The Dozenal Society of America
Duodenal System of Arithmetic, Measures, Weights and Coins

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The object of appending a treatise on a new system of arithmetic and metrology is to demonstrate what can be done with that subject, which demonstration might by that means be conveniently accessible to the student and to the public.

The problem of an international and complete system of metrology has at all times been esteemed an important desideratum, but no attempt has yet been made to remove the principal difficulty which is in the way, and we can expect no satisfactory metrology until its primary obstacle is removed.

The base ten, which is adopted in our present arithmetic, does not admit of binary and trinary divisions, as required in metrology. This is the principal difficulty in the way of establishing a satisfactory system of measures, weights and coins.

The number *10* is actually the worst even number that could have been selected as a base of numeration, for which either *8*, *12*, or *16* would have been better.

The inconveniences of the decimal base in metrology are well known, and have been explained at various times by various writers; but the present arithmetic is so thoroughly incorporated with civilization that it appears difficult to unlearn and get rid of the same for the substitution of something better.

The American Pharmaceutical Association appointed a committee, of which Alfred B. Taylor of Philadelphia was chairman, for the purpose of investigating the present condition of metrology with a view to its improvement, who gave the subject a very careful and deliberate consideration.

An elaborate report containing over *100* octavo pages of fine print was prepared and read before the annual session of the Association, held in Boston September *15*, *1859*. This report explains the inconveniences of the decimal arithmetic and of the French metrical system, illustrated by quotations from various authors of high authority.

In the course of this report Mr. Taylor proposed and elucidated an **Octonal System** of arithmetic and metrology.

Octonal System.

The octonal system has *8* to the base, which admits of binary division to unity without fractions. It would be an easy system to learn and manage in both arithmetical and mental calculations, but it requires a greater number of figures than the decimal system in expressing high numbers, and eight is too small as a base.

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The octonal system, moreover, does not admit of trinary division, as is required in the circle and time.

Decimal System.

The decimal arithmetic is of Hindoo origin, and was imported into Arabia some one thousand years ago, from which it was spread throughout Europe and the entire civilized world.

The base ten originated from the *10* fingers, which were used for counting before characters were formed to denote numbers.

The base *10* admits of only one binary division, which gives the prime number *5* without fraction. The trinary divisions give an endless number of decimals. The decimal system is therefore not well suited for metrology, in which binary and trinary divisions are required.

It is this defect of the decimal system which has caused confusion in metrology and discordance among nations respecting the adoption of one common system of measures; which problem will never be satisfactorily solved as long as decimal arithmetic is maintained.

By examining the tables of measures and weights of different nations we find that binary and trinary divisions are invariably preferred, notwithstanding that decimal arithmetic must be used in their calculation.

The French decimal metrology is perhaps the best that can be devised in connection with decimal arithmetic; it looks very inviting and simple on paper, but what is gained by the metrical system in calculations is lost in the shop and market.

The defects of the metrical system are the defects of our arithmetic itself, and as long as decimal arithmetic is maintained the French system is the best of all that have yet been devised.

The slow adoption of the metrical system by other nations is sustained by good reasons—namely, that it does not constitute a complete, uniform and convenient system of metrology. The decimal system, as before stated, is not applicable to the admeasurement of the circle, of time and of the compass, where binary divisions are indispensable. The circle requires both binary and trinary divisions, neither of which can be accommodated by the decimal base.

When the metrical system was first established in France, it was intended to decimate also the circle and the time, which was soon found to be impractical and the idea abandoned.

The French metrology is therefore not a complete system, and it has been renounced for all measures in astronomy, geography, navigation, time, the circle and the sphere, where it is inapplicable.

The decimal system is also inapplicable in music, where the binary and trinary divisions are invariably used.

Music represents the natural disposition of the mind to arrange or classify quantities. The musical bar is divided into *halves*, *quarters*, *eighths* and *sixteenths*; and also into *thirds*, *sixths*, *ninths* and *twelfths*; but we never find music divided into tenths.

The most natural or binary division of music is represented thus:*

A bar of music divided by the decimal system would appear thus:†

*The following figure, Orig. Fig. 232, has been moved to Figure 1 at page 3.

†The following figures, Orig. Fig. 233 and Orig. Fig. 234, have been combined and moved to Figure 2 at page 3.

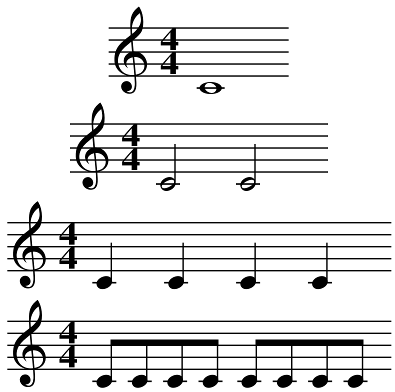


Figure 1: Orig. Fig. 232.

Orig. Fig. 233

or if you please

Orig. Fig. 234

Figure 2: Orig. Figs. 234 and 234.

No music could be produced by either of these last divisions, but a mechanical noise only could be made by it.

The lowest grade of man, and even animals, sing binary music. Even an Australian magpie can be taught to whistle any ordinary song as correctly as played on a musical instrument; whereas a decimal division of music could never be learned and appreciated even by the highest intelligence.

Such is also the comparison between binary and decimal arithmetic. Decimal arithmetic is a heavy burden upon the mind, and limits mental calculations within a very narrow compass; whilst binary or trinary arithmetic would become natural to the mind like music, and render mental calculations as easy as music played by the ear.

THE FOLDED FRENCH METRE.

The French metre is difficult to fold into a convenient shape for the pocket. The ten-folded metre with lap-joints is a very convenient form for approximate measurements, but cannot be relied upon for correctness, because the numerous lap-joints cannot be made permanently accurate, and moreover the lap-joints do not form a straight but a broken line. The metre folded into five parts with lap-joints is an odd affair.

The two-folded metre of five decimetres in each part, of about *20* inches long, is too large for the pocket.

The four-folded metre makes two and a half decimetres in each part of about *10* inches long, which will answer for the pocket; and is perhaps the best form of the French metre when made with regular hinges like the English four-folded rule, but it is still a broken measure.

An international association for obtaining a uniform decimal system of weights, measures and coins has been in existence for over thirty years, and has yet accomplished very little. The object of this association is wholly for the introduction of the French metrical system, which has met with the most natural and reasonable objections—namely, that it is not a complete system, and that it is inconvenient in the shop and in the market; but the strong influence of this association has induced many governments to force that system upon their people.

