A Dozenal Primer

compiled by the

Dozenal Society of America

Numeration Throughout is Dozenal (Base Twelve)

0 1 2 3 4 5 6 7 8 9 Ɛ Ɛ 10

where Ɛ is ten, Ɛ is eleven, and 10 is a dozen

Dozenal numeration is a system of thinking of numbers in twelves, rather than tens. Twelve is a much more versatile number, having four even divisors—2, 3, 4, and 6—as opposed to only two for ten. This means that such hatefulness as "0.333. . . " for 1/3 and "0.1666. . . " for 1/6 are things of the past, replaced by easy "0;4" (four twelfths) and "0;2" (two twelfths).

In dozenal, counting goes "one, two, three, four, five, six, seven, eight, nine, ten, elv, dozen; dozen one, dozen two, dozen three, dozen four, dozen five, dozen six, dozen seven, dozen eight, dozen nine, dozen ten, dozen elv, two dozen, two dozen one. . . " It's written as such: 1, 2, 3, 4, 5, 6, 7, 8, 9, ↊, ↋, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1↊, 1↋, 20, 21. . .

Dozenal counting is at once much more efficient and much easier than decimal counting, and takes only a little bit of time to get used to. Further information can be had from the dozenal societies, as well as in many other places on the Internet.

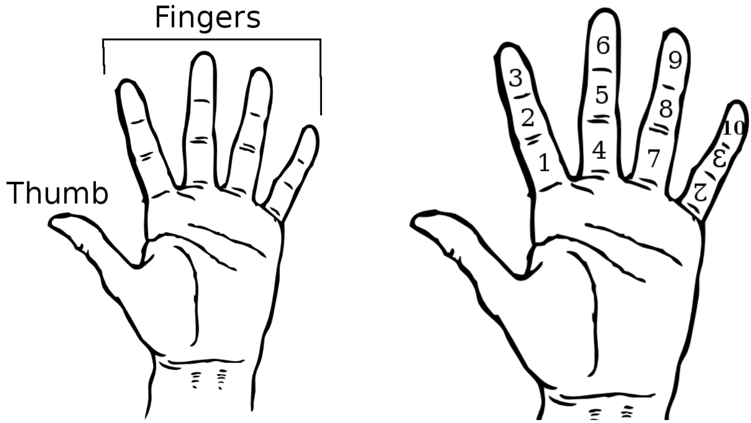
The Dozenal Society of America
http://www.dozenal.org

The Dozenal Society of Great Britain
http://www.dozenalsociety.org.uk

Learning to count in dozens is easy, but it does take some getting used to after having already learned to count in tens. So we will start at the very beginning, as if you'd never learned to count at all; soon you'll be counting dozens as quickly and easily—more so!—than you ever did in tens.

Counting from 1 to 10 on the Fingers

First, we count from 1 to 10; and the easiest way to count is on our *fingers*. Hold up your hand, and you see that you have four fingers and one thumb; the thumb can touch all the others. Notice also that each finger has three segments; the thumb can touch all of those, too.



This lets us count to a *dozen* very easily on our fingers. Here's how.

Hold out your hand—right or left, it doesn't matter—with the palm facing you. We are going to count the segments on your fingers.

Start by placing the tip of your thumb on the first segment of your first finger, your *index finger*. Now say "*one*" (1).

Now place the tip of your thumb on the second segment of your index finger. Say "*two*" (2).

Place your thumb on the top segment of your index finger; say "*three*" (3).

Let's keep going. Do the same thing with your next finger, your *middle finger*. Count off "*four*" (4), "*five*" (5), and "*six*" (6). Now you know how many segments are on your first two fingers, or half your hand. Move to the next, your *ring finger*. Count off "*seven*" (7), "*eight*" (8), "*nine*" (9). Now finally your *pinky finger*, or *little finger*. Count off "*ten*" (↊), "*eleven*" (↋), and "*twelve*" (10).

