1823	1X.XX35	01	02 30
87 30	2£	1X.XX35	20.£X51	23.5X89	26.6710	2X.4545	34.3441	39.X054	47.0172	58.91X7	77.8558	£5.8838	1£0.5352		00	00
Minettes		'0	£	X	9	8	7	6	5	4	3	2	1	0	Temins	Degrees & Minutes
Minutes		150	137.5	125	112.5	100	87.5	75	62.5	50	37.5	25	12.5	0		

DUODECIMAL NATURAL COTANGENTS

OPINIONS AND SUGGESTIONS

Lewis Carl Seelbach

is the sponsor of a synthetic world language which he calls, "Lingveroj". We were privileged to see some of his material on that subject when he sent us the number symbols of the Reuterdahl Alphabet for consideration in connection with the preparation of Kingsland Camp's recent paper on "Symbols".

Carl does not favor the use of "Mo" to represent the great-gross, nor does he see the need for any similarity of sound and form in these prefixes, such as Do, Gro, and Mo. He says:-

"Do, Gro, and Meg are sufficiently differentiated in sound to minimize confusion in listening to numbers. Avoid Modo Grombo. This makes me a heretic, but I shall not start a new church, believing that this point will become self-evident to others upon consideration and practice. Extreme logical consistency is a principle that has distinct limitations in linguistics. 'Meg' has the same meaning as 'Grand', which was Perry's choice of a word for the great gross. 'Mo' sounds like dialect of Amos and Andy for 'more'. 'Meg', by itself, is a complete Greek suffix, not copyrighted, ready for service. The differences in multiple or quantity should be marked and labeled by distinctly different sounds. I believe that 'Mo' was a hasty selection."

William Shaw Crosby

has sent us a number of additions to the Duodecimal Bibliography and we are grateful. He discovered the references in the Encyklopadie der Mathematischen Wissenschaften, and in its French revision, the Encyclopedie des Sciences Mathematiques. It was through him, too, that we learned that certain of the tables appearing in the magazine, Mathematical Tables and Other Aids to Computation, (issued by the National Research Council,) were published in all the bases from two to twelve.

"Bill" has never been happy about the use of X as our symbol for "dek". He has chosen as his preference the Irish "T". In a recent letter he enclosed a drawing of this character which we reproduce for your appreciation. He realizes the need for a separate-identity notation in addition to our least-change usage, and remarks:-

"The thing that stumps me every time I sit down and try to invent a good separate-identity notation is that it is almost impossible to improve on the existing symbols for *1* and *0*. And they are the most important numerals of the entire bunch. Moreover, if there are any two numbers that a separate notation should distinguish, they are 10 and *10*.

"I wonder whether anyone has suggested using the twelve signs of the zodiac. Or rather, the symbols for the signs. They are ancient enough and international enough to please anyone on that score, even if they are not particularly elegant or easy to draw. One caution is advisable: the symbol for Aries, the first sign, should not stand for *1*, but for *0*, so that Aries would continue to mark the astronomer's zero of right-ascension."

Harry C. Robert, Jr.,

surmises that we classify the new Aspirants as "angels who stay in their places, or fools who rush blindly about, tripping over aforesaid angels." We'd classify him as an angel who trips gayly about. With his first letter he submitted a table of cubes, decimal and duodecimal, commenting on the simple symmetry of the duodecimal pattern. He made some interesting suggestions on symbology and submitted the Table of Quarter Squares which appeared in last issue. In this issue there is the Coconut Problem and his sweet solution. If this proves as interesting to our readers as we expect, there will be more of his problems in subsequent issues. We feel like the oil prospector who struck the Big Inch.

Dr. R. A. Millikan

of the California Institute of Technology, in response to a letter of ours, said:- "Your appeal should have been directed to God about a million years ago, to give us twelve fingers and twelve toes instead of ten, in which case the advantages of the duodecimal system would have been attained. Perhaps it was too bad it was not thought of at the time the Great Architect was fashioning our hands and feet."

We thank Dr. Millikan for his kind acknowledgement of the advantages of duodecimals. May he live to enjoy the benefits of duodecimal numbers and measures, in determining the value of the physical constants.

