

The
Duodecimal Bulletin

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THE DUODECIMAL SOCIETY OF AMERICA

20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y.

THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve-base arithmetic. The lessons and examinations are free to those whose entrance applications are accepted. Remittance of \$6, covering initiation fee (\$3) and one year's dues (\$3), must accompany applications.

The Duodecimal Bulletin is the official publication of the Duodecimal Society of America, Inc., 20 Carlton Place, Staten Island 4, New York. George S. Terry, Chairman of the Board of Directors. F. Emerson Andrews, President. Ralph H. Beard, Editor. Copyrighted 1946. Permission for reproduction is granted upon application. Separate subscriptions \$1 a year.

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The Duodecimal Bulletin

REPORT OF PRESIDENT ANDREWS
at the Annual Meeting

When this Society held its first official Annual Meeting last January, we were embroiled in desperate efforts to recover from the German offensive in Belgium, and it seemed to most observers that probably both wars, and certainly the Japanese phase, would last throughout 1945. Some of our most promising younger members were with the armed forces. Professional mathematicians, in teaching and elsewhere, were doing double duty training bomber pilots, seagoing navigators, and engineers, or they had been spirited away for the mysterious Manhattan Project which produced the atomic bomb, or other enterprises only less critically important. All of us were burdened with extra tasks and extra strains growing out of the war. We asked ourselves, was it courage or mere rashness to launch a society of this nature in a year so troubled?

Whichever it was, the year 1945 has shown more progress than could reasonably have been expected. There have been difficulties. Some of our members from whom we expect much are still overseas. Efforts to build up our membership, particularly with younger men, encountered little success while the war was on, and need now to be redoubled. We are still too small a society to carry on the effective program for which your officers hope. But marked gains have been made, and it is proper that they should be recorded at this meeting.

The most notable achievement of the year, in my opinion, was the publication of the Duodecimal Bulletin. Issues were published in March, June, and October in editions of about 700 copies. In addition to our own membership, this Bulletin reaches strategic university and public libraries, editors of mathematical journals, and leaders in mathematical thought. The New York Public Library alone distributes 170 copies through its various branches. One hundred copies go to Teachers College of Columbia University for distribution to Dr. Lazar's classes on the history of mathematics. Through the medium of this Bulletin the stimulating, thoughtful, and constructive articles which some of you have contributed, have been given some permanence, and have become part of the growing duodecimal literature. Thanks of the Society go to all of the contributors, but they go in triple measure to Ralph H. Beard, editor, production man, and circulation manager. His faithful, arduous work is beyond praise.

During 1945, the Society's Annual Awards were announced for 1944 and 1945. Over his protest, the 1944 Award was made to your president, who claims no merit except that of bringing a valid and important idea to the attention of abler minds. The Annual Award for 1945 went to Mr. George S. Terry, whose Duodecimal Arithmetic is by far the most important work yet produced in this field of mathematics. One of the members of the Board of Directors wrote recently, "I am glad the Society is now on its feet - or at least, on Mr. Terry's feet." But Mr. Terry is an exceedingly modest man, and would not permit me to tell you to how great an extent this Society owes its continuing existence to his support, intellectual and financial. So I shall leave these facts unmentioned, but not unguessed.

The work of Vice-President F. Howard Seely with the aspirants and new members deserves special mention.

Advancement of the idea of counting by dozens has proceeded in many directions even in this year of tremendous news events. A notice in the Journal of the Franklin Institute brought us correspondence from Belgium, Ireland, Scotland and other countries. The letter of George Wetmore Colles in Mechanical Engineering resulted in an interview by our indefatigable Mr. Beard with Editor Stetson, and the appearance of an excellent presentation of the Society's objectives in the next issue of that journal. Several newspaper articles have appeared, based on official releases, or special letters by members.

The Library of Congress now uses "Duodecimal" as a heading in its Index. Your president, at the invitation of the Mathematics Section of the New York Society for the Experimental Study of Education, lectured on duodecimals for one evening at the Columbia University Faculty Club. We have had further confirmation of the use of duodecimal multiplication by the U. S. Army Transport Command in figuring cargo cubages. Thanks to Mr. Beard, an extensive bibliography of books, articles, and papers relating to duodecimal counting is now available, having been published in the October issue of the Bulletin.

These are some of the items of which we have record. But no record exists of the most important means of all for promoting the work of the Society - the word-of-mouth publicity given it by all of our members in casual conversation with friends and neighbors, with business associates, before small local groups.

As we enter a new year, and we trust a less troubled one, it needs emphasizing that the strength of this Society lies, not so much in what your officers may be able to do, but in the energy and the enthusiasm of our individual members.

Each One Teach One

F. HOWARD SEELY

We mourn the loss of another of the pioneers of duodecimals in the death of F. Howard Seely, Vice-President of the Duodecimal Society, and Chairman of the Committee on Member Qualification.

He was Treasurer of Oakland's First Congregational Church, and a member of the Sons of the American Revolution. A graduate of George Washington University, he was, three years ago, voted one of their three most outstanding alumni. At about that time, he and his wife, Mary Blachly Seely, celebrated their 50th Wedding Anniversary.

Besides Mrs. Seely, there survive two daughters, Mrs. Leonard D'Ooge, of Sacramento, and Mary Seely, of Piedmont, Calif., a grandson, John Seely, of Gilfilen, Calif., and a sister, Mrs. John A. Birge, of Shelton, Conn.

A dozen years ago, Mr. Seely became interested in duodecimals through reading Mr. Andrews' article, An Excursion in Numbers, in the Atlantic Monthly. The correspondence thus started led to the formation of the Duodecimal Society of America, and he has had a large part in the progress of the Society, and in the development of the duodecimal proposals. Many of the accepted practices of the Society originated with him. In recognition of his work, the highest honor at the disposition of the Society was given him, in the bestowal of the Annual Award for 1946.

