The Dozenal Society of America

Twelve versus Ten[1]

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> To the solid ground
> Of Nature trusts the mind that builds for aye.
>
> Wordsworth

In any developed system of numeration there must be a turning-point, at which the notation is folded back upon itself, and from which point on the higher numbers are to be expressed by additive and multiplicative processes through the lower. The number that forms thus at once a point of rest and a point of departure may be called the *base* or *radix* of the system. The rising powers of this base are set in order leftward, and the falling powers rightward, of the initial or unit's position. Any natural number other than unity may be taken as this radix, and, in fact, various integers, as two, five, ten, twelve, twenty, sixty, have been taken. Among all of these, however, *ten* has attained by far the widest and most complete recognition, and within the present century the metric decimal system has established itself firmly in western and central Continental Europe, as well as among men of science everywhere. Nor can there be any doubt of its immense superiority as a labor-saving device over every rival, so long as ten is made the turning-point in our notation. In fact, the thorough-going adoption of it or of its equivalent must follow in time as a natural logical outgrowth of a reversion of our notation back upon itself at the ten-point.

This preference for ten as radix does not, however, rest upon any natural or spiritual basis, upon any inherent fitness of ten itself to discharge this supreme function in arithmetic, but solely upon a physical peculiarity of the counting animal, man. He is a pentadactyl, he has ten fingers. Inasmuch as elementary reckoning is almost always done on the fingers in its first stages and by the young, whether in individual or educational or national life, it was almost inevitable that ten should be taken as the point of reflection. If only one hand be used, then five presents itself as radix, or if both hands and feet, then twenty, and both of these numbers have indeed served as radices.

But none of these related integers has any intrinsic fitness for the office in question. To be sure, ten is resolvable into the factors two and five, and the first of these is important—the operation of taking the half is so frequently necessary; but not so with five—we have comparatively little use for fifth parts, while on the other hand we meet with thirds and fourths at every turn and in every department of life. But ten is not divisible by either three or four and stands in no simple relation to either six or eight, two other important numbers. These are very serious defects in ten as a radix, and they encumber very gravely all reckoning and especially practical calculation in the denary system or with decimals. Thus, the constantly recurrent fraction $\frac{1}{3}$ is expressible only through an interminate decimal, $\frac{1}{4}$ requires two figures, $\frac{1}{6}$ is interminate as a decimal, $\frac{1}{8}$ requires three figures for its expression. These are very weighty burdens, which the general and final adoption of the decimal system will bind upon the back of humanity forever. Neither will any conceivable development of human intelligence lighten them in the least, for they inhere in the unalterable nature of the number ten.

But there is an altogether simple and easy deliverance lying ready at hand. It is the rejection of the unsuitable radix ten and the adoption

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of *twelve*, which is fitted perfectly and in every particular. It may be divided by two, by three, by four, by six; it is related simply to eight and nine. It is this superior divisibility that marks it out among all numbers as pre-eminently the *natural* radix, a divisibility shared with its own multiples only, as 24, 36, etc. It is the contention of this article that a change from the denary to the duodenary notation would be a great and much-needed simplification, above all to the practical man, and that it is entirely feasible. Let us examine the matter in detail.

1. Of course, the adoption of twelve as basis would necessitate the introduction of two new symbols for ten and eleven, since 10 would then no longer signify *ten* but *twelve*. This need, however, would be a mere trifle, and most easily supplied. Appropriate symbols may be readily devised, but we shall content ourselves for the present with the initals *t* and *e*.

2. The second step may be taken with equal facility: to invent a simple and consistent nomenclature. Manifestly the names thirteen, fourteen, twenty, thirty, hundred, thousand, must now lose whatever appropriateness some of them may once have had. But in devising new names we must obey the natural impulses of reason and linguistic sense as indicated in some of the names to be abolished. Instead of TY, the reminiscence of ten, in twen*ty*, etc., we may put *tel*, a like reminder of twelve. We recognize also the inevitable and irresistible tendency of frequent use to break down harder combinations of sound into easier ones, and we should adopt extremely simple forms at the outset. Here, then, is a body of terms that would at least answer perfectly our purpose:

One, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve; *tel* one, tel two, tel three, tel four, tel five, tel six, tel *sen*, tel eight, tel nine, tel ten, tel *len*, twen*tel*; twentel one, twentel two, etc.; thirtel, fortel, fiftel, sixtel, *sen*tel, eightel, ninetel, tentel, *len*tel; dipo, tripo, tetrapo, etc., N-po. The termination PO will be at once recognized as an abbreviation both of *power* and *position*, and in fact the second power of twelve occupies the second position to the left of the unit place, which latter counts not as the first but as the *zeroth* position. These names depart as little as possible from familiar ones and may all be learned in five minutes.

3. The superiority of twelve over ten as a radix will begin to be revealed in the multiplication table:

2×1	=	2	3×9	=	23	5×5	=	21	7×1	=	7	8×9	=	60	t×5	=	42
2×2	=	4	3×t	=	26	5×6	=	26	7×2	=	12	8×t	=	68	t×6	=	50
2×3	=	6	3×e	=	29	5×7	=	2e	7×3	=	19	8×e	=	74	t×7	=	5t
2×4	=	8	3×10	=	30	5×8	=	34	7×4	=	24	8×10	=	80	t×8	=	68
2×5	=	t	4×1	=	4	5×9	=	39	7×5	=	2e	9×1	=	9	t×9	=	76
2×6	=	10	4×2	=	8	5×t	=	42	7×6	=	36	9×2	=	16	t×t	=	84
2×7	=	12	4×3	=	10	5×e	=	47	7×7	=	41	9×3	=	23	t×e	=	92
2×8	=	14	4×4	=	14	5×10	=	50	7×8	=	48	9×4	=	30	t×10	=	t0[2]
2×9	=	16	4×5	=	18	6×1	=	6	7×9	=	53	9×5	=	39	e×1	=	e
2×t	=	18	4×6	=	20	6×2	=	10	7×t	=	5t	9×6	=	46	e×2	=	1t
2×e	=	1t	4×7	=	24	6×3	=	16	7×e	=	65	9×7	=	53	e×3	=	29
2×10	=	20	4×8	=	28	6×4	=	20	7×10	=	70	9×8	=	60	e×4	=	38
3×1	=	3	4×9	=	30	6×5	=	26	8×1	=	8	9×9	=	69	e×5	=	47
3×2	=	6	4×t	=	34	6×6	=	30	8×2	=	14	9×t	=	76	e×6	=	56
3×3	=	9	4×e	=	38	6×7	=	36	8×3	=	20	9×e	=	83	e×7	=	65
3×4	=	10	4×10	=	40	6×8	=	40	8×4	=	28	9×10	=	90	e×8	=	74
3×5	=	13	5×1	=	5	6×9	=	46	8×5	=	34	t×1	=	t	e×9	=	83
3×6	=	16	5×2	=	t	6×t	=	50	8×6	=	40	t×2	=	18	e×t	=	92
3×7	=	19	5×3	=	13	6×e	=	56	8×7	=	48	t×3	=	26	e×e	=	t1
3×8	=	20	5×4	=	18	6×10	=	60	8×8	=	54	t×4	=	34	e×10	=	e0

[2] This answer was erroneously printed "90" in the original edition.

We may add $10 \times 10 = 10^2 = 100 =$ Dipo.