Raymond Kassler

writes that he has brought home from his service in Germany a set of mathematical and surveying tables based on the centessimal system. That system of circular measure divides the right-angle into 100 grades (or grads), each of 100 minutes of 100 seconds. The unit point is represented by a comma instead of our customary period, and the log functions are expressed to five places of decimals for each of these minutes.

Paul Van Buskirk

is one of our Aspirants who is about to enter the service. He has been participating in a discussion of the Metric System in the "Our Readers Say" section, of the monthly, "Civil Engineering". An excellent paper of his on "Discrepancies in Metric System", appeared in their May issue.

In a recent letter he mentions "Men and Measures", by Edward Nicholson. This is an excellent study of the physiological origins of many of our measures and weights, which will richly repay the perusal of anyone interested in the duodecimal measures.

Mr. Nicholson gives the following definitions of some of our natural measures:-

Cubit = *16*" = length of bent forearm from elbow-point to fingertip.

Span = *9*" = tip of thumb of outspread hand to tip of little finger.

Palm = *3*" = breadth of four fingers.

Digit = $\frac{3}{4}$" = breadth of middle finger at its middle.

Yard = 2 Cubits.

Foot = *10*" = *14* Digits = *4* Palms.

Fathom = *6* Feet = *2* Yards = *4* Cubits.

A Roman statement, probably from Frontinus, gives the following:-

"Pes habet palmos iv, uncias xii, digitas xvi;
Palmus habet digitas iv, uncias iii."

Nicholson further says:- "A vessel containing a cubic foot of water afforded a standard, the Eastern Talent, both for weight and for capacity. The vessel containing a weight of corn equal to the weight of a talent (or amphora) of oil, became the bushel, about twenty-two to twenty-five percent more than a cubic foot." This is meaty stuff.

It is interesting to note that the Roman traders and merchants used twelve almost exclusively in the subdivisions of their weights and measures, while the Roman military used the multiples of ten in much of its work.

Dudley A. George writes:-

"Some twenty years ago, I thought it would be fun to figure in twelves. However, I had, in order to earn a living, (as a bank clerk then,) to continue to figure in the usual way in tens. The solution was the use of new symbols, both figure and sound, so as to avoid confusion. I find now that I have no difficulty doing your problems, but continue to think in my own symbols.

"For the record, my figures (which have changed a little year by year, but not much,) are as follows:-

1	/	on	pronounced	on
2	∠	tu		too
3	7	ri		ree
4	ϑ	ka		kah
5	V	fe		fay
6	y	si		see
7	⩘	poi		poi
8	ℰ	ge		gay
9	S	lai		lye
X	γ	vi		vee
Ɛ	ℓ	sho		show
10	/0	den		dane

"In the 'do'-place, I think of triple digits times 'do', as ending in 'n'; triple plus one, in 'r';: and triple minus one, in 'l', as follows:-

ner-tu (*12*)	tul-ri (*23*)	ren-fe (*35*)
kar-ge (*48*)	fil-ka (*54*)	sen-poi (*67*)
por-poi (*77*)	gel-sho (*8Ɛ*)	lin-vi (*9X*)
vir-tu (*X2*)	shol-ka (*Ɛ4*)	shol-di (*Ɛ0*)

"I have tried consonant-vowel combinations such as Mr. Seelbach suggests in 'Consovocalic', and decided that, although splendid in theory, the sounds do not have adequate body for practical use either in reading aloud or in memorizing. I believe that the mind and the tongue demand at least a syllable per symbol for efficient thinking.

"Please do not assume that I support the Principle of Separate Identity exclusively. It seems to me that popular approval requires that the Principle of Least Change be closely followed for most purposes. But sometimes 'Least Change' requires more than one would at first suppose. For instance, the use of 'do' for three different ideas (for *10*, for '-teen' as in 'do-three', and for the '-ty' as in *40*,) is contrary to our habit, and against the Principle of Least Change.