In practice, we want our units divided into the simplest and most natural fractions—namely, *halves*, *thirds*, *quarters*, *sixths*, *eighths*, etc.—which cannot be done by the metrical system, or decimal arithmetic without long tails of figures commencing with 0.

For instance, the simple fraction $\frac{1}{3}$ expressed by decimals is 0.33333....... without end, and will never be correct, and requires a good education to understand the true meaning of it. The good scholar manages the decimal fractions as easily as a musician plays on his hand-organ, but the fraction 0.33333 is not so easily understood by the majority of the people, who will naturally ask what it means. In the answer it is necessary to explain that the unit is divided into 100000 parts, and 33333 of those parts is nearly $\frac{1}{3}$ of the whole. The people will then surely reply that they are not willing to cut their things up into 100000 parts and lose a portion by the division in order to get it into three.

DUODENAL SYSTEM.

Charles XII. of Sweden proposed to introduce a duodenal system of arithmetic and metrology. The king complained of ten as a base, and said, "It can be divided only once by 2, and then stops." The number *12* can be divided by 2, 3, 4 and 6 without leaving fractions; and divided by 8 gives $\frac{3}{2}$, by 9 gives $\frac{4}{3}$, and by 10 gives $\frac{6}{5}$, all convenient fractions for calculation.

The number *12* has always been a favorite base in metrology.

The old French foot was divided into *12* inches, the inch into *12* lines, and the line into *12* points. The *dozen* is a well-known base adopted all over the world; *12 dozens* is a *gross*, and *12 gross* is a *great-gross*. We have *12* months in a year, *12* hours in a day, *12* signs in the zodiac, *12* musical notes in an octave. The old Roman metrology was based on *12*, like the English foot and the Troy pound.

A writer in the Edinburgh Review (Jan., *1807*, vol. *9*, page *376*) regrets that the philosophers of France, when engaged in making so radical a change in the measures and standards of the nation, did not attempt a reform in the popular *arithmetic*. He, being in favor of a duodenal system, says, "The property of the number *12* which recommends it so strongly for the purpose we are now considering is its divisibility into so many more aliquot parts than ten, or any other number that is not much greater than itself. Twelve is divisible by *2*, *3*, *4* and *6*; and this circumstance fits it so well for the purpose of arithmetical computation that it has been resorted to in all times as the most convenient number into which any unit either of weight or measure could be divided. The divisions of the Roman *as*, the *libra*, the *jugerum*, and the modern foot, are all proofs of what is here asserted; and this advantage, which was perceived in rude and early times, would have been found of great value in the most improved eras of mathematical science. . . . We regret therefore that the experiment of this new arithmetic was not attempted. Another opportunity of trying it is not likely to occur soon.

"In the ordinary course of human affairs such improvements are not thought of, and the moment may never again present itself when the wisdom of a nation shall come up to the level of this species of reform."

If man had been created with six fingers on each hand, we would have had in arithmetic a duodenal instead of the present cumbrous decimal system.

A uniform duodenal system of metrology, even with decimal arithmetic, would be much better in the shop and market than the French metrical system.

A duodenal system would be equally applicable in all branches of metrology, and it would include those which are excluded by the metrical system—namely, astronomy, geography, navigation, time and the circle.

The duodenal system would require two new characters to represent *10* and *11*, so as to place *10* at *12*. This change in the figures would appear strange at the first glance, but a little reflection, with due consideration, would soon lead to the satisfaction that these two new figures simplify the arithmetic and render it much easier for mental calculation than decimal arithmetic.

Senidenal System.

The senidenal system has 16 to the base. A full elucidation of this system has been worked out by the author and was published in the year 1862 by J. B. Lippincott & Co., Philadelphia. It is called the tonal system.

Systems.	Base.	100	1000	10,000	100,000	1,000,000
Octonary	8	64	512	4,096	32,768	260,744
Denary	*10*	*100*	*1,000*	*10,000*	*100,000*	*1,000,000*
Duodenary	12	144	1,728	20,736	238,832	2,865,984
Senidenary	16	256	4,096	65,536	16,777,216	268,435,456

Table 1: Scale of Four Arithmetical Systems.

The advantage of 16 as a base for arithmetic is that of its binary division to infinity. It is really the best system that could be devised for metrology and mental calculations.

The disadvantage of 16 as a base is that it requires six new figures to complete the base, which would be difficult to introduce, and also that it does not admit of trinary divisions, as is required in the circle and time, but it is under all circumstances far superior to the decimal system.

The difficulties with the decimal system are fully explained in the elucidation of the tonal system.

The names of the systems are Hindoo.

The octonary system requires the greatest number of places for expressing high numbers, for instance 1,000,000 octonal means only *260,744* of the decimal system.

The senidenary or tonal system uses less places; for instance, 1,000,000 senidenal means *268,435,456* of decimal numbers.

DUODENAL ARITHMETIC AND METROLOGY.

The base in the duodenal system is *12*, instead of *10* in the decimal system.

The Arabic system of notation is composed of ten simple digits, or characters—namely, *0*, *1*, *2*, *3*, *4*, *5*, *6*, *7*, *8*, *9*, and the base *10*.

These same characters can be used in the duodenal system by adding two numbers to complete the base—namely, *11* and *12*; then all the units of weights and measures should be divided and multiplied by *12*, but in order to render the system simple for calculation, it will be necessary to substitute new characters for the numbers *10*, *11* and *12*—namely,

Decimal system,	*1*,	*2*,	*3*,	*4*,	*5*,	*6*,	*7*,	*8*,	*9*,	*10*,	*11*,	*12*
Duodenary system,	1,	2,	3,	4,	5,	6,	7,	8,	9,	φ,	γ,	10

in which 10 denotes the base *12*, φ stands for *10*, and γ stands for *11*,

The Italic figures mean decimal numbers, and the Roman figures mean duodenal numbers.

In order to distinguish the two systems from one another, it will be necessary to give new names to the duodenary figures.

A duodenary system of arithmetic cannot be adopted by only one nation, but the whole civilized world ought to agree upon such a scheme. Different nations have different languages and names for the decimal figures and numbers; but in the adoption of a duodenary system of arithmetic, one common nomenclature might be agreed upon.

The new figures and nomenclature appear to be the greatest objection to the introduction of the duodenal system of arithmetic and metrology.

There is no difficulty in convincing the public of the utility of the duodenal system, and with that impression, a pride will be taken in using the new nomenclature, which could be taught in every school; and each individual would attempt to follow up the time of education.