Congratulations! You've counted to a *dozen*! You can see how we write these numbers from the palm on the right, above. This is a big step; the first and hardest leap is over.

Counting from 10 to 100 on the Fingers

Count from 1 (*one*) to 10 (dozen), just as before. While you do it, read the numbers off and look at the symbols we use for them below:

1 *one*	2 *two*	3 *three*	4 *four*
5 *five*	6 *six*	7 *seven*	8 *eight*
9 *nine*	↊ *ten*	↋ *eleven*	10 *dozen*

Now, take your other hand, whichever one you're not using, and put your thumb on the first segment of your index finger, just as you did when you were counting 1 (*one*). Remove your thumb from your pinky finger. Now you're pointing to the number 10 (one *dozen*)!

Your first hands counts *ones*; your second hand counts *dozens*. So the thumb on your second hand's index finger means 10; now you can keep counting up past 10 with your first hand, exactly the way you did when counting 1 to 10. So count the segments on your first hand again, keeping your second hand's thumb on 1, and count like this:

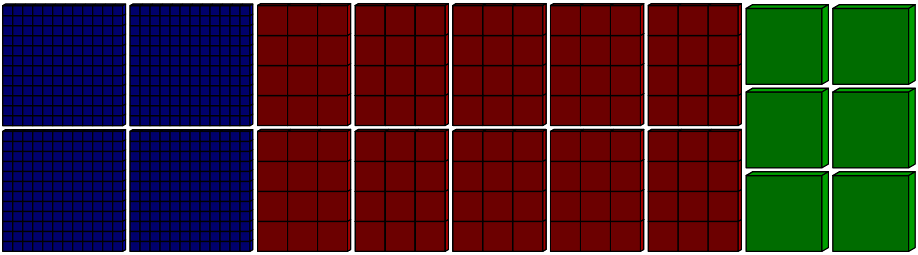
> Dozen *one*, dozen *two*, dozen *three*. . .

And keep going until you get to *dozen eleven*. Now you move your second hand to 2 and count *two dozen*, and you can begin counting *two dozen one* on your first hand again.

Every time you get to 10 (*dozen*) on your first hand, you count another *dozen* on your second hand. When you get to *eleven dozen eleven*, you can keep counting; but you'll have to use your head or some paper, unless you have a third hand.

Places and Powers

Every time you reach ↋ (*eleven*), you add a new place to the number. As an example, below we have boxes; green is a single box, red is a dozen boxes, and blue is a dozen dozen boxes. Let's look at a few groups of boxes and see how these translate to numbers.



We can translate this into digits very directly: because there are 4 groups of 100, ↊ groups of 10, and 6 groups of 1, we simply write them in:

$$4 \quad ↊ \quad 6$$

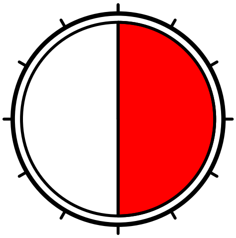
We can read this, for now, as "four gross ten dozen six."

Each time we add a digit to the left, we're counting groups of *twelve times* the number of the last place. So the first digit is groups of one; the second is groups of dozens; the third is groups of dozen dozens, or grosses; the fourth is groups of dozen grosses; and so on.

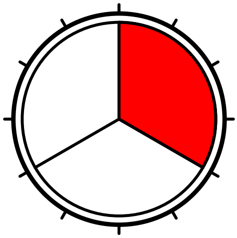
It can also go *backwards*, by adding digits to the *right*; this means that the digit is *one twelfth* of the last digit. We separate the ones digit from those less than one with a *dozenal point*, usually written as a semicolon (;) but sometimes, as in decimal, as a period (.). In other words, we count the number of digits after the dozenal point, multiply by 10, and make that the denominator of a fraction, with the numerator being the digits to the right of the point:

$$4;6 = 4\frac{6}{10} \qquad 4;↋6 = 4\frac{↋6}{100}$$

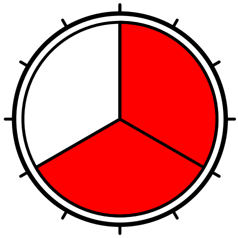
Dozenal makes a great many of these fractions *shorter* and *simpler* than in decimal; a few big examples are below.