"The use of 'edo', 'egro', etc., is a greater deviation from our usage than is necessary. Of course, it is not logical to use the same suffix for the fractional, partitive numbers and for the ordinals. But I believe that we have, in 'er', an English suffix which would adapt easily and naturally to this use, and at the same time broaden our mathematical language; as:- 'six doner' for *.6*, and 'four grosser' for *.04*. If you think of these as 'donair' and 'grosair', they sound more appropriate.

"We similarly need a suffix to denote the powers too, but this is enough for the present."

MATHEMATICAL RECREATIONS

Mary Lloyd, Editor

You'll get a laugh out of a contribution by Harry C. Robert, but you'll also learn an excellent bit of mathematical tactics from his handling of a problem. He writes:-

"This problem was pulled on me by one of the ex-service men in my office just to see if I was as good as he was at such things. He had spent two weeks working on it while returning from overseas service, and had won $50.00 from his Colonel. But imagine his surprise when I showed him that he had won with an inferior answer.

Problem of the Coconuts

"There are five men, one monkey, and a pile of coconuts. The first man gives the monkey one coconut and takes exactly one-fifth of the remainder. The second man likewise gives the monkey one coconut and takes exactly one-fifth of the remaining nuts. The third, fourth, and fifth men also, each in his turn, first give the monkey one coconut and take exactly one-fifth of the remainder. After the fifth man has taken his coconuts, the rest of the pile is divided equally among the five men. What is the least number of coconuts that can be in the pile at the start, for the foregoing division to be possible, using only whole coconuts at each step of the transaction?"

After you have worked out your answer, compare your operation with Harry Robert's clever method, shown on Page 1X.

Mr. Terry sent us a letter crammed with interesting items on numbers and number problems. We have selected three of them for this issue, and will keep the rest on ice for later use.

. . . . .

A. In how many ways can unity be expressed as the sum of 3 equal fractions, each integer being used once For instance:-

$$\frac{3}{9} + \frac{8}{20} + \frac{57X}{14£6} = 1$$

There are three other solutions, and the process of elimination is a good exercise in the use of the three-times table. Can you find the others? This construction is not possible for decimals.

. . . . .

B. The value of π, the ratio of the circumference of a circle to its diameter, has been calculated by many people to many places. Duodecimally, its value is 3.184 809 494 etc. The following fraction, with the same integers in the numerator and the denominator, gives a remarkably good approximation to π. You may be able to find others equally interesting.

$$\frac{103£}{03£1} = 3.184\ 80X$$

C. What sums of two-figure addends exhibit, with the addends, each integer from 1 to £ once? For example:-

1X£ = 32 + 54 + 68 + 79

Duodecimally, there are eight such sums, each divisible by £.

1X£ 218 245 263

1£X 236 254 281

Decimally, there are only four, each, of course, divisible by 9.

189 216 234 243

Can you determine why they are divisible by r - 1?

. . . . .

Mr. Andrews submits an amusing construction which is valid for decimals and duodecimals.

```
               1 x £ + 2 = 11
              12       3   111
             123       4   111 1
           1 234       5   111 11
          12 345       6   111 111
and 12 345 678 9X£    10   111 111 111 111
```

. . . . .

This time we have four puzzles, one in each of the four ordinate operations, and all four are about our favorite topic. Solve 'em and see! The one on division is a bit tougher than usual because it employs only £ letters. So don't forget to add a blank letter column for the numerical eliminations. You'll probably fill in the "blank" with a stronger and perhaps more appropriate word before the crypt is soluted.

```
  I Z S U O E N E S I          P R I D E I N O U R
+     Z A O E A E S V        - T R U E M E T H O D
  -------------------          -------------------
  V U R L L L L D U R          N H P U R E P Z I R
```

```
                        U U R
              ---------------
C I R C L E ) A M E R I C A N
              C I R C L E
              -----------
                L O D D E A
                C I R C L E
                -----------
                M I I M A R N
                M U D C M D N
                -------------
                  E A L R A N
```

```
            I D E A L
        x     K I N D
      ---------------
      E D O O R D D
    L I Z Q O L
  U E E Z D L
  -------------------
```