The following table contains the names of the figures and numbers up to twelve in different languages:

No.	English	French	German	Swedish	Spanish	Latin	Greek
0	Naught	Zéro	Null	Noll	Cero	Nihil	Zero
1	One	Un	Eins	En	Uno	Unus	Eis, En
2	Two	Deux	Zwei	Två	Dos	Duo	Duo
3	Three	Trois	Drei	Tre	Tres	Tres	Treis
4	Four	Quatre	Vier	Fyra	Cuatro	Quatuor	Tessares
5	Five	Cinq	Funf	Fem	Cinco	Quinque	Pente
6	Six	Six	Sechs	Sex	Seis	Sex	Hex
7	Seven	Sept	Siben	Sju	Siete	Septem	Hepta
8	Eight	Huit	Acht	Åtta	Ocho	Octo	Okto
9	Nine	Neuf	Neun	Nio	Nueve	Novem	Ennes
10	Ten	Dix	Zehn	Tio	Diez	Decem	Deka
11	Eleven	Onze	Elf	Elva	Once	Undecem	Endeka
12	Twelve	Douze	Zwölf	Tolf	Doce	Duodecem	Dodeka
No.	**Russian**	**Finnish**	**Welsh**	**Gaelic**	**Gothic**	**Turkish**	**Hebrew**
0	Nill	Nolli				Zuffer	Ayin
1	Odna	Yksi	Una	Unah	Ains	Bier	Aleph
2	Dva	Kaksi	Dou	Gha	Tvai	Icki	Beth
3	Tri	Koleme	Tri	Trec	Threis	Utch	Gimel
4	Tcheteri	Nelja	Pedvar	Cheir	Fidvor	Duert	Daleth
5	Piat	Viisi	Pump	Coag	Finf	Bach	He
6	Shest	Kunsi	Chewech	Seach	Saihs	Altoe	Vau
7	Sem	Seisemän	Saith	Sheach	Sibum	Yedi	Zain
8	Vosem	Kahdeksän	Wyth	Oacht	Ahtan	Seckiz	Bheth
9	Deviat	Yhdeksän	Nan	Nuegh	Niun	Dokus	Teth
10	Desiat	Kymmenän	Deg	Doach	Taihun	On	Yod
11	Odenstset	Yksitoista	Undeg	Undech	Ainstaihun	Onbier	Yodaleph
12	Dvenatset	Kaksitoista	Doudeg	Dhadech	Tvaitaihun	Onicki	Yodbeth
No.	**Arabian**	**Persian**	**Hindoo**	**Chinese**	**Japanese**	**Sanscrit**	**Duodenal**
0	Siforon			Bow	Ley		**Zero, 0**
1	Ahed	Yika	Ek Ache	Yat	Itchi	Aika	**An, 1**
2	Ishnan	Du	Do	Ge	Ni	Dwan	**Do, 2**
3	Sayisset	Seh	Ien	Sam	San	Tri	**Tre, 3**
4	Erbayet	Chehaur	Char	Tze	Tchi	Chatur	**For, 4**
5	Jemset	Pendj	Panoh	Ngnu	Go	Pancha	**Pat, 5**
6	Sittet	Shesh	Chha	Luck	Lock	Shash	**Sex, 6**
7	Saybet	Helft	Sath	Tchut	Sytchi	Saptan	**Ben, 7**
8	Saymaniet	Hesht	Ath	Pbat	Hatchi	Ashta	**Ott, 8**
9	Tiset	Nuh	Nau	Geo	Ku	Navan	**Nev, 9**
10	Eshbret	Deh	Das	Shop	Dgiu	Dashan	**Dis, φ**
11	Ahedeshere	Yikadeh	Gyarah	Shopyat	Dgiuitchi	Aikadashan	**Elv, γ**
12	Ishnaneshere	Dudeh	Barah	Shopgas	Dgiuni	Dwandashan	**Ton, 10**

Comments on Nomenclature.

On account of the different pronunciations of letters and words in different languages, the true sound of a name cannot be conceived without a knowledge of the language in which it is written.

The Japanese sound for 9 is written *ku* in the table, but for the English pronunciation it should be written *koo*.

There are some letters of the alphabet which have nearly the same sound in all languages, and only such letters should be used in the coining of names for the figures and numbers in the duodenary system.

The letters *th*, *w*, *o*, *wr*, *ght* in the English language, and also the letter *C*, which has two sounds in almost all languages, should not be used for the new names.

The names given to the duodenary figures in the last column are clear and distinct sounds, which would be well understood and pronounced alike in all languages.

It would be useless to attempt to introduce the names of the figures and numbers in either of the languages above given as a universal nomenclature, for not only that they are not suited for more than the language in which they are written, but prejudices would be against them. The introduction of the French metrical system has been greatly retarded by reason merely of its cumbersome nomenclature.

The best work on the etymology of numbers known to the author is that of Professor S. Zehetmayr, published in Leipsic, 1854. In the establishment of a new and universal nomenclature of numbers we ought to select clear and distinct sounds, which can be understood and pronounced alike in all languages, without regard to the etymology of numbers.

The Arabic notation of numbers is yet used only by about one-third of the population of the earth, and the other two-thirds use different kinds of irregular characters or hieroglyphics, which combinations are unfit for arithmetical calculations.

The Roman notation was used in England up to the beginning of the seventeenth century, when the Arabic notation was gradually gaining ground against very strong opposition; and at last caused the burning of the houses of Parliament. The Arabic notation was introduced into Germany in the twelfth century, and into Italy in the eleventh century.

Comparison of Numbers in the Duodenary and Decimal Systems, with the Corresponding New Names

New	Names	*Old*	New	Names	*Old*	New	Names	*Old*
0	Zero	*0*	1	An	*1*	2	Do	*2*
3	Tre	*3*	4	For	*4*	5	Pat	*5*
6	Sex	*6*	7	Ben	*7*	8	Ott	*8*
9	Nev	*9*	φ	Dis	*10*	Ɣ	Elv	*11*
10	Ton	*12*	11	Tonan	*13*	12	Tondo	*14*
13	Tontre	*15*	14	Tonfor	*16*	15	Tonpat	*17*
16	Tonsex	*18*	17	Toben	*19*	18	Tonott	*20*
19	Tonev	*21*	1φ	Tondis	*22*	1Ɣ	Tonenv	*23*
20	Doton	*24*	21	Dotonan	*25*	22	Dotondo	*26*
23	Dotontre	*27*	24	Dotonfor	*28*	25	Dotonpat	*29*
26	Dotonsex	*30*	27	Dotoben	*31*	28	Dotonott	*32*
29	Dotonev	*33*	2φ	Dotondis	*34*	2Ɣ	Dotonelv	*35*