We note that since 100 in this system equals 144 in the common system, the range and efficiency of our table are increased by 44 per cent. More than this, however, it is much simpler than the ordinary. The multiplication by five and ten is indeed less simple, but the multiplication by two remains as simple as before, multiplication by eleven becomes as simple as was multiplication by nine, while multiplication by three, four, six, seven, eight, nine is much simplified. For each of these, except seven, there is a simple cycle, through which the last figure in the product moves: for 3 the cycle is 3, 6, 9, 0; for 4 it is 4, 8, 0; for 6 it is 6, 0; for 8 it is 8, 4, 0; for 9 it is 9, 6, 3, 0. The cycles for 3 and 9 are conjugate, each the reverse of the other, and so are the cycles for 4 and 8, while the cycle for 6 is its *own reverse*, it is *self-conjugate*. The cycles for 5 and 7 are conjugate, and each involves all the primary numbers. It is plain that in any system the cycles of two conjugates (whose sum is the base) will be conjugate and that the sum of two corresponding digits will be the base. The successive even multiples of 7 end in the successive even numbers, while the first digit is half the multiplier—circumstances that greatly simplify multiplication by this number. Every third multiple of 5 ends in the multiplier. Squares of multiples of 6 end in 0, squares of all other even numbers end in 4; squares of prime numbers (except 3 and 2) all end in 1; squares of numbers ending in 3 or 9 end in 9. The ordinary table furnishes no parallels in simplicity to these relations. Factoring becomes comparatively easy in this duodenary scale. Even numbers are of course recognized at once; all ending in 0, 3, 6, or 9 are divisible by 3; ending in 0, 4, or 8 they are divisible by 4: ending in 0 or 6, they contain the factor 6; ending in 0 or 8 preceded by an even digit, or in 4 preceded by an odd digit, they may be divided by 8; and so on.

4. The common useful fractions are easily and simply expressed in duodecimals; thus, $\frac{1}{2}$ =.6, $\frac{1}{3}$ = .4, $\frac{1}{4}$ = .3, $\frac{1}{6}$ = .2, $\frac{1}{8}$ = .16, $\frac{1}{9}$ = .14—a most important consideration; while the expressions for the practically useless fractions are interesting though interminate: thus, $\frac{1}{5}$ = .24972. . . , $\frac{1}{7}$ = .186t351. . . , $\frac{1}{t}$ = .124972. . . We observe that the repetend in $\frac{1}{5}$ and in $\frac{1}{t}$ is the same, while in the repetend of $\frac{1}{7}$ we have 1 + t = 8 + 3 = 6 + 5 = e. Other curious relations may be traced out.

5. The expression of large numbers becomes measurably conciser; thus, all below 144 (in the present system) are expressed by two figures at most; all below 1728 by three figures at most, all below 20736 by four figures at most; and so on. Inasmuch as it is *at least* as easy to multiply in the twelve as in the ten scale, this reduction in the number of figures corresponds to a proportional reduction not only in mechanical but also in logical labor.

6. In the expression of irrational numerics through duodecimals a much higher degree of accuracy is obtained than by the use of the same number of digits in decimals. Thus, in dropping all decimals beyond the second place, the error committed is less than one-half of $\frac{1}{100}$, but in duodecimals it is less than half of $\frac{1}{144}$; so the other corresponding errors in the two systems are less than half of $\frac{1}{1000}$ and $\frac{1}{1728}$, $\frac{1}{10000}$ and $\frac{1}{20736}$ and so on. In a 7-place logarithmic table the error introduced in the use of the 7th digit is less than half of $\frac{1}{10{,}000{,}000}$, in the duodenary system it would be less than half of $\frac{1}{35{,}831{,}808}$; a fivefold accuracy is gained without additional labor. Similarly in the inverse use of the table.

All the foregoing advantages lie securely rooted either in the slightly superior size or in the greatly superior divisibility of the number twelve; they are all intrinsic and inalienable. The following have an importance at least equal, but of another nature—extrinsic and derived from certain immemorial prescriptions in which twelve has successfully asserted its higher claims as the *natural* radix. Thus, there are twelve months in the year and twelve hours in the half day. Both these divisions of time may—one might almost say must—stand, but they are inconvenient so long as ten dominates our calculations. Let us consider.

7. The circle of the clock-face is divided into twelve *hours*, each of these into 60 *minutes*, each of these into 60 *seconds*. On the other hand, the circle, or round angle, is divided into 360 degrees, each of these into 60 *minutes*, each of these into 60 *seconds*. The origin of this sexagesimal division is very interesting, but its perpetuation is very unreasonable, unnatural, and bewildering; it vexes and confounds the child's mind, and con-