Harry Robert's Solution of the Coconut Problem

Since the remainder is to be divided by *5*, and is also four-fifths of a larger number, and therefore divisible by *4*, we may write:

$$\text{Final Remainder} = R_5 = 4 \cdot 5a = 18a$$

$$\frac{5}{4} R_5 = \frac{5}{4}(18a) = 21a$$

$$R_4 = \frac{5}{4} R_5 + 1 = 21a + 1$$

$$\frac{5}{4} R_4 = \frac{X5a + 5}{4} = B$$

This is used each step to eliminate the fraction:

$$B = 27\,a + 1 + \frac{a + 1}{4}$$

If we let $a = 4b - 1$,

$$B = X4b - 27 + 1 + b = X5b - 26$$

$$R_3 = \frac{5}{4} R_4 + 1 = B + 1 = X5b - 25$$

$$\frac{5}{4} R_3 = \frac{441b - 101}{4} = C = 441c + X0$$

$$R_2 = \frac{5}{4} R_3 + 1 = C + 1 = 441c + X1$$

$$\frac{5}{4} R_2 = \frac{1985c + 425}{4} = D = 1985d - 446$$

$$R_1 = \frac{5}{4} R_2 + 1 = D + 1 = 1985d - 445$$

$$\frac{5}{4} R_1 = \frac{9061d - 19X1}{4} = E = 9061e + 1980$$

$$R_0 = \frac{5}{4} R_1 + 1 = E + 1 = 9061e + 1981$$

The smallest value is obtained when the arbitrary integer, e , equals *0*. This yields the Answer: *1981* coconuts.

. . . .

A final word as to last issue's answers. The solutions for Mr. Terry's puzzles were "Unphlegmatic" and "Ambidextrous". He must have been, to find another twelve-letter key word. The solutions for the others were "Butch Dozenal", and "Decimal Group".

Thanks, you all, for your letters. If you'll keep them coming along, we'll be able give you more of the kind of stuff you like best.

Mary Lloyd

Each One Teach One

THE MAIL BAG

You will find that much of the material which once reached your attention via The Mail Bag is now to be found under "Opinions and Suggestions." What you will find here, are items that defy other classification, and just what you might expect to find at the bottom of any mail bag, - bits of letters, scraps of comment, and crumbs from the editor's pockets and desk.

. . .

Being located at the cross-roads of the world, we enjoy the advantage of meeting some of our people now and then, generally on their way somewhere else.

In the early summer, we spent some delightful hours with Mrs. F. Howard Seely and her daughters, Mary Seely and Dorothy D'Ooge. They were on their way to visit family relatives and friends in Connecticut. They had stopped for several such visits on their northern route east, and planned to make others on their southern route back to their home in Piedmont, California.

Several weeks later, we had luncheon with Jamison Handy and his wife, who were making a similar circuit from their home in Los Angeles. President Andrews was able to join both these gatherings, and shared in the pleasure of welcoming these members of our duodecimal family.

We wish to accent the benefit we all derive from these personal contacts. Because of the wide dispersion of our members, the pleasure of talking things over together is only rarely possible and we have to exchange our ideas in rather formal written fashion. We wish to become personally acquainted whenever good fortune permits. Let us know when you plan to come to New York, and, in any event, don't fail to call when you are here. You will find the Beard home telephone listed in the New York telephone directory. But it might be well to note, that, during business hours, you can reach that man, (or leave word for him to call you,) over Exchange 4-4600, Extension 824.

. . .

Arthur Coldewe, of San Anselmo, Calif., has just informed us that, in Trautwine's Civil Engineer's Reference Book, there is an exposition of the use of the duodecimal base in which the characters t and e are used for X and £. This arrived too late for the necessary checking before inclusion in our bibliography. Thanks for the news, Mr. Coldewe.

. . .

We are enjoying a healthy mail response to our article, Duodecimal Mathematics, which appeared in the June issue of the magazine, Office Appliances. Among these inquiries were letters from Copenhagen, Denmark, and Gothenburg, Sweden. The filaments of our Society's co-operation are gradually weaving a network of duodecimal enthusiasts over the entire world. Try your hand at a little thread spinning of your own, and send a duodecimal article to your most used, and most favored magazine.