30	Treton	*36*	31	Tretonan	*37*	32	Tretondo	*38*
33	Tretontre	*39*	34	Tretonfor	*40*	35	Tretonpat	*41*
36	Tretonsex	*42*	37	Tretoben	*43*	38	Tretonott	*44*
39	Tretonev	*45*	3φ	Tretondis	*46*	3ϒ	Tretonelv	*47*
40	Forton	*48*	41	Fortonan	*49*	42	Fortondo	*50*
43	Fortontre	*51*	44	Fortonfor	*52*	45	Fortonpat	*53*
46	Fortonsex	*54*	47	Fortoben	*55*	48	Fortonott	*56*
49	Fortonev	*57*	4φ	Fortondis	*58*	4ϒ	Fortonelv	*59*
50	Paton	*60*	51	Patonan	*61*	52	Patondo	*62*
53	Patontre	*63*	52	Patonfor	*64*	55	Patonpat	*65*
56	Patonsex	*66*	57	Patoben	*67*	58	Patonott	*68*
59	Patonev	*69*	5φ	Patondis	*70*	5ϒ	Patonelv	*71*
60	Sexton	*72*	61	Sextonan	*73*	62	Sextondo	*74*
63	Sextontre	*75*	62	Sextonfor	*76*	66	Sextonpat	*77*
66	Sextonsex	*78*	67	Sextoben	*79*	68	Sextonott	*80*
69	Sextonev	*81*	6φ	Sextondis	*82*	6ϒ	Sextonelv	*83*
70	Benton	*84*	71	Bentonan	*85*	72	Bentondo	*86*
73	Bentontre	*87*	72	Bentonfor	*88*	77	Bentonpat	*89*
77	Bentonsex	*90*	77	Bentoben	*91*	78	Bentonott	*92*
79	Bentonev	*93*	7φ	Bentondis	*94*	7ϒ	Bentonelv	*95*
80	Otton	*96*	81	Ottonan	*97*	82	Ottondo	*98*
83	Ottontre	*99*	82	Ottonfor	*100*	88	Ottonpat	*101*
88	Ottonsex	*102*	88	Ottoben	*103*	88	Ottonott	*104*
89	Ottonev	*105*	8φ	Ottondis	*106*	8ϒ	Ottonelv	*107*
90	Nevton	*108*	91	Nevtonan	*109*	92	Nevtondo	*110*
93	Nevtontre	*111*	92	Nevtonfor	*112*	99	Nevtonpat	*113*
99	Nevtonsex	*114*	99	Nevtoben	*115*	99	Nevtonott	*116*
99	Nevtonev	*117*	9φ	Nevtondis	*118*	9ϒ	Nevtonelv	*119*
φ0	Diston	*120*	φ1	Distonan	*121*	φ2	Distondo	*122*
φ3	Distontre	*123*	φ2	Distonfor	*124*	φφ	Distonpat	*125*
φφ	Distonsex	*126*	φφ	Distoben	*127*	φφ	Distonott	*128*
φφ	Distonev	*129*	φφ	Distondis	*130*	φϒ	Distonelv	*131*
ϒ0	Diston	*132*	ϒ1	Distonan	*133*	ϒ2	Distondo	*134*
ϒ3	Distontre	*135*	ϒ2	Distonfor	*136*	ϒϒ	Distonpat	*137*
ϒϒ	Distonsex	*138*	ϒϒ	Distoben	*139*	ϒϒ	Distonott	*140*
ϒϒ	Distonev	*141*	ϒϒ	Distondis	*142*	ϒϒ	Distonelv	*143*
100	San	*144*	148	San-fortonott	*200*	200	Dosan	*288*
210	Dosan-ton	*300*	300	Tresan	*432*	358	Tresan-patonott	*500*
400	Forsan	*576*	420	Forsan-dotan	*600*	500	Patsan	*720*
568	Patsan-sextonott	*800*	600	Sexan	*864*	630	Sexan-treton	*900*
700	Bensan	*1008*	800	Ottsan	*1152*	900	Nevsan	*1296*
φ00	Dissan	*1440*	ϒ00	Elvsan	*1584*	1000	Tos	*14728*
1100	Tossan	*1872*	1200	Tosdosan	*2016*	1300	Tostresan	*2160*
1400	Tosforsan	*2304*	1500	Tospatsan	*2448*	1600	Tossexan	*2592*
1700	Tosbensan	*2736*	1800	Tosottsan	*2880*	1900	Tosnevsan	*3024*
1φ00	Tosdissan	*3168*	1ϒ00	Toselvsan	*3312*	2000	Dotos	*3456*
4000	Fortos	*6912*	6000	Sextos	*10368*	8000	Ottos	*13724*
φ000	Distos	*17180*	10000	Dill	*20736*			

Fractions							
Duodenary System				Decimal System			
$\frac{1}{2}$	0.6	$\frac{1}{4}$	0.3	$\frac{1}{2}$	*0.5*	$\frac{1}{4}$	*0.25*
$\frac{3}{4}$	0.9	$\frac{1}{8}$	0.16	$\frac{3}{4}$	*0.75*	$\frac{1}{8}$	*0.125*
$\frac{3}{8}$	0.46	$\frac{5}{8}$	0.76	$\frac{3}{8}$	*0.375*	$\frac{5}{8}$	*0.625*
$\frac{7}{8}$	0.φ6	$\frac{1}{3}$	0.4	$\frac{7}{8}$	*0.875*	$\frac{1}{3}$	*0.3333...*
$\frac{2}{3}$	0.8	$\frac{1}{6}$	0.2	$\frac{2}{3}$	*0.6666...*	$\frac{1}{6}$	*0.16666...*
$\frac{5}{6}$	0.φ	$\frac{1}{14}$	0.09	$\frac{5}{6}$	*0.83333...*	$\frac{1}{16}$	*0.0625*
$\frac{3}{14}$	0.23	$\frac{7}{14}$	0.53	$\frac{3}{16}$	*0.1875*	$\frac{7}{16}$	*0.4375*
$\frac{\gamma}{14}$	0.83	$\frac{1}{20}$	0.06	$\frac{11}{16}$	*0.6875*	$\frac{1}{24}$	*0.041666...*
$\frac{7}{20}$	0.36	$\frac{\gamma}{20}$	0.56	$\frac{7}{24}$	*0.2916666...*	$\frac{11}{24}$	*0.458333...*
$\frac{1}{28}$	0.046	$\frac{7}{28}$	0.976	$\frac{1}{32}$	*0.03125*	$\frac{7}{32}$	*0.21875*
$\frac{1}{9}$	0.14	$\frac{2}{9}$	0.24	$\frac{1}{9}$	*0.11111111...*	$\frac{2}{9}$	*0.22222...*
$\frac{4}{9}$	0.54	$\frac{5}{9}$	0.68	$\frac{4}{9}$	*0.44444...*	$\frac{5}{9}$	*0.55555...*
$\frac{8}{9}$	0.φ8	$\frac{\gamma}{32}$	0.416	$\frac{8}{9}$	*0.88888...*	$\frac{11}{32}$	*0.34375...*
$\frac{15}{28}$	0.646	$\frac{1}{54}$	0.023	$\frac{17}{32}$	*0.53125*	$\frac{1}{64}$	*0.015625*
$\frac{\gamma}{54}$	0.209	$\frac{33}{54}$	0.739	$\frac{11}{64}$	*0.171875*	$\frac{39}{64}$	*0.609375*