He possessed a lively imagination and a pungent humor. In a recent contribution to the Bulletin, the sweet irony of Ernest Stryver (né Seely,) bears witness to his enjoyment of tangy comment. His wide range of talents included an impressive knowledge of musical theory, and a considerable literary ability. In addition to his several addresses on duodecimals, and a number of articles, he has written a duodecimal arithmetic, (still in the manuscript stage,) which is unique in the element of humor that he has introduced into its problems.

One of his outstanding accomplishments was the perfection of a formula for the conversion of decimal and duodecimal numbers, which is particularly applicable to machine operation.

We will miss him. But in his seventy-eight years, he has accomplished many works that have earned wide acknowledgement, and will serve to keep his memory green, in his church, in his community, and in this Society which owes so much to him.

OFFICIAL TRANSACTIONS

At a meeting of the Board of Directors on 18 January 1946, Mr. Terry, as Chairman of the Finance Committee, raised a point as to the use of the Endowment Fund. The Fund has been regarded as of the permanent type, with only its income applicable to the current expenses of the Society, except by special approval of the Board. Mr. Andrews reviewed the various types of funds as covered in a recent Russell Sage study. The merits and faults of the several types were thoroughly discussed as applying to our Society and its future needs. It was the consensus that our Fund should be regarded as of the liquidating type, and that the principal of the Fund should be absorbed into operating expenses over a reasonable term of years

The Annual Meeting of the Society was held in Room 200, Russell Sage Foundation Building, 130 East 22nd St., New York City, on 18 January 1946. The President's Report to that meeting appears elsewhere in this issue.

The Treasurer reported that the balance carried forward from 1944 was \$396.22, receipts for 1945 were \$588.52, and expenditures \$667.20. The year's deficit of \$78.68 reduced the balance to be carried forward into 1946 to \$317.54.

He also advised that the Office of Internal Revenue of the U. S. Treasury Department had ruled, under date of 22 October 1945, that the Society was not required to file income tax returns, that it was exempt from Capital stock tax, and from employment and social security taxes, and that contributions to the Society are deductible from the taxable income of the donor.

It was announced that the Annual Award of the Duodecimal Society for 1946 had been conferred upon Mr. F. Howard Seely, for his notable achievement in devising a formula permitting machine conversions of decimal and duodecimal quantities, for his excellent work in developing the lessons and examinations for the Society, and in recognition of his long and devoted service to the progress of duodecimals, and of the Society.

The Committee on Awards also cited the good work of Mr. Lewis Carl Seelbach in his active correspondence to persons of prominence in the advancement of the interests of the Society.

The Secretary's report emphasized that the most productive of our efforts has been the publication of books and magazine articles, and recommended that we endeavor further this type of activity among our members. The Secretary proposed as future objectives: (a) the development of advanced courses in duodecimals, covering weights and measures, trigonometry, navigation, and astronomy, (b) the production for general use and

sale, of rules, slide-rules, micrometers, and other duodecimal tools and instruments, (c) the promotion of industrial applications of duodecimals, and (d) the stimulation of international correspondence and of duodecimal activities in other countries.

In accordance with the recommendations of the Nominating Committee, Mr. F. Morton Smith, whose term expires this year, was re-elected to the Board of Directors for another three-year term and the following members were elected as the new Nominating Committee:-

Wendell B. Campbell, Chairman
Herbert K. Humphrey,
Kingsland Camp

President Andrews has announced the following appointments, and changes in committee personnel.

Interim additions to the Board of Directors	William S. Crosby Paul E. Friedemann
Committee on Awards	George S. Terry, Chairman Lewis Carl Seelbach Eugene M. Scifres
Mathematical Recreations	Mary Lloyd, Chairman
Public Information	Lewis Carl Seelbach, Chairman Paul E. Friedemann F. Emerson Andrews

The new Committee on Public Information has been assigned responsibility for the preparation of special articles and similar material, designed for publication in newspapers and magazines. The articles that have appeared have proven that this field of effort is particularly productive in elicited response. Mr. Andrews has named himself as a member of this committee in his personal role as a writer.

DECIMAL POINT OR DUODECIMAL POINT

We continually stumble over the inappropriate term, "decimal point." To us, this usage is improper, since the point is the point whether used with the decimal base, the duodecimal base, or any other.

From time to time, the comment is made that the point is not properly placed between the units column and the tenths column, (in decimal notation,) but should be placed under the units col-

um figure, as herewith. (478 26) For several reasons, this practice fails to find much favor, but the suggestion does emphasize the improper terminology involved.

Writers of papers on duodecimals frequently use the phrase, "duodecimal point," which, though serving to avoid confusion, is quite as faulty as "decimal point." But W. K. Humphrey, in his paper in this issue, on "Machine Conversion of Decimals and Duodecimals," has found, and has used, the ideally appropriate terminology. He names it, "The Unit Point."

The more one considers this usage, the more fitting it seems. We congratulate Mr. Humphrey on its peculiar aptitude, and we would suggest that it become generally adopted.

ADDITIONS TO DUODECIMAL BIBLIOGRAPHY

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Quackenbos, G. P.

A Higher Arithmetic
Appleton and Co., New York, 1874

Each One Teach One

DUODECIMAL LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	X	£
1	0000	0478	08£2	10£2	1481	1823	1£60	2277	2573	2852	2£16	3185
2	3420	3665	3898	3X££	4113	4317	4512	4700	48X1	4X76	5043	5205
3	5380	5530	5697	5838	5993	5£25	6072	61£6	6336	6472	65X5	6715
4	6840	6964	6X85	6£X2	70£8	720£	731£	7428	7533	7636	7738	7836
5	7932	7X28	7£20	8012	8101	81XX	8296	837£	8463	8545	8625	8703
6	87X0	8877	8950	8X24	8X£7	8£88	9058	9126	91£3	927£	9345	940X
7	9492	9555	9617	9697	9756	9815	9892	994X	9X05	9X80	9£35	9£X9
8	X060	X113	X185	X235	X2X5	X354	X402	X470	X518	X584	X62£	X696
9	X73£	X7X4	X848	X8£0	X953	X9£5	XX57	XX£7	X£58	X££7	£056	£0£5
X	£153	£1£0	£249	£2X5	£340	£397	£432	£488	£521	£576	£60£	£662
£	£6£6	£749	£79£	£831	£883	£914	£965	£9£5	£X45	£X94	££23	££72

MACHINE CONVERSION OF DECIMALS AND DUODECIMALS

by R. K. Humphrey

In working with duodecimals, one is frequently confronted with the necessity of translating decimal figures to duodecimal, or the reverse. In these operations many errors occur. It will be possible, thanks to Mr. Seely's formula, to avoid these errors through the use of special conversion machines. At present, however, there is apparently only one duodecimal calculating machine in the world, the one built by Mr. Terry for production of his monumental book of tables, DUODECIMAL ARITHMETIC (Longmans Green).