The distances reached by a published book or article are amazing. The roof of the world echoes the sound of the printed word. President Andrews, in his article in this issue, mentions his correspondence with Mr. Gutmann of Whangarei, New Zealand. Another response to his book, New Numbers, is that of Professor J. C. Codrington, of Habana, Cuba.

Professor Codrington teaches night classes in mathematics. He became enthused by reading New Numbers, and inquired of Mr. Andrews about the Duodecimal Society. He has now about completed his tests as an Aspirant, and is translating New Numbers into Spanish. At Mr. Andrews' suggestion he has sent us an extensive roster of names for our mailing list, and may be able to develop a nucleus of dodekaneers in Habana, thus extending our contacts among our neighbor Americans.

A recent check of our files discovers mail responses from 38 of the United States, from 3 of the Canadian provinces, from Cuba and Panama, - from England, Scotland, Ireland, Belgium, France, Germany, Denmark and Sweden, in Europe, - from Egypt in Africa, - from Japan in Asia, - and from New Zealand and Australia.

. . .

Dr. Nathan Lazar of Teachers College, Columbia University, lectures on the History of Mathematics, including the development and use of other number bases than the ten. For some time, he has delivered to his students copies of our pamphlets and of the Duodecimal Bulletin. For the monthly magazine, The Mathematics Teacher, (official journal of the National Council of Teachers of Mathematics,) Dr. Lazar conducts a department called "In Other Periodicals," in which he lists the contents of the latest issues of the other mathematical publications. We were delighted to learn that Dr. Lazar has covered the contents of each Duodecimal Bulletin in the succeeding issue of The Mathematics Teacher. We are very grateful to Dr. Lazar for his courtesies, and for his generous co-operation.

. . .

We frequently indulge in the pleasant pastime of searching for references to duodecimals in the mathematical works in New York's libraries. Several of our recent discoveries will interest you. Leon Bassot, in his "Historical Sketch of the Foundation of the Metric System," writes the following:-

"On 8 May 1790, the French National Assembly rendered a decree which is, in a way, the 'acte de naissance' of the Metric System, which says, page 5 and 6, - 'Finally, the Academy shall indicate the scale divisions which it believes most convenient for all weights, measures and coins.'

"The Academy at once set itself to work, and on 27 October 1790, the first commission, composed of Borda, Lagrange, Lavoisier, Tillet, and Condorcet, presented their report on the title of coined metals, and the scale of uniform division, and gave the preference to the decimal division over the duodecimal. Laplace, in his Systeme du Monde, has explained in a masterly way the reasons which determined the commission.

" 'The identity of the decimal calculation and that of the integral numbers admits of no doubt of the advantages of the division of every kind of measures into decimal parts; it is sufficient, to convince oneself, to compare the difficulties of complex multiplications and divisions with the facility of the same operations under integral numbers, a facility which becomes even greater by means of logarithms, of which, by simple and inexpensive instruments the use could be made extremely popular. In truth, our arithmetical scale is not divisible by three and four, two divisions whose simplicity render them very common. The addition of two new characters is sufficient to procure this advantage, but a change so important would have been infallibly rejected with the system of measures which was attached to it. Moreover, the duodecimal scale has the inconvenience of requiring that we retain the products of twelve numbers which surpasses the ordinary length of memory to which the decimal scale is proportionate.

" 'Finally, we should lose the advantage which probably gave rise to our arithmetic, that which causes the fingers of the hand to be used in enumeration. We did not hesitate to adopt the decimal division and for the purpose of uniformity in the whole system of measures, we decided to derive them all from the same linear measure, and its decimal divisions. The question is then reduced to the choice of this universal measure, which has been given the name of meter.' "

Then we turned up some material on Lagrange in Eric Temple Bell's "Men of Mathematics." He reports that Lagrange played an important part in the planning of the metric system, and that it was due to his opposition that 12 was not chosen as the base instead of 10.

Mr. Bell indulges in some heavy irony about the propagandists for the use of the duodecimal base, whose " . . disadvantages are numerous and obvious to anyone who understands short division." He credits Lagrange with proposing 11 as a better base than 12, and says that " . . any prime number would have the advantage of giving all fractions in the system the same denominator."