Table 2: Fractions

The above table* of fractions shows the simplicity of the duodenary system, which requires few figures where the old system requires a great number of decimals. For 3ds, 6ths, 9ths, 12ths and 24ths the duodenary system finishes the fraction with one or two places where the number of decimals is endless.

Addition Table											
1	2	3	4	5	6	7	8	9	φ	γ	10
2	4	5	6	7	8	9	φ	γ	10	11	12
3	5	6	7	8	9	φ	γ	10	11	12	13
4	6	7	8	9	φ	γ	10	11	12	13	14
5	7	8	9	φ	γ	10	11	12	13	14	15
6	8	9	φ	γ	10	11	12	13	14	15	16
7	9	φ	γ	10	11	12	13	14	15	16	17
8	φ	γ	10	11	12	13	14	15	16	17	18
9	γ	10	11	12	13	14	15	16	17	18	19
φ	10	11	12	13	14	15	16	17	18	19	1φ
γ	11	12	13	14	15	16	17	18	19	1φ	1γ
10	12	13	14	15	16	17	18	19	1φ	1γ	20

*This table has been moved to Table 2 on page ↊.

Multiplication Table

1	2	3	4	5	6	7	8	9	φ	γ	10
2	4	6	8	φ	10	12	14	16	18	1φ	20
3	6	9	10	13	16	19	20	23	26	29	30
4	8	10	14	18	20	24	28	30	34	38	40
5	φ	13	18	21	26	2γ	34	39	42	47	50
6	10	16	20	26	30	36	40	46	50	56	60
7	12	19	24	2γ	36	41	48	53	5φ	65	70
8	14	20	28	34	40	48	54	60	68	74	80
9	16	23	30	39	46	53	60	69	76	83	90
φ	18	26	34	42	50	5φ	68	76	84	92	φ0
γ	1φ	29	38	47	56	65	74	83	92	φ1	γ0
10	20	30	40	50	60	70	80	90	φ0	γ0	100

The duodenal multiplication table of the single figures is 44 per cent. more extensive than that of the decimal, but the binary and trinary properties makes it much easier to learn and to remember.

Examples in Addition.

38φ45γ3	8φ093	0.3
6108γφ5	9γ6φ	γ3.06
99γ1598	φ3γ	4φ9.φ1
	3φ	674γ.60
	98φγφ	39φ06.00
		44γ36.4φ

Examples in Subtraction.

74γ8φ6	3φ43.γ1	0.γ48φ6
314364	1γ9φ.0φ	0.003γφ
437542	1φ65.φ3	0.γ44φ8

Examples in Multiplication.

8694	φ4γ63	36γ.3φ45
24	63γ	0.0γ6
2φ314	956689	1957γ226
5168	272φ69	32442507
7γ994	525916	34.19φ4296
	55φ36759	

Examples in Division

$$\begin{array}{l}
42)136\Upsilon 0\varphi(38\Upsilon 4 \\
\quad \underline{106} \\
\quad\ \ 30\Upsilon \\
\quad\ \ \underline{294} \\
\quad\quad 370 \\
\quad\quad \underline{358} \\
\quad\quad\ \ 14\varphi \\
\quad\quad\ \ \underline{148} \\
\quad\quad\ \ 002
\end{array}
\qquad
\begin{array}{l}
4\varphi.\gamma)3\varphi 057.63\varphi(946.38 \\
\quad\quad \underline{3823} \\
\quad\quad\ \ 1\varphi 27 \\
\quad\quad\ \ \underline{1778} \\
\quad\quad\quad 26\Upsilon 6 \\
\quad\quad\quad\ \ \underline{2556} \\
\quad\quad\quad\quad 1603 \\
\quad\quad\quad\quad \underline{1289} \\
\quad\quad\quad\quad\ \ 3364 \\
\quad\quad\quad\quad\ \ \underline{333\varphi} \\
\quad\quad\quad\quad\quad\ \ 36
\end{array}$$

On account of the binary and trinary properties of the duodenary system, these arithmetical operations are much easier to the mind than those with decimal arithmetic. The only difficulty about it is to unlearn the decimal system.

The duodenary system has all the advantages and none of the disadvantages of the decimal system; it is also better adapted to mental calculations, which are very difficult with our present arithmetic.

METROLOGY.

The utility of a duodenary system of arithmetic consists in its combination with a similar system of metrology—namely, that all units of measure should be divided and multiplied by the same base, twelve.

Units of measure are required for the following fifteen quantities.

Length.	Weight.	Heat.	Force.	Power.
Surface.	Mass.	Light.	Velocity.	Space.
Volume.	Money.	Electricity.	Time.	Work.

MEASUREMENT OF LENGTH.

Assume the mean circumference of the earth to be the primary unit of length, and divide it by twelve repeatedly until the divisions are reduced to a length which would be a convenient unit to handle in the shop and in the market.

The mean circumference of the earth is about *24851.64* miles, which, multiplied by *5280*, will be

			Duodenal.
0	*131216659.2*	feet	1 circum.
1	*10934721.6*	feet	1 hour.
2	*911226.8*	feet	1 grad.
3	*75935.56*	feet	1 minute.
4	*6327.96*	feet	1 mile.
5	*527.33*	feet	1 cable.
6	*43.944*	feet	1 chain.
7	*3.772*	feet	1 metre.
The required unit of length	*43.944*	inches	1 metre.

The length of the circumference of the earth, divided by the seventh power of *12*, gives a length of *43.944* inches, which is assumed as a unit for all measurements of length, and which we will call a **metre**.

Twelve duodenal metres is a length of *43.944* feet, which is a convenient measure in the field or in surveys, and which we will call a **chain**.

Twelve duodenal chains is a length of *527.33* feet, which we will call one **cable**.

Twelve duodenal cables is a length of *6327.96* feet, which we will call one **mile**. The duodenal mile will be about *300* feet longer than our present knot or sea-mile.