With one exception, for every conversion there is a choice of methods, one involving decimal arithmetic, the other duodecimal. It is only necessary that the rest of us use the decimal method in each case, since that is the method which our regular decimal machines will handle. There is one conversion, however, dozenal fraction to decimal fraction, for which the only instructions usually available require multiplication by X. A method involving division by twelve is presented. Meantime, for machine work, slight variations of the usual methods are more convenient than straightforward work, and since these may be of interest, they are presented in order.

INTEGERS, Decimal to Dozenal.

The method to be chosen for machine work is that involving repeated division by twelve. For this purpose, a method used by Comptometer operators has special convenience, and since it is not universally known, will be given in some detail. It uses repeated addition of the complement of the divisor, and is based

upon a method given by Gerbert about a thousand years ago, probably the oldest method of which we have written record, and at the same time, the newest to be taught.

The dividend is entered in the machine. First are considered the smallest number of places at the left of the dividend into which the divisor will go, and to these the complement of the divisor is added repeatedly. At the same time, the number appearing in the dial just to the left of the divisor (complement) is watched, and the number of times the complement has been added are counted. When these are equal, attention is turned to the dials directly above the complement, and additional operations are performed until the number in these dials will no longer contain the divisor. Then the whole operation is moved one column to the right, and so on until all columns of the dividend have been treated. At the end of this entire operation, the two right hand columns give the remainder, and the rest of the columns give the first quotient.

The advantage of this method now appears, for the first quotient becomes the second dividend in the conversion, and it is already entered in the machine, ready for the second division. The conversion $23,325 = 11,529$ is shown in detail on Tape No. 1.

The fastest machine for this computation is a non-listing machine of the Comptometer type, but in order to show the dial readings a listing machine has been used for this example (and for all the examples). It is almost essential that the machine have dials, else a great number of sub-totals must be taken; with dials, it is not necessary to take sub-totals at all, but they have been included to show the process.

Tape No. 1.	23325
	88000
	111325s
	(1) 8800
2>1	→
	120125s
	(2) 8800
2=2	→*
	128925s
But 12 can be taken from 89	8800
	8800
	8800
	8800
	8800
	8800
	880
	880
	880
	880
	88
	88
1st quotient and	88
2nd dividend	→
	194309s
1st remainder	→
	880000
	88000
	88000
	88000
	88000
	88000
	88000
	88000
2nd quotient	→
	1611109s
2nd remainder	→
	8800000
	880000
	880000
	880000
3rd quotient	→
	13031109s
Limit of machine reached; move dividend to right side of machine	
	13
	88
23,325 = 11,529	(10)*

INTEGERS, Dozenal to Decimal.

For this conversion, the method of repeated multiplication by twelve must be used, but it is simpler to move the product one column to the right, as if the multiplier had been 1.2. As usually given, each digit is multiplied by twelve in its turn; the 10 part of the multiplier moves this digit one column to the left; then the next multiplication moves it another column to the left. Consequently, each digit must be introduced in its proper turn. But in multiplying by 1.2, the 1. part of the multiplication does not move the digit; therefore, since each digit is going to be multiplied by 1. sometime, we can accomplish this all at once by merely entering the number to be converted into the machine. Special care must be taken if any of the digits are X or Z, but that will come later; for the present it is assumed that the duodecimal number happens to contain no X or Z.

Having entered the number, and so taken care of all multiplications by 1., it is now necessary to take care of those by .2; for the first (left hand) digit, this is done by adding that digit twice in the second column; at this point, a sub-total must be taken, or the dials must be consulted, and the first two digits of the sub-total are added in the second and third columns; again the resulting sub-total is examined and its first three digits are added twice in the second, third, and fourth columns. This process is repeated until finally the next to last digit has been added in the last (right hand) column, whereupon the number in the machine is the answer sought.

If the number to be converted contains X or Z, these must be represented in the decimal machine by 10 and 11. This introduces an extra 1 in the immediately preceding column, which must be accounted for during the process; that is, when it is the turn of the next preceding number to be added in, a number smaller by 1 must be used, after which the process goes as before. It is safer, however, to enter at first not the entire number, but only those digits which precede (lie to the left of) the

Tape No. 2	11500
11,529	119
	11619s
	1000
	1000
	13619s
	1300
	1300
	16219s
1 less, on account of using 11 for Z	1610
	1610
	19439s
	1943
	1943
11,529 =	23325*
	11500
	1000
	1000
	13500s
	1300
	1300
	16100s
	1610
	1610
	119
	19439s
	1943
	1943
11,529 =	23325 *

Finally, the first step in the conversion, multiplication by 12, just undoes the last division by 12, so both may be omitted. So, divide the mixed number by 12^{w-1} ; of this result, the integral part is the first digit in the conversion; succeeding digits are found one at a time by multiplication by 12, as on Tape No. 3. When all digits have been found, the resulting duodecimal fraction must be changed to the equivalent mixed number by moving the unit point w columns to the right (multiplying by 12^w). The conversion $35.87 = 2\mathcal{E}.X5$ by this method appears on Tape No. 5.

Tape No. 5.