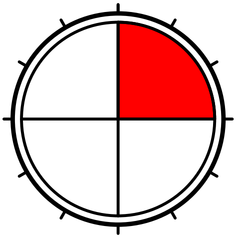
$^1/_2 = 0;6$



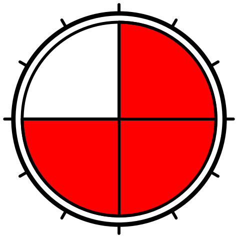
$^1/_3 = 0;4$



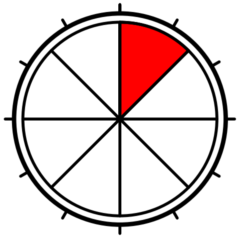
$^2/_3 = 0;8$



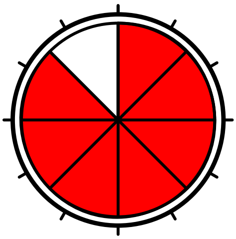
$^1/_4 = 0;3$



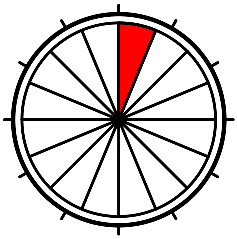
$^3/_4 = 0;9$



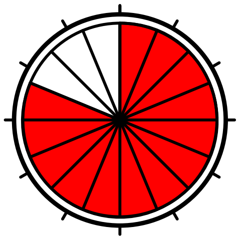
$^1/_8 = 0;16$



$^7/_8 = 0;↊6$



$^1/_{14} = 0;09$



$^{11}/_{14} = 0;99$

This is dozenal, so 14 is a *dozen-four* (sixteen).

So Why Bother?

But why should we bother? We already know how to count in decimal; why learn to count in dozenal? What benefits would come from that? Is there any use in counting by twelves?

Absolutely there is. The biggest benefit is that twelve is divisible by 2, 3, 4, and 6, while ten is divisible by only 2 and 5. This means that, using twelve, math is easier from things as complex as mechanics and as simple as dividing up a restaurant check.

It also means that fractions are much more regular. Below is a table showing single-digit fractionals, double-digit fractionals, triple-digit fractionals, and infinitely recurring fractionals.

Fraction	Dozenal	Decimal
1/2	0;6	0.5
1/3	0;4	0.333...
1/4	0;3	0.25
1/5	0;2497...	0.2
1/6	0;2	0.1666...
1/7	0;186↊35...	0.142857...
1/8	0;16	0.125
1/9	0;14	0.111...
1/↊	0;124972...	0.1
1/↋	0;111...	0.0909...
1/10	0;1	0.083...

Notice that dozenal has two more single-digit fractionals; twice as many double-digit fractionals; no triple-digit fractionals; and only four infinite fractionals. Decimal has fewer single- and double-digit, does have a triple-digit (for a common fraction, no less), and two more infinite fractionals.

The fraction 1/3 is particularly damning for decimal. Every third number is a multiple of three; yet decimal can never exactly express a third.

Multiplication tables also become much simpler.