Perplexedly, we confess that we are unable to construe this last statement of Mr. Bell's so that it makes sense. We know of no way to prevent the use of vulgar fractions of the form of 1/3 or 5/6. That cannot be what he means. Nor can he be referring to point-form fractions. For, regardless of the base used, - prime, or satisfactory, - point-form fractions all use the number-base as the agreed denominator, unless other usage is expressly stated. So that interpretation is eliminated.

The evidence of some degree of heat in Mr. Bell's remarks would seem to indicate a sensitivity to the point in question. Perhaps it were well to dismiss his statement as without particular meaning.

. . .

In the Scientific Monthly for August appears a letter from our revered friend, George Wetmore Colles, in criticism of Professor Oystein Ore's article on "Our Everyday Reckonings" which was published in the Scientific Monthly for November, 1945. Though somewhat belated, we rejoiced that The Scientific Monthly gave Mr. Colles' letter the publicity it deserved. We know that the letter was written much earlier than our comment on "Unscientific Science" in the May Duodecimal Bulletin, and we looked for its appearance. Our congratulations to all concerned, including the editors of the Scientific Monthly.

Mr. Colles does not entirely approve of the use of a duodecimal notation for general purposes, but he does highly approve of the use of the duodecimal subdivision of the scales of our weights and measures. In 1898 he presented a paper before a meeting of the American Society of Mechanical Engineers on, "The Metric versus the Duodecimal System," which inaugurated a thorough review of the metric proposal by that society, ending in the decision that no action toward compulsory use of the metric system could be countenanced. Through the kindness of Mr. Colles, we have still a few copies of his classic paper, which we will be glad to place in hands that appreciate its value.

. . .

Mr. M. Irving Chriswell, author of the article on "Duodecimal Tools for the Machinist" in this issue, has just graduated from Aspirant to Member. He is a teacher of mathematics in one of Buffalo's high-schools, and was introduced to the Society by Lewis Carl Seelbach. We think that his article deserves high praise, and is an important contribution at just this time.

. . .

The number symbols proposed by Mr. Dudley A. George, Aspirant, are presented through the able assistance of William Shaw Crosby, who redrew them in form permitting reproduction. We wish to thank him for his kindness, and to praise his workmanship.

. . .

Each One Teach One

Harry C. Robert, Jr., recently did some extemporaneous philosophizing about weights and measures on his typewriter, and you will enjoy his thinking, as I did. He says:-

"The history of the subject indicates that the various units of weights and measures have been, almost invariably, the result of the activities of tradesmen in facilitating the flow of commerce, the operations of building craftsmen in fitting their structures together, or of land surveyors in partitioning land for various purposes. At centers of such activities, units of various sorts were adopted by common consent, and since their primary purpose was to simplify the work at hand, these units generally were well adapted to their requirements, and fitted the arithmetic of their day.

"As long as these units remained more or less informal, they could be, and were, modified to meet changing requirements. This happy and ideal situation lasted only until some queer quirk of the laws of evolution produced the 'city slicker.' Or maybe this fellow was produced by the first anti-lynching law, instead of the law of evolution, and until we gave him too much legal protection, he did not last long enough to get his name in the history book. At any rate, when this short-weight, short-measure, short-change cheat appeared on the scene, with sufficient legal protection to save his hide from his victims, the public demanded protection from the same law, and units of weights and measures were given legal definitions.

"Once the units were on the statute books, progress in keeping the units in step with changing requirements was at an end. Such has been the stagnant condition of our standards while trade grew from a relatively local affair to world-wide proportions, - while central governments developed which controlled great portions of the earth's surface, - while men generally adopted the symbol for 'nothing', and rebuilt their arithmetic using positional notation.

"During this time, with the exceptions of calendar reform, and the French metric system, the reforms in our standards have amounted to nothing more than compromises of minor quibblings over the second and third decimal-places of equivalents, and the legalistic wording in the definitions of units.