35870		29892s	
88000		99980000	
88000	These two	9892	
8800	steps may	98920	
8800	be omitted		
8800		118704s	
8800	298902s	99890000	
8800	880000	8704	
8800	880000	87040	
8800	88000		
8800	88000	104448s	
8800	88000	99900000	
880	88000	4448	
880	8800	44480	
880	8800		
880	8800	53376s	
880	8800	99950000	
880	8800	3376	
880	8800	33760	
880	8800	40512*	
88	8800		
88	8800		
88		$35.87 + 12^2 = .2491$	
88	2490102*	.2491 = .2\mathcal{E}.X54	
88	2491	.2491 x $12^2 = 2\mathcal{E}.X54$	
88	2491		
88	24910		
88			
298902s	29892s		
		Thus $35.87 = 2\mathcal{E}.X54$	

MIXED NUMBERS, Dozenal to Decimal.

The mixed number is changed to a duodecimal whole number by simply pointing off, which is equivalent to multiplying by 12^1 . Then this whole number is converted to decimal form by repeated multiplication by 12, as outlined above. The resulting decimal whole number must finally be divided by 12^1 . The conversion $2\mathcal{E}.X5 = 35.87$ is shown on Tape No. 6.

Tape No. 6.

	5165000
	8560000
	8560000
$2\mathcal{E}.X5 \times 12^2 = 2\mathcal{E}.X5$	8560000
	856000
	856000
	856000
	856000
	856000
	856000
	85600
2000	85600
200	856000
200	856000
1100	856000
	85600
3500s	85600
350	85600
350	85600
105	85600
	85600
4305s	85600
430	85600
430	85600
	8560
5165s	8560
	8560
	8560
$2\mathcal{E}.X5 = 5165$	8560
	8560
Divide by 144	856
	856
$5165 + 12^2 = 35.867$	856
	856
and $2\mathcal{E}.X5 = 35.867$	856
	856
	856
	35867152*

REVISED ADDRESS LIST

In view of the many changes that have occurred in the addresses of our members, an up to date list is presented to facilitate the exchange of correspondence among us.

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MATHEMATICAL RECREATIONS

The Problem of the Four Fours

In a recent letter, Mr. George Terry inquired:- "Is the Four Fours Problem suitable for the Bulletin? Using the four arithmetical processes and duodecimals, with repetition permitted, we can express the consecutive numbers from 0 to 26, - the last two of which may be expressed as:

$$\frac{4}{.4(.4)} - 4 = 25$$

$$\frac{4 - .4}{.4(.4)} = 26$$

"Perhaps a simpler problem is to use three fours and allow the use of the square root sign. Under these conditions, I was able to reach 14, and failed to find a solution for 15."

Then, in the first issue (February) of the new monthly magazine, "Holiday," issued by the Curtis Publishing Co., an item appeared under the caption of "The Numbers Racket," describing a different handling of the same problem. This article proposed starting with any number, using it four times with mathematical symbols, to get an expression that would equal zero. Then the same digit is used again four times, in different arrangement, to get an expression that would equal one. And so forth, running the series, without gaps, as far as possible.

Thereafter, another series is started, using the next consecutive digit four times, with mathematical symbols, to derive an equation that would equal zero. Then, the next one to equal one, and so on, running this series as far as possible.

The claim was made that, using each digit except zero and one, it is possible to run each series to, at least, sixteen. And that for the Four Fours series, it was possible to develop an equation for every number up to 200.

In looking up this problem in "Mathematical Recreations and Essays," by W. W. Rouse Ball, we found this comment:- "Another traditional recreation is, with the ordinary arithmetic and algebraic notation, to express the consecutive numbers from 1 upwards as far as possible in terms of four '4's." Everything depends on what we mean by ordinary notation.

"If (a) this is taken to admit only the use of the denary scale, decimals, brackets, and the symbols for addition, subtraction, multiplication, and division, we can thus express every number up to 22 inclusive.

"If (b) also we grant the use of the symbol for square root, we can get to 30; but note that though by its use a number like 2 can be expressed by one "4," we cannot for that reason say that .2 is so expressible.

"If further (c) we permit the use of symbols for factorials, we can express every number to 112."

Mr. Ball goes on to cover the possibilities using integral indices and sub-factorials, and concludes:- "Thus, under condition (c,) with four '1's" we can get to 34, with four "2's" to 36, with four "3's" to 46, with four "5's" to 36, with four "6's" to 30, with four "7's" to 25, with four "8's" to 36, and with four "9's" to 66."

Of course, using the duodecimal scale changes the problem considerably, but the possibilities should not be greatly different. Note that, for duodecimals, Mr. Terry reached 26 without using the square root sign. Using square roots and factorials, Mr. Andrews and Mary Lloyd have run the series up to 67, and are still going. It would be interesting to know whether the figure of 94 for condition (c) can be equalled, or exceeded.

TABLE OF QUARTER SQUARES Harry C. Robert, Jr.

	0	1	2	3	4	5	6	7	8	9	X	£
0	0.0	0.3	1.0	2.3	4.0	6.3	9.0	10.3	14.0	18.3	21.0	26.3
1	30.0	36.3	41.0	48.3	54.0	60.3	69.0	76.3	84.0	92.3	X1.0	£0.3
2	100.0	110.3	121.0	132.3	144.0	156.3	169.0	180.3	194.0	IX8.3	201.0	216.3
3	230.0	246.3	261.0	278.3	294.0	2£0.3	309.0	326.3	344.0	362.3	381.0	3X0.3
4	400.0	420.3	441.0	462.3	484.0	4X6.3	509.0	530.3	554.0	578.3	5X1.0	606.3
5	630.0	656.3	681.0	6X8.3	714.0	740.3	769.0	796.3	804.0	832.3	861.0	890.3
6	900.0	930.3	961.0	992.3	X04.0	X36.3	X69.0	XX0.3	£14.0	£48.3	£81.0	££6.3
7	1030.0	1066.3	10X1.0	1118.3	1154.0	1190.3	1209.0	1246.3	1284.0	1302.3	1341.0	1380.3
8	1400.0	1440.3	1481.0	1502.3	1544.0	1596.3	1609.0	1650.3	1694.0	1718.3	1761.0	17X6.3
9	1830.0	1876.3	1901.0	1948.3	1994.0	IX20.3	IX69.0	IX£6.3	1£44.0	1£92.3	2021.0	2070.3
X	2100.0	2150.3	21X1.0	2232.3	2284.0	2316.3	2369.0	2400.3	2454.0	24X8.3	2541.0	2596.3
£	2630.0	2686.3	2721.0	2778.3	2814.0	2870.3	2909.0	2966.3	2X04.0	2X62.3	2£01.0	2£60.3