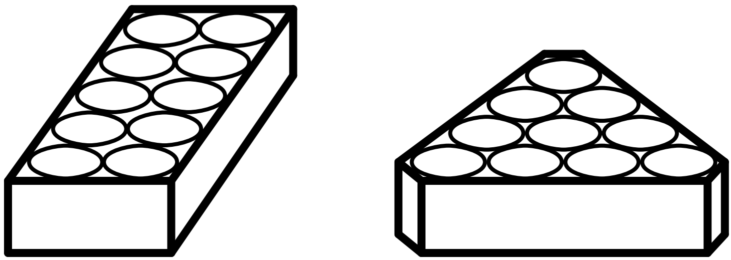
1	2	3	4	5	6	7	8	9	↊	↋	10
2	4	6	8	↊	10	12	14	16	18	1↊	20
3	6	9	10	13	16	19	20	23	26	29	30
4	8	10	14	18	20	24	28	30	34	38	40
5	↊	13	18	21	26	2↋	34	39	42	47	50
6	10	16	20	26	30	36	40	46	50	56	60
7	12	19	24	2↋	36	41	48	53	5↊	65	70
8	14	20	28	34	40	48	54	60	68	74	80
9	16	23	30	39	46	53	60	69	76	83	90
↊	18	26	34	42	50	5↊	68	76	84	92	↊0
↋	1↊	29	38	47	56	65	74	83	92	↊1	↋0
10	20	30	40	50	60	70	80	90	↊0	↋0	100

Thanks to Michael deVlieger for designing this table.

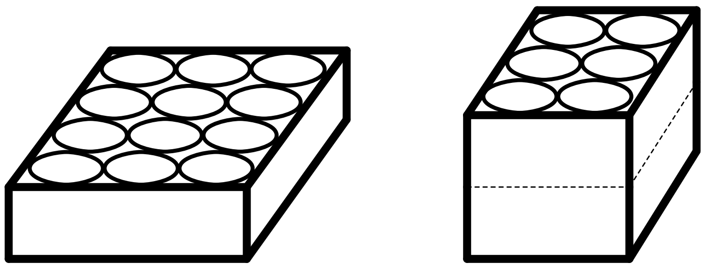
As this table makes clear, nearly every column in the dozenal multiplication table has a clear and repeating pattern. In the decimal multiplication table, by contrast, there are comparatively fewer columns with such patterns, and what patterns are there tend to be longer. Most notably, in the decimal table 3 and 9 don't have any pattern at all, and 4 and 8 have patterns that are five items long. In dozenal, 3 and 9 have four-item patterns, and 4 and 8 have 3-item patterns.

It's also worth noting that this doesn't just make our own use of arithmetic easier; it has many practical applications. A very easy one to demonstrate is *packing*. Because twelve is more divisible than ten, we can efficiently pack twelve items in more ways than we can pack ten items.

For example, ten cans can really only be packed in two ways, one of which is triangular and therefore doesn't store nearly as compactly as the others; so really, only one arrangement is practical:



Twelve cans, on the other hand, can be packed 2×6, 3×4, or even double-stacked as 2×3, giving two more options for efficient storage and shipping. The two most efficient of these arrangements, in terms of packing material, are shown below:



The second of these, double-stacked 2×3, is so efficient that it is actually cheaper to pack ten cans in a box like this, leaving two empty spaces, than to pack a rectangular ten-box full!

Other practical applications where counting in dozens gives great advantages are statistics (parts per gross is much more flexible than parts per hundred, or "percent"); time (we have twelve hours per half-day for a very good reason); carpentry and other building (metric carpenters know what they're doing when they use a "metric yard" of 1.2 meters); and a host of others.

Speaking in Dozens

Once we are counting in dozens, though, how do we *speak* in dozens? We clearly can't go on saying "twenty," "thirty," and so on; these are designed specifically around tens. Fortunately, dozenal has a lot of offer here, too.

We can, of course, simply continue speaking in dozens the way we occasionally do in the decimal world: use plain, old English words. When we need two dozen eggs, we simply say "two dozen." Why do anything else?

So 24 is *two dozen four*, and 89 is *eight dozen nine*. When we remember that a dozen dozen is traditionally called a "gross," we can move on to three digits, where 7↊4 is *seven gross ten dozen four*. We can take this one place further with the more unusual, but still traditional, term "great-gross" for a dozen gross; some have called this a "grand." So ↋735 is *eleven great-gross seven gross three dozen five*.