sumes time to no purpose in every conversion of longitude and time into each other. Let the division into twelve hours stand; but let it be of the half-circle, so that the whole circum-angle shall be divided into 24 (or in our notation 20) hours; divide each of these into twelve grades, each of these into twelve primes, each of these into twelve seconds, and so on. The metric number of the half-circle or straight angle, radian being unit, is π; hence the quasi-equation $\pi = 10$. In a round angle there will be 200 grades, in a straight angle 100 (dipo), in a right angle 60, in a sextant 40, in an octant 30; the other important angles (of 30 and 15 degrees) will contain 20 (twentel) and 10 (twelve) grades. Surely this would be a great simplification, shunting an enormous amount of labor and confusion. The grade, corresponding to five minutes of time, would be a very convenient standard of reference. We should then say "in a grade," as we now say "in five minutes" or "in a minute" (meaning "five minutes").

8. Our present division of the year into twelve months of unequal length is puzzling, irrational, and inconvenient. Let there be twelve months of thirty days each, and let the year begin at the vernal equinox, the natural starting-point. There remain five days to be disposed of. Let these be legal holidays with special names, extra-mensual, belonging to no month. Let them mark the stations of the sun's progress through the sky and be: New Year's day, first quarter-day, mid-year's day, second quarter-day, Old Year's day. They might otherwise be named *Vernequid*, upper *Solstid*, *Autumnequid*, lower *Solstid*, *Vernequin*. Leap year would require the intercalation of a day at mid-year, which might be called *Autumnequin*. The lower *Solstid* would coincide practically with Christmas, and the sacred festival might be transferred without perceptible jar to that day, for symbolic reasons the most appropriate in the year for the feast in question. On the other hand the upper *Solstid* would nearly coincide with the Fourth of July, and both our national celebration and the French fète of the Fourteenth might easily be slightly shifted without offense to the patriotic conscience. The two solstids would then stand, the one for our political, the other for our religious, faith. The names of the months need not be changed; it would suffice to push their beginnings ten days backward. Even in modern times greater changes have been made in the calendar.

9. Our coinage could be made duodecimal with very little trouble. We need only reduce our *quarter* to twenty-four cents, or count it as twenty-four cents, and adopt it as unit. The twelfth would be a penny, which might of course be still further sub-divided, though there would be little use for lower denominations. The twelve-quarter (tel-kar?), about three dollars, would take the place of the current V,[3] which is rather inconveniently large for most of us in these days of low prices and lower salaries. The ease of dealing with thirds and fourths would be greatly valued by the man of business, while the reduction in value of the unit would put the whole Anglo-Saxon world in line (quarter = shilling = mark, practically or with slight adjustment), and what is far more, it would be an important lesson to the masses in domestic economy. *Four marks* may not quite equal *one dollar*, but it sounds bigger and we should be more careful in spending it. *Experto create*.

10. The fundamental measurement is of *length* and the most important of all units is the *linear*. The hunt for a natural unit of length is to be sure a very interesting and instructive one, especially inspiring when led on by such "winged hounds of Zeus" as Maxwell and Thompson. As for practical results, however, it is a kind of red-fox chase, yielding much fun but little fur. It is in vain that the *meter* professes to be the ten-millionth of a quadrant through Paris; it is no such thing, and even if it were,—*cui bono?* It is really a purely arbitrary length carefully preserved. Any other standard will in last analysis prove to be equally arbitrary. The British *yard*, familiar to all of us, is as good as any. It is already divided into twelfths, each of three inches. Call this twelfth a *trinch*. The twelfth of this last would be our well-known quarter-inch, a very convenient unit for small lengths and, of course, capable of any degree of sub-division. For greater measurements we have already at hand the *mile*, 1760 yards, or, with a slight accommodation, 1728, that is 1000 (tripo) yards. For the immense stretches

[3]The Roman numeral five, for a five-dollar bill, many of which in this era had prominent Roman numeral fives on them.

of astronomy the square, or even the cube, of this would be suited. So we may pass from the decimal to the duodecimal system in this cardinal matter with hardly a jar and with loss of naught but impedimenta. The areal and voluminal units would, of course, be the squares and cubes of the linear units; the mass-units would be the mass of the voluminal unit of some standard substance under standard conditions. Other derivative units require no notice.