To multiply 75 by 3£: 75 + 3£ = £4
 75 - 3£ = 36

Tab. Value of £4 = 2814.0
 Tab. Value of 36 = 309.0

Subtracting 2507.0 = 75 x 3£

(x - y)² = 4xy

NUMBER SYMBOLS

by Kingsland Camp

Symbols for duodecimal numbers fall, for present purposes, into two classes, transitional and ideal. For the former we are adhering to the principle of least change. The latter are subject to discussion and revision until consensus of opinion in our Society settles upon a consistent and reasoned-out scheme as affording the best all-around facility for practical use. The deliberate purpose of this note is to provoke discussion and suggestion.

Readers of the Bulletin need no explanation of our present transitional number symbols, used throughout our Society's literature. Of course it is entirely optional with individual members and non-members to use, privately, any symbols they find convenient. In particular, Mr. Humphrey's suggestion of always indicating the duodecimal point, and doing so by means of a semicolon, is a very neat one, especially when typing duodecimal numbers.

The desirability of radically new permanent notation arises from the clumsiness and just-grown-like-Topsy nature of our present number symbols. No age realizes the stupidity of its accepted usages, and few people outside of primary-school children and their teachers notice the clumsiness of the present two-stroke written symbols for four and five. The children, as we all once did, learn well enough how to write them but completely forget their awkwardness.

Furthermore, a system of symbols should strongly aid, by suggestion, the elementary processes of addition and subtraction. For example, the very form of the duodecimal symbol for nine should suggest its complementary relationship to three; similarly with all complement pairs. Then the majority of us who are not naturally quick at arithmetic would more easily think of and use the alternate device, when adding, of increasing by do and subtracting the complement of the digit being added, when that double operation is easier than straight addition.

Any symbol should be easy to write. Since it will not be possible to find symbols on our typewriters, or among the variety of type fonts at our printers, which have the characteristics that we seek, new symbols will have to be invented. Those presented here are submitted to provoke discussion, and to stimulate alternate proposals.

The following symbols can all be easily written with one flowing motion of the pen or pencil, and if assigned to the proper digits, possess the complementary relationship just described.

The figure of a pointing hour-hand seems particularly appropriate for the cardinal points of a clock or compass dial, and the upward pointing symbol may be appropriate for an alternate monogram for do, thus:-

<u>3</u>	<u>6</u>	<u>9</u>	<u>10</u>
⊕	⊙	⊖	⊕

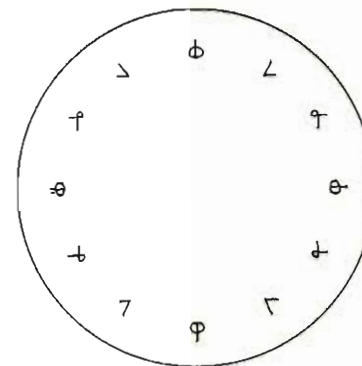
The symbol for seven might be retained with its present significance. Since it is just as suitable as the foregoing for version and inversion, we may assign the following four symbols at once:-

<u>1</u>	<u>5</u>	<u>7</u>	<u>8</u>
∟	∟	7	∟

There are at least two suitable sets of easily-written symbols for the remaining four digits. The first is very unfortunate, since the decimal system has already appropriated two forms of it, 6 and 9. The other, if carelessly written, could be confused with the 1-5-7-8 set of digits. Both are shown for comparison:-

<u>2</u>	<u>4</u>	<u>8</u>	<u>11</u>
6	9	9	8
†	‡	‡	†

The clock-face diagram below shows the essential symmetry and relationship of these symbols.



IDEAS AND OPINIONS

It has been our practice to publish all comments and suggestions, which were too limited in scope to constitute an article, in The Mail Bag. It has become apparent that much of this material could be profitably segregated in a section devoted to the announcement of new proposals, and comment and criticism of our current practices. Thus, this new department is introduced.

For instance, George S. Terry comments, "In the discussion of symbols, it might be well to consider the Reverse Notation. It is not necessary to have ten symbols to count by tens. We can count to 5 and then back from ten, 6 being regarded as ten less 4, perhaps written $\bar{4}$; 7 written $\bar{3}$; and so on. We thus have six symbols only, and count:

1 2 3 4 5 $\bar{4}$ $\bar{3}$ $\bar{2}$ $\bar{1}$ 10

"Duodecimally we need only seven symbols, and count by dozens, thus:

1 2 3 4 5 6 $\bar{5}$ $\bar{4}$ $\bar{3}$ $\bar{2}$ $\bar{1}$ 10

"There are obvious advantages in this system for addition and subtraction, since 'carries' are largely eliminated. But division and multiplication, I find difficult. However, without using the negative digits as such, we have an alternative suggestion for getting enough symbols for counting by dozens without inventing new ones, and getting them on the typewriter by putting five of them upside down. We could then count as follows:

1 2 3 4 5 6 9 $\bar{8}$ $\bar{7}$ $\bar{6}$ $\bar{5}$ 10

"This shows up the useful complementary numbers, and would lend itself to the Reverse Notation, if desired. I would suggest that those interested refer to Johnson's 'Reverse Notation,' where there are names suggested for the symbols."

In line with our material on the Principle of Least Change, and the Principle of Separate Identity, Jamison Handy agrees that "it seems best, for the time being, to carry on two separate lines of thought - final unity may have to be worked out much later. Accordingly, I comment on nomenclature from both the conservative and the radical points of view.