Twelve duodenal miles	=	1 **minute,**	on the earth's great circle.
Twelve duodenal minutes	=	1 **grad,**	
Twelve duodenal grads	=	1 **hour,**	
Twelve duodenal hours	=	1 **circum,**	

The duodenal metre to be divided into twelve equal parts of *3.772* inches each, and called **metons**. The meton into twelve equal parts of *0.31433* of an inch each, called **mesans**. The mesan into twelve equal parts of *0.0262* of an inch each, called **metos**.

Fig. 235[*] shows the full size of a meton with its divisions.

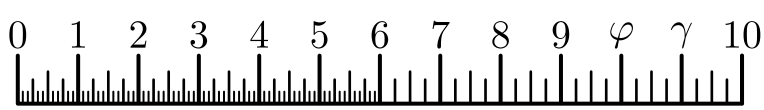


Figure 3: Orig. Fig. 235

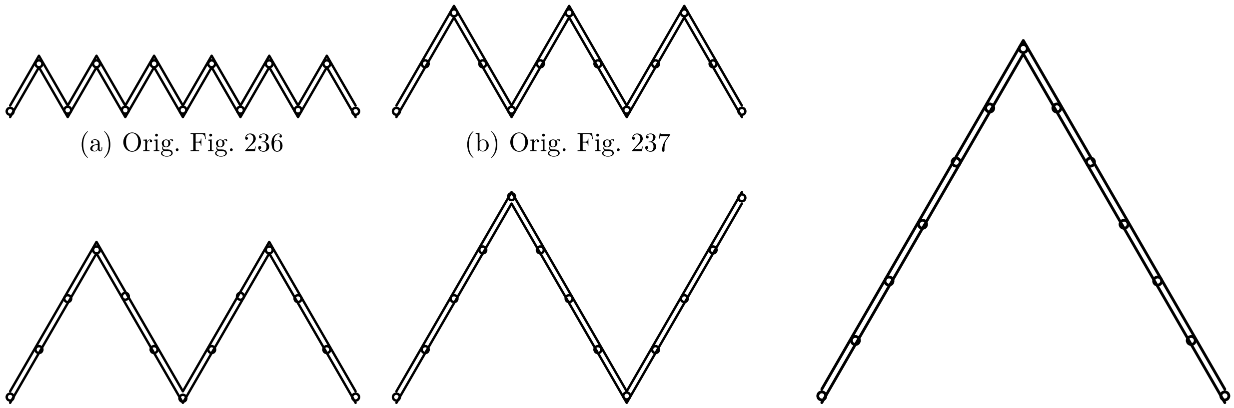
The first 6 **mesans** are divided into **metos**, and the last into quarters of **mesans**. The ordinary shop-metre need not be divided finer than into quarters of **mesans**, for in so small divisions the **metos** can easily be approximated.

The **metons** and **mesans** would be the most convenient for expressing short measures in the mechanic arts.

Fig. 236[†] represents a twelve-folded duodenal metre with lap-joints, like the ten-folded French metre; each part is one meton of *3.772* inches.

[*] This figure is now Figure 3 on page 11.

[†] These figures, and the others with it, have been moved to Figure 4 on page 12.



(a) Orig. Fig. 236 (b) Orig. Fig. 237

(c) Orig. Fig. 238 (d) Orig. Fig. 239 (e) Orig. Fig. 240

Figure 4: Various folding rules (orig. Figs. 236–240).

Fig. 237 represents a six-folded duodenal metre with lap-joints, of *7.544* inches in each. This form could be made with regular hinges like the English rule.

Fig. 238 represents a four-folded duodenal metre, with 3 metons in each part of *11.316* inches. This would be the most convenient form for the shop when folded with regular hinges like the English four-folded rule rule.

Fig. 239 represents a three-folded metre, with four metons in each part of *15.088* inches.

Fig. 240 represents a two-folded metre, with six metons in each part of about *22* inches.

We see here that the duodenal metre can be folded into five different forms, with even measures in each part.

The longest unit of measure is the circumference of the earth, which ought to be termed a **circum**. The circum should be used in expressing astronomical distances.

The duodenal grad is 100 duodenal miles, or 0.01 of the earth's great circle, which would be a proper measure for expressing long distances on the earth's surface; and which would convey a correct idea. of the real magnitude of such distances compared with the great circle.

The mile would be the common road measure and for traveling distances on land and sea.

Circum.	Grad.	Mile.	Cable.	Chain.	Metre.	Meton.	Mesan.	Metos.
1	100	10000	100000	1000000				
0.01	1	100	1000	10000	100000	1000000		
0.0001	0.01	1	10	1000	10000	100000	1000000	
	0.0001	0.1	1	10	1000	10000	100000	1000000
	0.000001	0.01	0.1	1	10	1000	10000	100000
		0.001	0.01	0.1	1	10	1000	10000
		0.0001	0.001	0.01	0.1	1	10	1000
		0.00001	0.0001	0.001	0.01	0.1	1	10
			0.00001	0.0001	0.001	0.01	0.1	1

Table 3: Duodenal Measures of Length

Division of the Circle

The circle to be divided into 100 equal parts (144 decimal).*

Duodenal System			*Old System*
One circle	=	100 grads.	*360* degrees.
One grad	=	100 lents.	*2* degrees *30* minutes.
One lent	=	100 ponts.	*1* minute *2.5* seconds.
One pont	=		*0.43418* of a second
One quadrant	=	30 grads.	*90* degrees.

Table 4: Duodenal division of the circle.

One duodenal mile on the earth's surface corresponds with an angle to one lent.

One duodenal chain on the earth's surface corresponds with an angle of one pont.

The latitude and longitude to be divided as the circle.

The angular measures correspond with the linear measures on the earth's surface. The terms minute and second are omitted in the division of the circle, so as not to confound angles with time.

The circle can thus be divided into *2*, *8*, *4*, *6*, *8*, *9*, *12* or *16* parts, without leaving fractions of a degree or grad.

The quadrant of the circle, containing 30 grads (*36*), can be divided into *2*, *3*, *4*, *6*, *9* or *12* parts without leaving fractions of a grad. These advantages with the duodenal division of the circle are of great importance in geometry, geography, trigonometry, astronomy and in navigation.

Either of the divisions corresponds with an even linear measure on the earth's surface.

Duodenal Division of Time.

The division of time should conform to that of the circle.

The time from noon to noon, including one night and day, to be divided into twelve equal parts, called hours.†

Either of these three divisions can be used in practice. The first division includes the second and third.

If the duodenal division of time was introduced all over the world, some nations would probably use the second expression, and others the third, but the third division is the best, because the hands on the watch would show the number of grads.