11. The operations of mercantile and commercial as well as of mechanical life would be greatly shortened and facilitated by the change proposed. For example, a movement of the duodecimal point one place leftward would convert yearly into monthly interest. Multiplication and division by three and four, so important in "business," at every step would check each other.

The construction of instruments for all kinds of measurement, whether of commerce or of scientific precision, would be made easy and their correct use facilitated.

Most of all, however, the enormous burden of learning, retaining, and using our tables of weights and measures would be rolled off once and forever in one huge mass from the minds of the coming generation. How great is the load that our forefathers, unknowing what they did, have bound upon the shoulders of their descendants is felt keenly, if only in part, by every pupil and every teacher. Like the superincumbent atmosphere, it presses on us continually and continuously, at every point both of time and of space. For this very reason we cannot come to perfect consciousness of the oppression. But once relieve it, and see how the elastic mind of youth will revive and erect itself, once more a freeman. Adopt the duodenary system *in toto*, and you despoil at one stroke the great giant Arithmos of his most formidable terrors. Nay more, you will add a full year, now so greatly desiderated, to the life of every youth that attains majority.

12. The complete triumph, either of ten or of twelve, is assured. The incoherences of our present system are universally recognized, and the structure cannot long withstand the blows of Example and of Reason. But the victory of ten would certainly be deplorable, for it would exalt body over spirit, and fasten clogs forever on the feet of humanity. The denary system can lay no claim to finality, as being the best either conceivable or practicable; it leaves much to be desired. But the duodenary *is* the best conceivable, the best that the *nature of number* admits. It and it only can pretend to be an absolute finality.

But some one will say, "after all, is the change *practicable?*" The discerning reader might be left to answer for himself. The change to the decimal system has been proved by actual experiment to be practicable throughout western and central Europe. The change to duodecimals would be even easier for us, because twelve already lies deep-rooted in many of our notions and modes of measuring. The only peculiar difficulty about this change is in reality a very slight and unimportant one, namely, we need two new symbols (for ten and eleven), a new termination (for multiples of twelve), and a half-dozen new names (for powers of twelve). But any foundry can cast the symbols, the ending *tel* is ready at hand, and the new names are coined and learned in a moment. This done, learning to count, add, subtract, multiply, divide in the new system is not the work of a day for the adult mind, and would be easier for the child than the tasks he now has to master.

To be sure, there would be a great inertia of custom, ignorance, and prejudice to overcome, but these oppose themselves alike to all progress.

A thorough-going adoption, either of the decimal or of the duodecimal system, is unavoidable and impending. If we choose the latter, all future generations will rise up and call us blessed; if the former, then we shall put into the mouth of posterity, but with sharper emphasis, with wider application, and with deeper meaning than ever Mephistopheles dreamed of, the just complaint:

Es erben sich Gesetz, und Rechte,	Statutes and laws through all the ages
Wie eine ew'ge Krankheit fort;	Like a transmitted malady you trace;
Sie schleppen von Geschlecht sich zum Gescblechte,	In every generation still it rages
Und rücken sacht von Ort zu Ort.	And softly creeps from place to place.
Vernunft wird Unsinn, Wohlthat Plage;	Reason is nonsense, right an impudent suggestion;

Weh dir, daß du ein Enkel bist !
Vom Rechte, das mit uns geboren ist,
Von dem ist, leider ! nie die Frage.

Alas for thee, that thou a grandson art!
Of inborn law in which each man has part,
Of that, unfortunately, there's no question.

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This article was completely remastered and retypeset from a scanned copy of the original using the LaTeX document preparation system. It is offered here in its predominantly original form. A part of Part 13, discussing the matter as a racial conflict between Anglo-Saxons and "Kelts," was removed. While this sort of talk was quite common among many segments of Anglo-Saxon society of the time, it is neither scientific nor mathematical, and had no justified place in this otherwise delightful piece. The poem at the end is an excerpt from Goethe's *Faust*, Act I, Scene IV; the translation offered is that of Charles Brook, 10Ɛ8, available at http://www.gutenberg.org/cache/epub/14460/pg14460.txt. This document is proudly made available by the Dozenal Society of America (http://www.dozenal.org).