"The conservative line of thought is best for getting ideas across to the newcomer. I feel that the policy should be to avoid artificial words as much as possible . . . use words in the dictionary that can be stretched to cover; e.g., read 16 as "a dozen and six", .1 as "one twelfth", .01 as "one per gross", in such a manner that a stranger overhearing by chance may think it odd or unusual, but, inasmuch as it is perfectly

good english, can figure out what you mean without further initiation if he wants to. A beginner who has reached this stage fluently will naturally find it easy to shorten terms to the prefixes and suffixes of Mr. Terry, or might even think of better cuts along the lines of evolution of slang that becomes acceptable.

"Along the radical line of thought, I am not 'sold' yet on the necessity of new symbols throughout. But I certainly like the approach to naming and reading the numbers suggested by Kingsland Camp, for its freshness, and its possibility for compact, clear, and universal expression. Commenting only on details, I believe that e as in they is considered by some to be a diphthong as generally spoken in English. e = a in senate is a better example. Fortunately, mispronouncing the vowel sound for 3 as e (in debt) plus ee (in seen) still indicates the same quantity; i.e., three from six is still three, so that this bad chance for ambiguity is cancelled out by the rules. On the other hand, a diphthongized pronunciation of 2 (o as in go), likely by most English-speaking people, is apt to be mistaken for X (oh-oo).

"In regard to the consonant equivalents, difficulties are likely to arise, especially in telephoning or whispering, from the use of both voiced and unvoiced versions of related consonants to represent different numbers; e.g., v and f for 5 and 7, g and k for 8 and X. If you don't see what I mean, try to distinguish very from fairy, or goal from coal, when whispered or telephoned without context.

"Mr. Crosby's remarks on X and 2, lead me to comment that Mr. Norland has independently selected the same symbols that I've been using for fifteen years, 3 upside down, or 2 for ten, and 7 upside down, or L for eleven, though my arbitrary reasons were different from his. Naturally, if there are only two of us in the Society who have been using the character elf for ten, while everybody else has become accustomed to elf for eleven, it is up to the two of us to change, rather than be 'Johnny, the only one in the parade who is in step.' Since becoming somewhat familiar with the Society's usages, I've been using t for ten, and a for eleven, which serve the purpose well. Mr. Crosby's remarks attracted my attention, because I agree with him about the symbol for eleven, but would like to have something else for ten, and noted that his symbol, which looks like a 2 upside down, isn't much different from my initial t, as I generally write it.

"Supplementing my remarks, carried on page 23 of Bulletin 2, I wish to add:- For longitude, I was planning to measure in hours from Greenwich, to coincide with the 24 hour day for clocks and watches. However, I hadn't yet fully developed this point, and realized it was inconsistent with dividing, or denoting, latitude by degree measure (i.e., fraction of a full turn, or revolution.) But, for the time being, I dismissed it, thinking that I might have later changed my mind about angular measure, ex-

pressing angles as a duodecimal fraction of a half-turn, or half revolution, in which case the relation to radian measure would be obvious and easy; e.g., a right angle would be written $.6\pi$ (radians.) I wish to bring this additional remark to the attention of other members who might find it attractive to develop into a consistent proposal. If it should prove the most convenient and best angular, time, and earth measure, then maybe we won't have to buy new watches and clocks after all, but need only remove, or disregard, the second hand, and the divisions between the five minute periods."

Harry Robert comments on names and symbols, "Σ for 'el' is fine for hand written work. Written with two short curving strokes and one change in direction, without lifting the writing instrument from the paper, it is on a par with 2, 3, 7, and 9, as usually written, and is better than 4, or 5, regardless of how they are written. The distinctive straight stroke at the bottom would be a very legitimate formalization of the figure in type, or for decorative work.

"I cannot say the same for the use of X for dek. Aside from no apparent connection with the name "dek", the use of X appears to constitute an unnecessary recognition and perpetuation of the Roman number system, the clumsiness of which does not justify such recognition. Also, the use of X with Y and Z as designations of the axes of three dimensional space, and the resulting use of lower case x as the symbol for a variable quantity, will certainly result in some confusion.

"Further, its production as a hand-written symbol has the objection that it requires two separate and distinct strokes, with the writing instrument being removed from the paper between strokes. Granting that the two strokes required are the simplest available, the X is still a more unsatisfactory number symbol than any of those used in the ten system except 4 and 5.

"Taking into account all of the desirable qualities of a symbol, it appears to me that δ, the lower case Greek delta, is worthy of very careful consideration. Not only is the relationship to "dek" obvious, but the origin of the symbol would constitute a justifiable measure of homage to the great Archimedes, peer of our pioneers. Neither does δ have any widespread and deep-rooted use elsewhere in the field of mathematics to compare with the use of X.

"For hand-written work it would be hard to improve on δ, since it is made by a single flowing stroke without angular changes in direction. It can be started from either end, starting in either case with a downward curved counter-clockwise stroke, one of the most common and least awkward of penmanship and lettering strokes. This feature puts δ on a par with 6, 8, and 0, and it is bettered only by 1.

"It would also appear that δ meets the requirement of distinctiveness. I grant that some people make queer looking δ's and others don't finish their δ's and such figures might occasionally be confused with 3, but then a lot of people can't even read their own writing. The fact that our type designers have neglected δ in the past, does not mean that it can't be delightfully formalized for decorative work. So far as our typewriters go, why not use lower case d and e for "dek" and "el" respectively, until we can get an extra pair of keys on our machines."

DOSENILE DEPARTMENT

Mary Lloyd, Editor

We were really excited over getting a response to the arithmocryptograms from Mr. Terry. Anyone who has seen a copy of his "Duodecimal Arithmetic," will get a giggle from the notion of a housewife making puzzles too hard for its author. He writes:-

"In Bulletin No. 2, p. 2, it asks, 'When $ab=c$, what are the possibilities?' As stated, there are infinite possibilities. But if a and b are integers, and c is the unit-place integer of the product, then the number of solutions can be seen from the multiplication table. Is this what is intended? (Yes, Ed.) There are 26 solutions in that case, and these illustrate well the tendency of duodecimal products to round off at zero.