In the notation of time, say 3 hours and 46 minutes, will appear 3.46 hours, or 34.6 grads, or 346 minutes.

5 hours, 36 minutes and 15 seconds will appear 5.36.15 hours, or 53.6l.5 grads.

The conversion of angle into time, or time into angle, is only to move the point one place.

There is no necessity of *A. M.* and *P. M.* in the duodenal time.

*See Table 4 on page 13.

†This table has been placed in Table 5 on page 14.

htbp

	Duodenal System			*Old System*	
1.	One day	=	10 hours.	*24*	hours
	One hour	=	10 grads.	*2*	hours
	One grad	=	10 minutes.	*10*	minutes.
	One minutes	=	10 lents.	*0.83333*	of a minute
	One lent	=	10 seconds.	*4.1666*	seconds
	One second	=	10 ponts.	*0.3472*	of a second
2.	One day	=	10 hours.		
	One hour	=	100 minutes.		
	One minute	=	100 seconds.		
3.	One day	=	100 grads.		
	One grad	=	100 lents.		
	One lent	=	100 ponts.		

Table 5: Duodenal system of time.

Astronomers would surely use the third expression of time, which corresponds with the divisions of the circle.

DUODENAL CLOCK-DIAL.

Fig. 241* represents a duodenal clock-dial.

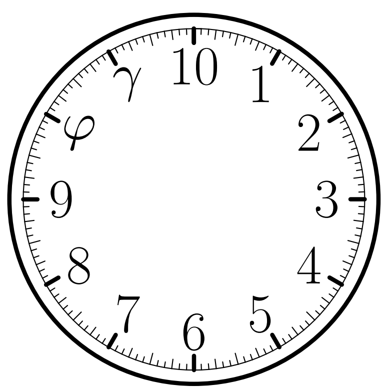


Figure 5: Orig. Fig. 241.

The hour-hand makes one turn in one night and day.

The minute-hand goes round once per hour, and the second-hand once per minute.

The hour-hand will point to 10 at noon, to 3 at *6* o'clock in the evening, to *6* at midnight, and to 9 at *6* o'clock in the morning.

The length of the pendulum vibrating duodenal seconds will be

*This figure has been placed in Figure 5 on page 14.

$$l = \mathit{39.1} \times \mathit{0.3472}^{\mathit{2}} = \mathit{4.711} \text{ inches, or}$$

$$= \mathit{1.3} \text{ metons}$$

The duodenal metre will vibrate

$$n = \frac{\mathit{6.254} \times \mathit{60}}{\sqrt{\mathit{43.944}}} = \mathit{56.6} \text{ times per old minute.}$$

$$= \mathit{41.55} \text{ times per duodenal minute.}$$

Duodenal Year.

The year is already divided into twelve months, but the division is unnecessarily irregular.

The days in the year ought to be divided so as to make the months of nearly equals lengths.

The two months following one another—namely, December and January—have both *31* days, and then comes February with only *28* days.

There is no good reason why the months should not be divided so as to have *30* days in seven months and *31* days in five months of the year, as shown by the accompanying table.*

No.	Days	Months	*Days*	*Old*
1	26	January,	*30*	*31*
2	26	February,[1]	*30*	*28*
3	26	March,	*30*	*31*
4	27	April,	*31*	*30*
5	26	May,	*30*	*31*
6	27	June,	*31*	*30*
7	27	July,	*31*	*31*
8	26	August,	*30*	*31*
9	27	September,	*31*	*30*
φ	26	October,	*30*	*31*
γ	27	November,	*31*	*30*
10	26	December,	*30*	*31*
	265	Year.	*365*	*365*

[1] In leap years February should have *31* days, or 27 duodenal.

Table 6: Duodenal Year

Different calendars are also used in different parts of the world, which ought to be only one common calendar.

Duodenal Compass.

*This table has been placed at 6 at 15.

The compass to be divided into grads like the circle, but numbered from North and South toward East and West, making 30 grads in each quadrant. Fig. 242[*] represents a duodenal compass.

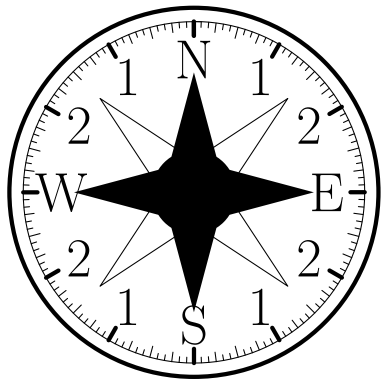


Figure 6: Orig. Fig. 242

The hours 1 and 2, corresponding each with 10 grads, are marked on the dial in each quadrant.

The nomenclature will be nearly the same as for the old compass, only the expression of fractional points would be changed to grads; for example, **South South-East**, **one-half South**, would be called simply **South ott East**.

Our present compass is divided into *32* points, and each point into four quarters, making *32* divisions in each quadrant, which shows the natural tendency toward binary divisions; but it is accompanied with a clumsy nomenclature. A course of $3\frac{1}{4}$ points from North toward East is termed **North-East by North, one-quarter East**. The duodenal expression would be simply **North an tre Est**, meaning one hour and three grads from North toward East, without expression of fractions; and the course is given with greater precision than by the present nomenclature.

Duodenal Measurement of Surface.

Small surfaces can be expressed in square metres, square metons or square mesans.

Duodenal System.			*Old System.*	
One square chain	=	1 lot.		
10 chains square	=	1 acre.		
One square cable	=	1 acre.	*6.3925*	acres.
One acre	=	100 lots.	*278075*	square feet.
One lot	=	100 square metres.	*193.1*	square feet.
One square mile	=	100 acres.	*920.52*	acres.
One square grad	=	10,000 square miles.		
One square grad	=	1,000,000 acres.		

[*]This figure has been moved to Figure 6 on page 16.

Duodenal Measure of Capacity.

The cubic metre to be the unit for capacity.

Duodenal System.			*Old System.*	
One cubic metre	=	1 tun.	*49.113*	cubic feet.
One tun	=	10 barrels.	*49.113*	cubic feet.
One barrel	=	10 pecks.	*4.0927*	cubic feet.
One peck	=	10 gallons.	*643.92*	cubic inches.
One gallon	=	10 glasses.	*53.66*	cubic inches.
One glass	=	10 spoons.	*4.47*	cubic inches.

The duodenal gallon is one cubic meton, or about one quart. An ordinary quart bottle would contain one duodenal gallon.

Dry and wet measures of capacity ,should be measured by the same units. A cord of wood 10 cubic metres.