This works perfectly well as far as it goes, and is usually the easiest way to introduce someone to dozens. However, after four places, it breaks down; and it doesn't help us at all with dozenal fractions. Therefore, more robust options have arisen over the years to take its place.

The *do-gro-mo* system has a venerable dozenal pedigree, taking shape in the years preceding the foundation of the Dozenal Society of America.

10	Do	0.1	Edo
100	Gro	0.01	Egro
1000	Mo	0.001	Emo
1 0000	Do-mo	0.0001	Edo-mo
10 0000	Gro-mo	0.0000 1	Egro-mo
100 0000	Bi-mo	0.0000 01	Ebi-mo
1000 0000	Tri-mo	0.0000 001	Etri-mo

Obviously, using this can take a number of different forms, most simply by naming the highest power and listing the digits afterwards.

E.g., Ɛ735 6823 is *eleven tri-mo seven three five six eight two three*. Fractions forms are taken by adding an *e* to the normal form; so 100 is *gro* and 0.01 is *egro*. Often, though, we pronounce fractions just as we do in dozenal, though we generally say *dit* rather than "point": 0.01 is *zero dit zero one*.

More recently, *Systematic Dozenal Nomenclature* has become quite popular. This system uses well-known Greek and Latin number roots to form dozenal number words. It is a robust and powerful system; for basic counting, though, it reduces to the following:

Num.	Root	Pos.	Neg.
0	Nil	Nilqua	Nilcia
1	Un	Unqua	Uncia
2	Bi	Biqua	Bicia
3	Tri	Triqua	Tricia
4	Quad	Quadqua	Quadcia
5	Pent	Pentqua	Pentcia
6	Hex	Hexqua	Hexcia
7	Sept	Septqua	Septcia
8	Oct	Octqua	Octcia
9	Enn	Ennqua	Enncia
Ꭓ	Dec	Decqua	Deccia
Ɛ	Lev	Levqua	Levcia
10	Unnil	Unnilqua	Unnilcia
11	Unun	Ununqua	Ununcia

Each place in the number has an easy word defining it, which can be determined by simply counting the places and subtracting one. A four-digit number is a *triqua*; a seven-digit is a *hexqua*. For fractional places, no need to subtract one; just count the digits to the right of the dozenal point. Six fractional places is a *hexcia*. E.g., Ɛ735 6823 is *eleven septqua seven three five six eight two three*.

For More Information

The Dozenal Society of America is at your service for all things dozenal, and offers a great deal of information about dozenals and mathematics in general. You may be interested in such basic classics as James Malone's *Eggsactly a Dozen*, which explains dozenal counting beautifully in only a single page. You might be a history buff, interested in learning the origins of this dozenal idea; for that, you can find classics like Thomas Freeman's *Systems of Numeration*, Leech's *Dozens vs. Tens*, or Leclerc's and Laplace's discussions of the dozen, in the original French and in English translations. These and almost limitless other resources, particularly the archive of nearly six dozen years of the DSA's flagship publication *The Duodecimal Bulletin*, are all available at the DSA's website, http://www.dozenal.org.

If you'd like a little more detailed discussion of what we talked about in this little booklet, consider reading *The Manual of the Dozenal System*, which exists in its original 1174 (1960.) version, as well as in a fully updated and revised version, on our website.

To see just how easy it is to do math in dozens (the same as doing them in tens, only easier), consider reading *Fundamental Operations in the Duodecimal System*. Here you'll see that adding, subtracting, multiplying, and dividing in dozenal require practically no relearning; you get the benefits of easier arithmetic with little additional work.

It's impossible to catalog the riches of what you can find from the DSA and other sources, or the riches and benefits of the dozenal system. Simply try it out! Next time you count something, count it in twelves; think of your height in feet and inches as dozens and ones; multiply two numbers you see on a license plate every now and then. Before long, you'll be as convinced of dozenal's benefits as we are. And enjoy! Twelve will make math easier; and maybe even fun.

Each One, Teach One