"I seem to be very bad at solving the cryptograms. Are you making them too hard, or am I just dumb? The making of them seems to present no difficulty aside from finding twelve-letter words without duplicate letters. Here are two that may be too easy, as the first gives away O, and the second shows O, and I.

Sincerely, George S. Terry."

E P A		P N G		D R A
		C N L T H		E X R T U S
		I E L		D O M D
		A U T		O T I U
		E P A		O X B R
		P I N E		M M B S
		P M L U		
		P T H		

The solution of arithmocrypts depends upon identifications and eliminations. There are many methods of attack, but all methods will include the following steps:

Set up a grid with the numerals across the top, and the letters arranged vertically. (Be sure there are 10 letters.)

Check for the letter that represents 0.

Check for the letters that cannot be 0, and block out those squares representing impossibilities.

Check for 1.

Check for the letters that cannot be 1.

Check for 2.

Check for sequences; for example:- when M is subtracted from W and leaves nothing, then this identifies a sequence, MW, because there must have been 1 to carry. This also establishes which of the preceding figures is the larger, and any such indications should be noted.

Check terminal figures for product possibilities.

1. When a times a ends in a :-

a = 0, 1, 4, or 9.

2. When a times a ends in b :-

b = 0, 1, 4, or 9.

a ≠ 0, 1, 4, or 9.

3. When a times b ends in a :-

a ≠ 1, and b ≠ 0.

If a ≠ 0, then b ≠ 2 or 6.

If b ≠ 1, then a ≠ 5, 7, or 2.

4. When a times b ends in c :-

a, b, and c ≠ 1.

a and b ≠ 0.

If c = 0, a and b ≠ 5, 7, or 2.

If c ≠ 0, a and b ≠ 6.

Check initial figures of divisor and product; when product has as many figures as divisor, there are limitations on the size of the multiplier and of the initial figure of the divisor; when product has more figures, the reverse is true.

Check subtractions for limitations of size.

Mr. Terry had used the same do-letter key word for both of his puzzles. So we transposed to a new word, with thanks to Miss Dorothy Littleford of Mare Island, California, for finding it.

The answer to the third puzzle is my hero, a slugging youngster with big ideas. He took on the answer to Puzzle No. 4 as his opponent, and, though greatly outnumbered, the correctness of his thinking, and the rightness of his principles gave him what it takes to fight and win. 'Ray for our side!

TND		LENE		GEM		RIAR
		TEEBEA				MUPDRU
		TOZZ				MRUR
		TBDE				CDMR
		UACH				EOMD
		TTHA				IDRU
		TUBN				CPLR
		UHU				ECC

This is a lot of fun, and as you work with these puzzles, you will develop many tricks of your own that will facilitate their solution. You should be able to crack any crypt in an hour. At least, that makes a good bogey. Some of them will fall right apart. The important thing is that they keep your duodecimals limber.

BASIC TWELVE, THE RULE FOR MAN

An Acrostic, based on Revelations 21, 10-17.

'Tis strange that when The Planner drew
His pattern for the city new,
Exactly measured by the rule,
Not taught us in our childhood school,
Exchanged the use of decimal
With Basic Duodecimal.

Jerusalem anew will come,
Emerging with the Basic Sum,
Reminding of Apostles old
Unfolding Israel's twelve-tribe fold.

So, gates and angels twelve we count,
And walls to gross of cubits mount.
Length, breadth, and height, - amazing man,
Exact twelve thousand furlongs span,
Make Basic Twelve the rule of man.

(Written by Philip Haendiges, the major minor, mass-production poet, without intention to suggest the Church of the New Jerusalem either by DEception, ELision, or innuendo.)

We love poetry, yet poets always confound us. They are a strange and wonderful people. They thrill us with the chromesthesia of the rich organ overtones of the meaning of life; - they play with words, and in their playing make profound sense.

We, too, deeply feel the loss of Mr. Seely. When the dislocations of the war assigned us to a nearby base, he and his family were a touch of home to us. We will never forget his kindness.

Responding to our importunings, he typed this poem as a contribution to our page.

Mary Lloyd.

BABBLE ON

In ancient Babylon were men
Who would not tie themselves to ten.
They had ten fingers like the rest,
But still they knew that twelve was best.

They knew a cube must have six faces,
And so they used that for a basis,
And if a single cube was nice,
With two, they made a pair-a-dice.

But though intelligence was mounting,
Their neighbors still used finger-counting,
And since with them they had to trade,
Another base must needs be made.

Least common multiple they found
Of ten and twelve, and that was sound,
Though I am sure I'd hate to face
The use of sixty as a base.

Still they succeeded vary well,
As we in modern times can tell.
They fixed twelve hours to mark the light,
Another twelve to pass the night.

Minutes and seconds we today
Use sixty, just the same as they.
And six times sixty for degrees;
We've always been content with these.

And so you see how much we owe
To those old guys who stole the show
So many centuries ago.
If we had sense as much as they
We would be using twelve today.

P. Howard Seely.

Diameter of the Earth	.	.	.	=12.8	M	dometric miles
Distance to the Sun	.	.	.	=27.1	M ²	dometric miles
Distance to the nearest star	.	.	.	= 2.88	M ⁴	dometric miles
Distance to farthest ^o photographed galaxy				= 7.4	M ⁶	dometric miles
Diameter of the Milky Way	.	.	.	=31.2	M ⁵	dometric miles

UNSCIENTIFIC SCIENCE

By Ralph H. Beard

The Scientific Monthly for November, 1945, carried an article on "Our Everyday Reckonings," by Oystein Ore, Professor of Mathematics at Yale University. Certain elements in that argument, we feel, cannot be permitted to stand without comment.

The article is strongly pro-metric. It reviews the advantages of the French Metric System, the disadvantages of any change in our number system, and prophesies the adoption of the metric system as the international standard.