The volume of solids should be measured by the cube of the linear units.

Duodenal Division of Money.

The unit of money ought to be the value of one duodenal dram of fine gold, which is about one dollar.

Duodenal System.			American Money.	
One dollar	=	10 shillings.	*1*	dollar.
One shilling	=	10 cents.	*8.3333*	cents.
One cent	=		*0.7*	of a cent.

The American dollar is divided into ten dimes, but that expression is rarely used in the market. The same is the case with the French franc and dixiéme. The reason of that is that the decimal base does not admit of binary divisions. In a duodenal system the name of a twelfth part of a dollar would be used.

Dolls.		*Cts.*	*Dolls.*		*Cts.*	*Dolls.*		*Cts*
$\frac{1}{2}$	=	60	$\frac{7}{8}$	=	φ6	$\frac{3}{14}$	=	23
$\frac{1}{4}$	=	30	$\frac{1}{3}$	=	40	$\frac{7}{14}$	=	53
$\frac{3}{4}$	=	90	$\frac{2}{3}$	=	80	$\frac{\gamma}{14}$	=	83
$\frac{1}{8}$	=	16	$\frac{1}{6}$	=	20	$\frac{1}{20}$	=	6
$\frac{3}{8}$	=	46	$\frac{5}{6}$	=	φ0	$\frac{7}{20}$	=	36
$\frac{5}{8}$	=	76	$\frac{1}{14}$	=	9	$\frac{\gamma}{20}$	=	56

The 14ths in the duodenal system are the same as *16*ths in decimals. The 20ths duodenal are *24*ths decimal.

The duodenal system admits of binary division of the dollar as far as required in commerce and in the market.

DUODENAL MEASURE OF WEIGHT.

The weight of one cubic metre of distilled water is assumed to be the unit of weight, and called one *ton*.

The duodenal ton will weigh about *3063.8* pounds, or *1.368* old tons.

Duodenal System.			*Old System Avoirdupois.*	
One ton	=	10 pud.	*3063.8*	pounds avoirdupois.
One pud	=	10 vegts.	*255.3166*	pounds avoirdupois.
One vegt	=	10 ponds.	*21.276*	pounds avoirdupois.
One pond	=	10 ounces.	*1.773*	pounds avoirdupois.
One ounce	=	10 drachms.	*2.3640*	ounces avoirdupois.
One drachm	=	10 scruples.	*0.1969*	ounces avoirdupois.
One scruple	=	10 grains.	*0.0164*	ounces avoirdupois.
One grain	=		*0.598*	grains Troy.

Ton.	Pud.	Vegt.	Pond.	Ounce.	Dram.	Scruple.
1	10	100	1,000	10,000	100,000	1,000,000
0.1	1	10	100	1,000	10,000	100,000
0.01	0.1	1	10	100	1,000	10,000
0.001	0.01	0.1	1	10	100	1,000
0.0001	0.001	0.01	0.1	1	10	100
0.00001	0.0001	0.001	0.01	0.1	1	10
0.000001	0.00001	0.0001	0.001	0.01	0.1	1

UNITS OF FORCE.

Force can be measured by either one of the units of weight.

The pond would be the most convenient unit in estimating power and work in machinery.

UNIT OF VELOCITY.

Metons per second would be the most appropriate expression of velocity in machinery.

A velocity of metons per second is the same as miles per hour.

UNIT OF TIME.

The second is the best unit of time to be used in the operation of machinery and falling bodies.

Unit of Power.

A force of one pond moving with a velocity of one meton per second to be one unit of power, and called **Effect**.

A power of one pond moving with a velocity of one meton per second would be = *1.605* foot-pounds per old second. This will make 30 duodenal effects per man-power, and 300 effects per horse-power.

Unit of Space.

The unit of linear space in the operation of machinery should be the meton or metre.

Unit of Work.

The work of lifting one ton through a height of one metre is a proper unit for estimating heavy work; it is equal to *11375* foot-pounds. This unit should be termed metreton and be used in the estimate of work of heavy ordnance.

The work which a laborer can accomplish per day would be about 100 metretons, which unit ought to be called a **Workmanday**.

The unit of work corresponding to velocity and effects should be one pond lifted one meton, which is *0.5567* of a foot-pound.

Unit of Mass.

The duodenal unit of mass would bethe amount of matter in one cubic meton of distilled water, to be called one **Matt**, which is *53.668* cubic inches of water.

Unit of Gravity.

The velocity which a falling body would attain at the end of the first duodenal second is $g = 2.\Upsilon 33$ metres per second, which would be the acceleratrix of gravity.

Unit of Temperature.

The thermometer scale should be divided into 100 duodenal parts (*144*) between the freezing and boiling points of distilled water at the level of the sea in latitude 16 grads (45°).

One duodenal grad = 125° Fahrenheit scale.

One duodenal grad = 0.69° Centigrade.

Unit of Heat.

The heat required to raise the temperature of one pond of distilled water from φ° to Υ° to be one unit of heat, which answers to *1713* foot-pounds of work.

Each kind of measure has different grades of units varying with the duodenal base, and any one of the units divided by 2, 3, 4 or 6 gives aliquot numbers in the quotient, which property renders the duodenal system very easy and clear to the mind for mental calculations and estimations of quantities.

In the establishment of a duodenal system of arithmetic and metrology it would perhaps be best to introduce the metrology first, and work it with decimal arithmetics until fairly established, after which the duodenal arithmetic would become more easy to learn and to apply.

The transition would not last long, for when one becomes imbued with the advantages and simplicity of the duodenal principles he would not bother his brain any more with the unnatural decimal base, but encourage others to take up the new system.

This document was made available by the Dozenal Society of America in August 11Ɛ9, being newly typeset from the original text by Donald P. Goodman. In the course of typsetting, the following changes were made. 1. On the fraction table, the original edition listed $\frac{17}{28}$, when really $\frac{15}{28}$ was meant; this has been corrected. 2. A caption was added to the figure showing the duodenal division of the circle. 3. A caption was added to the figure showing the duodenal divisions of time. 4. In the second "Duodenal Year," the decimal number "31" was italicized, to mirror the formats of decimal numbers elsewhere in the text. 5. Some superfluous footnote markers were eliminated from the table delineating the Duodenal Year. 6. In the weight table, ditto marks were replaced with proper words. Other changes made should be evident from the text itself; e.g., the renumbering of figures.

The figures have been completed redesigned and reproduced for this text.

To learn more about the dozenal system (which Mr. Nystrom calls "duodenal" in this text), including other ideas for symbols, number names, metric systems, and more, please contact the Dozenal Society of America (http://www.dozenal.org).