"The French metric system, however much we decry its adequacy today as an universal standard, represents an heroic attempt by man to put his legal standards back into step with the requirements of the times. It is truly remarkable that the French mathematicians were able to produce such a system in the midst of a tremendous upsurge of revolutionary ideology. Their concept of units in each system arranged in a geometrical progression with a ratio equal to the number base, and of consistent inter-relationships between the various systems have proven remarkably sound. That certain of these consistencies no longer hold, and are therefore fallacies in the common metric system propaganda, is not the fault of the concept, but should be blamed on the lawyers who tried to tie these standards to various chunks of entombed metal with legalistic language.

"Now the defects in the metric system are due almost entirely to the fact that the learned French mathematicians, like hothouse plants, knew too little of the facts of life. The phenomenal development in all branches of mathematics during the two centuries preceding the revolution could not have made them very acutely aware of the defects in their number system based on ten, or of the potential superiority of other bases. If they were aware of the historic involvement of the factor 3 in the life of the common man, and particularly in linear measure, they disregarded this knowledge, possibly on the assumption that if man could learn to live without kings, he could also do without his even third. They also disregarded the size of the unit that man had found most convenient for his every day ordinary uses.

"Most ancient systems had principal units ranging from ten inches to twenty-five inches, with many systems containing apparently two units within this range, - related generally by ratios of 1/2, 2/3, or 3/5. Not only in these ancient systems, but in many present day usages, and evidence from our rather complex and overlapping English system, I believe we have a clew to the matter. It appears that man has always tried to find units of various kinds that would permit him to give a reasonably exact description of any object, using only one unit, without resorting to fractional parts, or using too great a number of these units.

"When his concept of number was limited to something in between the number of his fingers and the sum of his fingers and toes, he felt the need of units spaced not more than ten or twenty numbers apart. To some extent he still prefers this. But as his number concept grew, and the common man had dollars frequently enough to learn how to make change, the need for such closely spaced units decreased.

"I suspect that for at least two centuries, each succeeding generation made less use of the yard than the preceding one. The rod or pole, the fathom, and many other old measures have practically disappeared except for very limited uses. Now, we can almost count to one hundred! But we are not much, if any, beyond that point. And now we have come to where we must change our standards, and reconstitute them in line with our requirements of today. But we should consider all phases of the problem very carefully before we even talk about new units, or new nomenclature."

. . .

Thanks, Mr. Robert. There are a number of our people getting their ideas on weights and measures into coherent form. Let's spread them on the record, so that others may have the use and benefit of your work and ideas.

Whiskers.

Our common number system is decimal - based on ten. The dozen system uses twelve as the base. This requires two additional symbols: X, called *dek*, is used for ten, and £, called *el*, is used for eleven. Twelve is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

Modern numeration employs one of the greatest of man's inventions, the zero - symbol for nothing. It permits the use of place values. In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and 2 dozen-dozen, or gross. This number would be called 2 *gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	136	Five ft. nine in.	5.9'
31	684	Three ft. two in.	3.2'
86	3£2	Two ft. eight in.	2.8'
18£	1000	Eleven ft. seven in.	£.7'

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only 2£, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal number.

```
12 ) 365
  12 ) 30  + 5
    12 ) 2  + 6
         0  + 2      Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by X, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or X.

Numerical Progression

1	One		
10	Do	*.1*	Edo
100	Gro	*.01*	Egro
1,000	Mo	*.001*	Emo
10,000	Do-mo	*.000,1*	Edo-mo
100,000	Gro-mo	*.000,01*	Egro-mo
1,000,000	Bi-mo	*.000,001*	Ebi-mo
1,000,000,000	Tri-mo	and so on.	

Multiplication Table

1	2	3	4	5	6	7	8	9	X	£
2	4	6	8	X	10	12	14	16	18	1X
3	6	9	10	13	16	19	20	23	26	29
4	8	10	14	18	20	24	28	30	34	38
5	X	13	18	21	26	2£	34	39	42	47
6	10	16	20	26	30	36	40	46	50	56
7	12	19	24	2£	36	41	48	53	5X	65
8	14	20	28	34	40	48	54	60	68	74
9	16	23	30	39	46	53	60	69	76	83
X	18	26	34	42	50	5X	68	76	84	92
£	1X	29	38	47	56	65	74	83	92	X1