But Professor Ore fails to escape two of the habitual faults of the protagonists of the metric system. He emphasizes the highly scientific basis of the metric system, while minimizing the fact that it is based on two fundamental inaccuracies. The length of the meter is purportedly established on the length of the meridian of Paris, but it is now acknowledged that errors were made in the measurement of this meridian. It is also acknowledged that the liter is not exactly the volume of one cubic decimeter.

The other fault has become equally general among metric protagonists. They lift one sentence from the scholarly and unbiased report of John Quincy Adams, as paying "glowing tribute to the metric system," and omit to state that the report contains much forceful argument against the metric system, and advised against its adoption.

These are not innocent errors. Repeatedly, these mistakes have been criticized in the public press, yet they are constantly reiterated. In the present case, either Professor Ore has not read the masterly John Quincy Adams report, or his use of the citation from that report as a "glowing tribute to the metric system" is questionable.

For instance, Mr. Adams says:- "The decimal numbers applied to the French weights and measures, form one of its highest theoretical excellences. It has, however, been proved by the most decisive experience in France, that they are not adequate to the wants of man in society; and for all the purposes of retail trade, they have been formally abandoned. The convenience of decimal arithmetic is, in its nature, merely a convenience of calculation; it belongs essentially to the keeping of accounts, but it is merely an incident to the transactions of trade. It is applied, therefore, with unquestionable advantage to moneys of account, as we have done; yet even in our application of it to the coins, we have not only found it inadequate, but in some respects inconvenient.

" . . . A glance of the eye is sufficient to divide material substances into successive halves, fourths, eighths, and sixteenths. A slight attention will give thirds, sixths, and twelfths. But divisions of fifth and tenth parts are among the most difficult to be performed without the aid of calculation. Among all its conveniences, the decimal division has the great disadvantage of being itself divisible only by the numbers two and five.

"Thus, then, it has been proved that the principle of decimal divisions can be applied only with many qualifications to any general system of metrology; and that time, space, gravity and extension inflexibly reject its sway. The new metrology of France, after trying it in its most universal theoretical application, has been compelled to renounce it for all the measures of astronomy, navigation, time, the circle, and the sphere; to modify it even for superficial and linear measure, and to compound with vulgar fractions in the most ordinary and daily uses of all its weights and measures.

"Yet a system of weights and measures, which excludes all geography, astronomy, and navigation from its consideration, must be essentially defective in the principle of uniformity."

If Professor Ore had said that Mr. Adams paid a glowing tribute to the idea of integrating the weights and measures with the numbering system, his statement would have been immaculate. And to this idea we, too, wish to pay tribute. This is the scientific principle that the partial application of the French metric system has proven to be essential.

This principle is requisite for a uniform metrology. Numeration is but one form of mensuration, and a system must be applicable to all measurement. The decimal subdivision of time, the circle, and the sphere was a dismal failure, but there is a naturalness about their subdivision by twelve. Twelve makes an excellent base for our numbers and our arithmetic, and offers extensive advantages over ten for this purpose. Our accustomed weights and measures form the elements of an ideal metric system based on the duodecimal subdivision of the yard.

There is no quick and easy way out of this dilemma. To change the numbering practice of any great part of the peoples of the world will take many years. Yet the change from counting by tens to counting by twelves is necessary if we are to attain a uniform system of metrology for the world. The scientific mind would suggest that no great change is possible at this time, but that all possible measures to facilitate, and to expedite, the general knowledge and use of duodecimals would be desirable.

Each One Teach One

Our common number system is decimal - based on ten. The dozen system uses twelve as the base. This requires two additional symbols: *Z*, called *dek*, is used for ten, and *2*, called *2l*, is used for eleven. Twelve is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

Modern numeration employs one of the greatest of man's inventions, the zero - symbol for nothing. It permits the use of place values. In our customary counting, the places in our numbers represent successive powers of ten; that is, in *365*, the *5* applies to units, the *6* applies to tens, and the *3* applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as *365*.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

<u>84</u>	<u>136</u>	Five ft. nine in.	5.9'
<u>31</u>	<u>694</u>	Three ft. two in.	3.2'
<u>96</u>	<u>322</u>	Two ft. eight in.	2.8'
<u>182</u>	<u>1000</u>	Eleven ft. seven in.	11.7'

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, *7* times *9* is *63*, which is *5* dozen and *3*; so set down *53*. Using this "which is" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are *35* years old, dozenally you are only *22*, which is two dozen and eleven. For larger numbers, keep dividing by *12*, and the successive remainders are the desired dozenal number.

$$\begin{array}{r}
 12 \overline{) 365} \\
 \underline{12 30} + 5 \\
 \underline{) 2} + 6 \\
 + 2
 \end{array}$$

Answer: 265

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it *12* times the second figure, plus *12²* (or *144*) times the third figure, plus *12³* (or *1728*) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *Z*, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by *12* or *Z*.

Numerical Progression				Multiplication Table											
1	One			1	2	3	4	5	6	7	8	9	Z	2	
10	Do	.1	Edo	2	4	6	8	Z	10	12	14	16	18	1Z	
100	Gro	.01	Egro	3	6	9	10	13	16	19	20	23	26	29	
1,000	Mo	.001	Emo	4	8	10	14	18	20	24	28	30	34	38	
10,000	Do-mo	.000,1	Edo-mo	5	Z	13	18	21	26	2Z	34	39	42	47	
100,000	Gro-mo	.000,01	Egro-mo	6	1C	16	20	26	30	36	40	46	50	56	
1,000,000	Bi-mo	.000,001	Ebi-mo	7	12	18	24	2Z	38	41	48	53	5Z	63	
10,000,000	Tri-mo	and so on.		8	14	20	28	34	40	48	54	60	68	74	
				9	16	23	30	38	46	53	60	69	76	83	
				Z	18	26	34	42	50	5Z	68	76	84	92	
				2	1Z	29	36	47	56	65	74	83	92	Z1	